A [illegible] potentia [illegible] pœniten-
tia non ſit [illegible] iuſtitiæ. [illegible] non eſt virtus theologi-
ca, ſed moralis, vt in Secunda [illegible] patet. [illegible] Pœnitentia autem
videtur [illegible] theologica [illegible] Deum [illegible] ſa-
tis [illegible] etiam [illegible] Ergo vide- [illegible]
[illegible] ſpecies iuſtitiæ.

[illegible] Præterea [illegible] iuſtitia cum ſit virtus moralis [illegible]
pœnitentia non conſiſtit [illegible] ſec [illegible]
[illegible] Ergo pœni-
tentia non [illegible]

[illegible] (ſicut dicitur [illegible] Ethic.) ſe-
[illegible] pœnitentia
[illegible] non [illegible] iuſtitiæ.
[illegible] s, dicen
[illegible] terrer [illegible] ſera,
[illegible] Ergo pœnitenti
[illegible] dicit in lib. de pœnitentia,
[illegible] ſemper puniens [illegible]
[illegible] pertinet ad iuſtitiam: vnde
[illegible] ſpeciem iuſtitiæ. Ergo [illegible]

[illegible] tia eſt
[illegible] commiſiſſe
[illegible] ſit Rhetor. pon-
quod pœnitentia ſit [illegible]

C O N C L [illegible] O.

Virtus pœnitentiæ [illegible] eſt iuſtitiæ commutatiua, non quia [illegible]
[illegible] es eſt, ſed [illegible] quia ſecundum [illegible]
[illegible] appellatur.

[illegible] m, quò (ſicut ſuprà dictum eſt) [illegible]
[illegible] ecialis ex hoc ſolo quod [illegible]
[illegible] ex eo [illegible]

[illegible] ſubiecto.

[illegible] Præterea [illegible] Sed vindicta [illegible]
[illegible] quia [illegible] vindictæ. Ergo [illegible]
iraſcibili.

§ 3 Præterea [illegible] ſecun-
dum Philoſophum [illegible] de Memoria [illegible]
præterito [illegible] Ergo pœnitentia eſt [illegible]
ſubiecto.

§ 4 Præterea [illegible] Sed pœni-
[illegible] pœniten-
anima [illegible]
[illegible] pœnitentia [illegible]
illud Pſal. [illegible] crificium Deo ſpiritus [illegible]
[illegible] actus voluntatis ſecundum [illegible]
[illegible] Ergo pœnitentia [illegible] untate.

C O N C L V S I [illegible]

[illegible] tia [illegible] quædam ſpecies iuſtitiæ, in [illegible]
[illegible] quod [illegible]

PONDEO dicendum [illegible]
[illegible] Vno modo [illegible]
[illegible] tiæ, eſt [illegible] concupiſci [illegible]
[illegible] eſt virtus. Et ſic ſicut [illegible]
[illegible] in Secunda parte habitum [illegible]
[illegible] eſt voluntas. [illegible] eſt qu
[illegible] od [illegible] eſt in [illegible]
[illegible] s actus eſt propoſitum.

AD [illegible] M [illegible] quod ratio i
[illegible] tentia ſecundum [illegible]

AD [illegible] quod [illegible]
[illegible] alio, pert [illegible]

RUDD'S
EUCLIDE

N
III
l

1651

Ewaldus 21924/B/3

7 c/s

81646

EUCLIDES ELEMENTS *OF* GEOMETRY:

The firſt VI Books:

In a compendious form contracted
and Demonſtrated.

By Captain THOMAS RUDD,
Chiefe Engineer to his late Majeſty.

Whereunto is added,
The *Mathematicall Preface* of
Mr. JOHN DEE.

⁂

LONDON:
Printed by ROBERT and WILLIAM LEYBOURN
for RICHARD TOMLINS and ROBERT BOYDELL, at
the *Sun* & *Bible* neer *Pie-corner*, and at the *Bulwark*
neer the *Tower*. *MDCLI.*

TO THE READER.

Friendly Reader,

HEre is preſented to thy view an exact and excellent *Compendium* of the firſt ſix Books of the *Elements* of *Geometry*, written by that learned *Philoſopher* & *Geometrician Euclide*. I ſhal in this place ſay little or nothing concerning the excellencie of the *Mathematicall Sciences* in generall, nor particularly of *Geometry*, that pious and learned Mathematician Mr. JOHN DEE in his following *Preface* (which deſerves perpetual commendations) having beene ſo large in the explanation and uſe of all the parts thereof, ſo that in it you may diſcern the whole *Body* of the *Mathematicks*.

Now *Geometry* (next after *Arithmetick*) is the firſt thing to be ſtudied in the courſe of the *Mathematicks*, and of all the *Books* of *Geometry* hitherto publiſhed, none are comparable to thoſe of *Euclide*, *The firſt ſix* of which are here in a moſt compendious form *contracted* and *demonſtrated*, then they have heretofore been in our *Language*: there being laid down, Firſt, the *Probleme*, or *Theoreme* it ſelfe, then the *Figure*, and the *conſtruction* thereof; and laſtly, the *Demonſtration* of the ſame.

The uſe of theſe *Elements* of *Euclide* are ſo frequent in all the parts of the *Mathematicks*, that whatſoever is done in them, hath dependance upon theſe for proofe.

As he can be accounted no good *Gramarian* that cannot prove his Latine by *Lillies Rules*, ſo he can be accounted no good *Geometrician*, that cannot prove his Propoſition by *Euclides Elements*.

I will not, in this place, goe about to apologize for the Work, only thus, the Author of this Epitomie having finiſhed his Manuſcript, before he could provide for the printing thereof, was tranſported from this life to a better, ſo that the Copie, in many places, was deficient, but ſome time hath been beſtowed in the ſupplying of thoſe defects. But notwithſtanding all this, ſome miſtakes may yet remain, which when thou chanceſt to meet with correct with thy pen, ſo ſhalt thou do right to the deceaſed Author, and ſupply the defects of the Reviſor, and in both doe good to thy ſelf. Theſe things thus premiſed, I commend you to the Work it ſelfe, in the peruſall and practiſe whereof, thou ſhalt receive abundance both of pleaſure and profit, in that before your eyes you ſhall ſee the *demonſtration* and *proofe* of all your workings, and be thereby able to diſcern and judge of other things of the like nature. And ſo I leave it with you, as a Clew which will lead you by degrees through the whole *Treaſury* of the MATHEMATICKS.

Farewell.

To the unfained lovers of Truth, and conſtant *Students* of Noble *Sciences*, JOHN DEE of *London*, heartily wiſheth grace from Heaven, and moſt proſperous ſucceſs in all their honeſt attempts and *exerciſes*.

DIvine *PLATO*, the great Maſter of many worthy Philoſophers, & the conſtant avoucher, and pithy perſwader of *Vnum*, *Bonum*, & *Ens*: in his School and Academie, ſundry times (beſides his ordinary Scholers) was viſited of a certain kind of men allured by the noble fame of *Plato*, and the great commendation of his profound and profitable doctrine. But when ſuch Hearers, after long harkening to him, perceived, that the drift of his diſcourſes iſſued out, to conclude, this *Vnum*, *Bonum*, and *Ens*, to be Spirituall, Infinite, Æternall, Omnipotent, &c. Nothing being alledged or expreſſed, How, wordly goods: how, worldly dignity: how, health, ſtrength, or luſtineſſe of body: nor yet the means, how a marveilous, ſenſible and bodily bliſſe and felicity hereafter, might be atteined: Straightway, the fantaſies of thoſe hearers were dampt: their opinion of *Plato,* was clean changed; yea his doctrine was by them deſpiſed; and his School, no more of them viſited. Which thing, his Scholer, *Ariſtotle*, narrowly conſidering, found the cauſe thereof, to be, For that they had no forewarning and information, in generall, “ whereto his doctrine tended. For, ſo, might they have had occaſion, either to have forborne his School haunting: (if they, then, had miſliked his Scope and purpoſe) or conſtantly to have continued therein: to their full ſatisfaction: if ſuch his finall ſcope and intent, had been to their deſire. Wherefore, *Ariſtotle*, ever, after that,uſed in brief,to forewarn his own “ Scholers and hearers, both of what matter, and alſo to “ what end, he took in hand to ſpeak, or teach. While “ I conſider the diverſe trades of theſe two excellent Philoſophers (and am moſt ſure, both, that *Plato* rightwell, otherwiſe could teach: and that *Ariſtotle* might boldly, with his hearers, have dealt in like ſort as *Plato* did) I am in no little pang of perplexity: Becauſe, that which I miſlike, is moſt eaſie for me to perform (and to have *Plato* for my example.) And that, which I know to be moſt commendable: and (in this firſt bringing, into common handling, the *Arts Mathematicall*) to be moſt neceſſary: is full of great difficulty and ſundry dangers. Yet, neither do I think it meet, for ſo ſtrange matter (as now is meant to be publiſhed) and to ſo ſtrange an audience,to be bluntly at firſt put forth, without a peculiar Preface: Nor (Imitating *Ariſtotle*) well can I hope,that according to the ampleneſſe and dignity of the *State Mathematicall*, I am able, either plainly to preſcribe the material bounds: or preciſely to ex-

expreſſe the chief purpoſes, and moſt wonderfull applications thereof. And though I am ſure, that ſuch as did ſhrink from *Plato* his School, after they had perceived his finall concluſion, would in theſe things have been his moſt diligent hearers (ſo infinitely might their deſires, in fine and at length, by our *Arts Mathematicall*, be ſatisfied) yet, by this my Præface and forewarning, Aſwell all ſuch, may (to their great behoof) the ſooner, hither be allured: as alſo the *Pythagoricall*, and *Platonicall* perfect ſcholer, and the conſtant profound Philoſopher, with more eaſe and ſpeed, may (like the Bee) gather, hereby, both wax and hony.

Wherefore, ſeeing I find great occaſion (for the “ cauſes alledged, and farther, in reſpect of my *Art* “ *Mathematick generall*) to uſe a certain forewarning “ and Præface, whoſe content ſhall be, that mighty, “ moſt pleaſant, and fruitfull *Mathematicall Tree*, with his chief arms and ſecond (grafted) branches: Both, what every one is, and alſo, what commodity, in generall, is to be looked for, aſwell of griff as ſtock: And “ foraſmuch as this enterprize is ſo great, that, to this “ our time, it never was (to my knowledg) by any at“chieved: And alſo it is moſt hard, in theſe our drery “ dayes, to ſuch rare and ſtrange Arts, to win due and common credit: Nevertheleſſe, if for my ſincere endevour, to ſatisfie your honeſt expectation, you will but lend me your thankfull mind a while: and to ſuch matter as, for this time, my pen (with ſpeed) is able to deliver, apply your eye or ear attentively: perchance, at once, and for the firſt ſaluting, this Præface you will finde a leſſon long enough. And either you will, for a ſecond (by this) be made much the apter: or ſhortly become, well able your ſelves, of the Lyons claw, to conjecture his royall Symmetrie, and farther property. Now then, gentle, my friends and countrey men, Turn your eyes, and bend your minds to that doctrine, which for our preſent purpoſe, my ſimple talent is able to yield you.

The intent of this Præface.

All things which are, and have being, are found under a triple diverſity generall. For, either, they are deemed Supernaturall,Naturall,or of a third being. Things Supernaturall,are immateriall, ſimple, indiviſible, incorruptible, and unchangeable. Things Naturall, are materiall, compounded diviſible, corruptible, and changeable. Things Supernaturall,are of the mind onely comprehended : Things Naturall,of the ſenſe exterior,are able to be perceived. In things Naturall, probability and conjecture hath place: But in things Supernaturall,chief demonſtration and moſt ſure Science is to be had. By which properties and compariſons of theſe two, more eaſily may be deſcribed, the ſtate, condition,nature and property of thoſe things, which, we before termed of a third being: which, by a peculiar name alſo, are called *Things Mathematicall*. For, theſe, being (in a manner) middle,between things ſupernatural and natural:are not ſo abſolute and excellent, as things ſupernatnrall ; Nor yet ſo baſe and groſſe, as things naturall : But are things immateriall, and nevertheleſſe, by materiall things able ſomewhat to be ſignified. And though their perticular Images, by Art, are aggregable and diviſible : yet the generall *Forms* notwithſtanding, are conſtant, unchangeable, untransformable and incorruptible. Neither of the ſenſe, can they, at any time, be perceived or judged. Nor yet, for all that in the royall mind of man, firſt conceived. But ſurmounting the imperfection of conjecture, weening and opinion ; and comming ſhort of high intellectuall conception, are the Mercuriall fruit of *Dianœticall* diſcourſe, in perfect imagination ſubſiſting. A marveilous newtrality have theſe things *Mathematicall*, and alſo a ſtrange participation between things ſupernaturall, immmortall, intellectuall, ſimple and indiviſible : and things naturall, mortall, ſenſible, compounded and diviſible. Probability and ſenſible proof, may well ſerve in things naturall, and is commendable: In Mathematicall reaſonings, a probable Argnment, is nothing regarded: nor yet the teſtimony of ſens,any whit credited : But onely a perfect demonſtration, of truths certain,neceſſary,and invincible:univerſally and neceſſarily concluded : is allowed as ſufficient for an Argument exactly and purely Mathematicall.

Number.

Note the word, *Vnite*, to expreſſe the Greek Monas, and not *Vnity*, as we have all, commonly, till now, uſed.

Of *Mathematicall* things, are two principall kinds, namely, *Number* and *Magnitude*. *Number*, we define to be a certain Mathematicall Summe, of *Vnits*. And an *Vnit* is that thing Mathematicall, Indiviſible, by participation of ſome likeneſſe of whoſe property, any thing, which is in deed, or is counted One, may reaſonably be called One. We account an *Vnit*, a thing *Mathematical*,though it be no *number*,& alſo indiviſible becauſe of it materially, Number doth conſiſt : which principally , is a thing *Mathematicall*. *Magnitude* is a thing *Mathematicall*,by participation of ſome likenes of whoſe nature, any thing is judged long, broad,or thick. A thick *Magnitude*,we call a *Solid*, or a *Body*. What *Magnitude* ſo ever is Solid or Thick, is alſo broad and long. A broad Magnitude we call a *Superficies* or a Plain. Every plain Magnitude, hath alſo length. A long Magnitude, we term a *Line*. A *Line* is neither thick nor broad, but onely long : Every certain Line, hath two ends : The ends of a Line, are *Points* called. A *Point* is a thing *Mathematicall*, indiviſible, which may have a certain determined ſituation. If a Point move from a determined ſituation, the way wherein it moved, is alſo a *Line*, mathematically produced : whereupon, of the ancient Mathematicians, a *Line* is called the race or courſe of a *Point*. A Point we define, by the name of a thing Mathematicall: though it be no Magnitude, and indiviſible : becauſe it is the proper end, and bound of a *Line* : which is a true *Magnitude*. And *Magnitude* we may define to be that thing *Mathematicall*, which is diviſible for ever, in parts diviſible, long, broad, or thick. Therefore though a Point be no *Magnitude*, yet *Terminatively* we reckon it a thing *Mathematicall* (as I ſaid)

Magnitude.

A Point.

A Line.

Magnitude.

ſaid) by reaſon it is properly the end and bound of a line.

Neither *Number* nor *Magnitude* have any Materiality. Firſt, we wil conſider of *Number*, and of the Science *Mathematicall*, to it appropriate, called *Arithmetick*, and afterward of *Magnitude*, and his Science, called *Geometrie*. But that name contenteth me not: whereof a word or two hereafter ſhall be ſaid. How Immaterial, and free from all matter, *Number* is, who doth not perceive? yea, who doth not wonderfully wonder at it? For, neither pure *Element*, nor *Ariſtoteles Quinta Eſſentia*, is able to ſerve for Number, as his proper matter. Nor yet the purity and ſimpleneſſe of Subſtance Spiritual or Angelical, will be found proper enough thereto. And therefore the great and godly Philoſopher *Anitius Boetius*, ſaid: *Omnia quæcunque à primæva rerum natura conſtructa ſunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo Conditoris Exemplar*. That is: All things (which from the very firſt originall being of things, have been framed and made) do appear to be Formed by the reaſon of Numbers. For this was the principall Example or pattern in the mind of the Creator. O comfortable allurement, O raviſhing perſwaſion, to deal with a Science, whoſe Subject is ſo ancient, ſo pure, ſo excellent, ſo ſurmounting all creatures, ſo uſed of the Almighty and incomprehenſible wiſedome of the Creator, in the diſtinct creation of all creatures: in all their diſtinct parts, properties, natures, and vertues, by order, and moſt abſolute number, brought from *Nothing*, to the *Formality* of their being and ſtate. By *Numbers* property therefore, of us, by all poſſible means, (to the perfection of the Science) learned, we may both wind and draw our ſelves into the inward and deep ſearch and view, of all creatures diſtinct vertues, natures, properties, and *Formes*: And alſo, farther, ariſe, clime, aſcend and mount up (with Speculative wings) in ſpirit, to behold in the Glaſſe of Creation, the *Form* of *Forms*, the

Exemplar Number of all things *Numerable*: both viſible and inviſible: mortal and immortal, Corporal, and Spiritual. Part of this profound and divine Science, had *Joachim* the Propheſier atteined unto: by *Numbers Formall*, *Naturall*, and *Rationall*, foreſeeing, concluding, and foreſhewing great perticular events, long before their comming. His books yet remaining, hereof, are good proof. And the noble Earl of *Mirandula*, (beſides that) a ſufficient witneſſe: that Joachim *in his propheſies, proceeded by no other way, then by Numbers Formall.* And this Earl himſelf, in *Rome*, ſet up 900 Concluſions, in all kind of Sciences, openly to be diſputed of: and among the reſt, in his Concluſions *Mathematicall* (in the eleventh Concluſion) hath in Latine this Engliſh ſentence. *By Numbers, a way is had, to the ſearching out, and underſtanding of every thing, able to be known. For the verifying of which Concluſion, I promiſe to anſwer to the 74 Quæſtions, under written by the way of Numbers.* Which Concluſions, I omit here to rehearſe: aſwell avoiding ſuperfluous prolixitie: as, becauſe *Joannes Picus*, *Works*, are commonly had. But, in any caſe, I would wiſh that thoſe Concluſions were read diligently, and perceived of ſuch, as are earneſt Obſervers and Conſiderers of the conſtant law of Numbers: which is planted in things Naturall and Supernaturall: and is preſcribed to all Creatures, inviolably to be kept. For, ſo, beſides many other things, in thoſe Cõcluſions to be marked it would appear, how ſincerely, and within my bounds, I diſcloſe the wonderful myſteries by numbers, to be atteined unto.

Anno 1488.

Of my former words, eaſie it is to be gathered, that *Number* is a treble ſtate: One, in the Creator: another in every Creature (in reſpect of his compleat conſtitution:) and the third, in Spirituall, and Angelicall Minds, and in the Soul of man. In the firſt and third ſtate, *Number*, is termed *Number Numbring*. But in all Creatures otherwiſe, *Number* is termed *Number Numbred*

bred. And in our Soul, Number beareth ſuch a ſway, and hath ſuch an aſſinity therewith; that ſome of the old *Philoſophers* taught, *Mans ſoul, to be a Number moving it ſelf.* And indeed, in us, though it be a very Accident, yet ſuch an Accident it is, that before all Creatures it had perfect being, in the Creator, Sempiternally. *Number numbring* therefore, is the diſcretion diſcerning, and diſtincting of things. But in God the Creator, This diſcretion, in the beginning, produced orderly and diſtinctly all things. For his *numbring*, then, was his Creating of all things. And his Continuall *numbring* of all things, is the Conſervation of them in being. And, where and when he will lack an *Unit:* there and then, that perticular thing ſhall be *Diſcreated.* Here I ſtay. But our Severalling, diſtincting, and *numbring*, createth nothing: but of Multitude conſidered, maketh certain and diſtinct determination. And albeit theſe things be waighty, and truths of great importance, yet (by the infinite goodneſſe of the Almighty *Ternarie,*) Artificial Methods and eaſie ways are made, by which the zealous Philoſopher may win neer this Riveriſh *Ida*, this Mountain of Contemplation: and more then Contemplation. And alſo, though *number*, be a thing ſo Immaterial, ſo divine, and æternal: yet by degrees, by little, and little, ſtretching forth, and applying ſome likeneſſe of it, as firſt, to things Spiritual: and then bringing it lower, to things ſenſibly perceived, as of a momentany ſound iterated: then to the leaſt things that may be ſeen, numerable: And at length (moſt groſſely) to a multitude of any corporal things ſeen, or felt: and ſo, of theſe groſſe and ſenſible things, we are trained to learn a certain Image, or likeneſſe of numbers, and to uſe Art in them to our pleaſure and profit. So groſſe is our converſation, and dull is our apprehenſion, while mortal Senſe, in us, ruleth the common wealth of our little world. Hereby we ſay, Three Lyons are three or a *Ternarie*. Which * *Ternaries*, are each, the

Union, knot and *Uniformity* of three diſcreet and diſtinct *Units*. That is, we may in each *Ternary*, thrice ſeverally, point, and ſhew a part, *One, One,* and *One*. Where, in Numbring, we ſay One, Two, Three. But how farre, theſe viſible Ones, do differ from our Indiviſible Units (in pure *Arithmetick*, principally conſidered) no man is ignorant. Yet from theſe groſſe and material things, may we be led upward, by degrees, ſo informing our rude Imagination, toward the conceiving of *numbers*, abſolutely: (Not ſuppoſing, nor admixting any thing created, Corporal or Spiritual, to ſupport, contein, or repreſent thoſe *numbers* imagined:) that at length, we may be able, to finde the number of our own name, glorioufly exemplified and regiſtred in the book of the *Trinity* moſt bleſſed and æternall.

But farther underſtand, that vulgar Practiſers, have Numbers, otherwiſe, in ſundry Conſiderations: and extend their name farther, then to Numbers, whoſe leaſt part is an *Unit*. For the common Logiſt, Reckenmaſter, or Arithmetician, in his uſing of Numbers: of an Unit, imagineth leſſe parts: and calleth them *Fractions*. As of an *Unit*, he maketh an half, and thus noteth it, $\frac{1}{2}$, and ſo of other (infinitely diverſe) parts of an *Unit*. Yea and farther, hath *Fractions of Fractions, &c.* And for aſmuch as *Addition, Subſtraction, Multiplication, Diviſion*, and *Extraction of Roots*, are the chief, and ſufficient parts of *Arithmetick*: which is, the *Science that demonſtrateth the properties of numbers, and all operations, in numbers to be performed*. *Arithmeticke Note.* How often therefore, " theſe five ſundry ſorts of Operations, do, for the " moſt part of their execution, differ from the five ope-" rations of like generall property and name, in our " Whole numbers practiſable. So, often, (for a more di-" ſtinct doctrine) we, vulgarly account and name it, another kind of *Arithmetick*. 1. And by this reaſon: the Conſideration, doctrine, and working in whole numbers onely: where, of an *Unit*, is no leſſe part to be allowed:

allowed: is named (as it were) an *Arithmetick* by it ſelf. And ſo of the *Arithmetick of Fractions.* In like ſort the neceſſary, wonderfull, and Secret doctrine of Pro-
2. portion, and proportionality hath purchaſed unto it ſelf a peculiar manner of handling and working: and ſo may ſeem another form of *Arithmetick.* Moreover
3. the *Aſtronomers*, for ſpeed, and more commodious calculation, have deviſed a peculiar manner of ordering numbers, about their circular motions, by Sexagenes and Sexageſmes. By Signes, Degrees, Minutes, &c. which commonly is called the *Arithmetick* of *Aſtronomical* or *Phiſical Fractions.* That have I briefly noted by the name of *Arithmetick Circular.* Becauſe it is alſo uſed in Circles, not *Aſtronomical, &c.* Practiſe hath led
4. *Numbers* farther, and hath framed them, to take upon them, the ſhew of *Magnitudes* property: Which is *Incommenſurability* and *Irrationality.* (For in pure *Arithmetick*, an *Unit*, is the common Meaſure of all Numbers.) And here, Numbers are become, as Lines, Plains, and Solids, ſometimes *Rationall*, ſometimes *Irrationall*: And have proper and peculiar characters, (as √ʒ. √ꝯ. and ſo of other. Which is to ſignifie *Root Square, Root Cubick: and ſo forth:*) and proper and peculiar faſhions in the five principal parts: Wherefore the practiſer, eſteemeth this a diverſe *Arithmetick* from the other. Practiſe bringeth in, here, diverſe compounding of Numbers: as ſometime, two, three, four (or more) *Radicall* Numbers diverſly knit by ſigns, of More and Leſſe: as thus √ʒ 12 + √ꝯ 15. Or thus √ʒʒ 19 + √ꝯ 12 − √ʒ 2, &c. And ſometime with whole numbers, or fractions of whole Number, among them: as 20 + √ʒ 24. √ꝯ 16 + 33 − √ʒ 10. √ʒʒ 44 + 12¼ + √ꝯ 9 And ſo, infinitely, may hap the varietie. After this; Both the one and the other hath fractions incident: and ſo is this *Arithmetick* greatly enlarged, by diverſe exhibiting, and uſe of Compoſitions and mixtings. Conſider how, I (being deſirous to deliver the Student from

from errour and Cavillation) do give to this *Practiſe*, the name of the *Arithmetick of Radicall numbers:* Not of *Irrationall* or *Surd numbers:* which other while, are Rationall: though they have the Sign of a Root before them, which *Arithmetick* of whole Numbers moſt uſuall, would ſay they had no ſuch Root, and ſo account them *Surd numbers*, which, generally ſpoken, is untrue: as *Euclid*'s tenth book may teach you. Therefore, to call them, generally *Radicall numbers*, (by reaſon of the ſign √ prefixed,) is a ſure way, and a ſufficient general diſtinction from all other ordering and uſing of Numbers: And yet (beſide all this) Conſider; the infinite deſire of knowledg, and incredible power of mans Search and Capacity: how, they, joyntly have waded farther (by mixing ſpeculation and practice) and have found out, and atteined, to the very chief perfection (almoſt) of *Numbers* Practical uſe. Which thing, is wel to be perceived in that great Arithmetical Art of *Æquation:* commonly called the *Rule of Coſs*, or *Algebra*. The Latines termed it, *Regulam Rei & Cenſus*, that is, the **Rule of the thing, and his value.** With an apt name, comprehending the firſt and laſt points of the Work. And the vulgar names, both in Italian, French. and Spaniſh depends (in naming it,) upon the ſignification of the Latin word *Res:* **a thing:** unleaſt they uſe the name of *Algebra*. And therein (commonly) is a double errour. The one of them which think it to be of *Geber*, his inventing: the other of ſuch as cal it *Algebra*. For, firſt, though *Geber* for his great ſkil in Numbers, Geometry, Aſtronomy, and other marvellous Arts, might have ſeemed able to have firſt deviſed the ſaid Rule: and alſo the name carrieth with it a very neer likeneſſe of *Geber* his name: yet true it is, that a *Greek* Philoſopher and Mathematician, named *Diophantus*, before *Geber* his time, wrote 13 books thereof (of which, ſix are yet extant: and I had them to * uſe, of the famous Mathematician, and my great

Anno 1550.

friend, *Petrus Montaureus*:) And ſecondly, the very name, is *Algicbar*, and not *Algebra*: as by the Arabian *Avicen*, may be proved: who hath theſe preciſe words in Latine, by *Andreas Alpagus* (moſt perfect in the Arabick tongue) ſo tranſlated. *Scientia faciendi Algiebar & Almachabel.* i. *Scientia inveniendi numerum ignotum, per additum numeri, & diviſionem, & æquationem.* Which is to ſay: **The Science of working Algiebar, and Almachabel**: that is: the **Science of finding an unknown Number, by Adding of a Number, and Diviſion, and Æquation.** Here have you the name: and alſo the principall parts of the Rule, touched. To name it, *The rule, or art of Æquation*, doth ſignifie the middle part and the State of the Rule. This Rule hath his peculiar Characters, and the principall parts of *Arithmetick*, to it appertaining, do differ from the other *Arithmeticall operatations*. This *Arithmetick* hath Numr-
5. bers Simple, Compound, Mixt, and Fractions accodingly. This Rule and *Arithmetick* of *Algiebar*, is ſo profound, ſo general and ſo (in manner) conteineth the whole power of Numbers, Application practical: that mans wit, can deal with nothing more profitable about numbers: nor match, with a thing, more meet for the divine force of the Soul, (in humane ſtudies, affairs or exercizes) to be tried in. Perchance you looked for, (long ere now) to have had ſome perticular proof, or evident teſtimony of the uſe, profit, and Commodity of Arithmetick vulgar, in the common life and trade of men. Thereto, then I will now frame my ſelf: But herein great care I have, leaſt length of ſundry proofs, might make you deem, that either I did miſdoubt your zealous mind to vertues ſchool, or elſe miſtruſt your able wits, by ſome, to gueſſe much more. A proof, then four, five, or ſix, ſuch, will I bring, as any reaſonable man, therewith may be perſwaded, to love and honour, yea learn and exerciſe the excellent Science of *Arithmetick*.

And firſt: who, neerer at hand, can be a better witneſſe of the fruit received by *Arithmetick*, then all kind of Merchants? Though not all alike, either need it, or uſe it How could they forbear the uſe and help of the Rule, called the Golden Rule? Simple and Compound: both forward and backward? How might they miſſe *Arithmeticall* help in the Rules of Fellowſhip: either without time, or with time? and between the Merchant and his Factor? The Rules of Bartering in wares onely: or part in wares, and part in money, would they gladly want? Our Merchant Venturers, and Travailers over Sea, how could they order their doings juſtly and without loſſe, unleaſt certain and general Rules for Exchange of money, and Rechange, were, for their uſe, deviſed? The Rule of Alligation, in how ſundry caſes, doth it conclude for them, ſuch preciſe verities, as neither by naturall wit nor other experience they were able elſe to know? And (with the Merchant then to make an end) how ample and wonderfull is the Rule of Falſe poſitions? eſpecially as it is now, by two excellent Mathematicians (of my familiar acquaintance in their life time) enlarged? I mean *Gemma Friſius* and *Simon Jacob*, Who can either in brief conclude, the general and Capital Rules? or who can Imagine the Myriades of ſundry Caſes, and perticular examples in Act and earneſt, continually wrought, tried and concluded by the forenamed Rules, onely? How ſundry other *Arithmeticall practiſes*, are commonly in Merchants hands, and knowledg: They themſelves can at large teſtifie.

The Mintmaſter, and Goldſmith, in their Mixture of Metals, either of diverſe kinds, or divers values: how are they, or may they, exactly be directed, and marvelouſly pleaſured, if *Arithmetick* be their guide? And the honourable Phiſicians wil gladly confeſſe themſelves much beholding to the Science of *Arithmetick*, and that ſundry ways: But chiefly in their Art of

Graduation, and compound Medicines. And though *Gallenus*, *Averrois*, *Arnoldus*, *Lullus*, and others have publiſhed their poſitions, aſwel in the quantities of the Degrees above Temperament, as in the Rules, concluding the new *Form* reſulting: yet a more preciſe, commodious, and eaſie *Method*, is extant: by a Countreyman of ours (above 200 years ago) invented. *R. B.* And foraſmuch as I am uncertain, who hath the ſame: or when that little Latin treatiſe (as the Author writ it,) ſhal come to be Printed: (Both to declare the deſire I have to pleaſure my Countrey, wherein I may: and alſo, for very good proof of Numbers uſe, in this moſt ſubtile, and fruitful Philoſophical Concluſion,) I intend in the mean while, moſt briefly, and with my farther help, to communicate the pith thereof unto you.

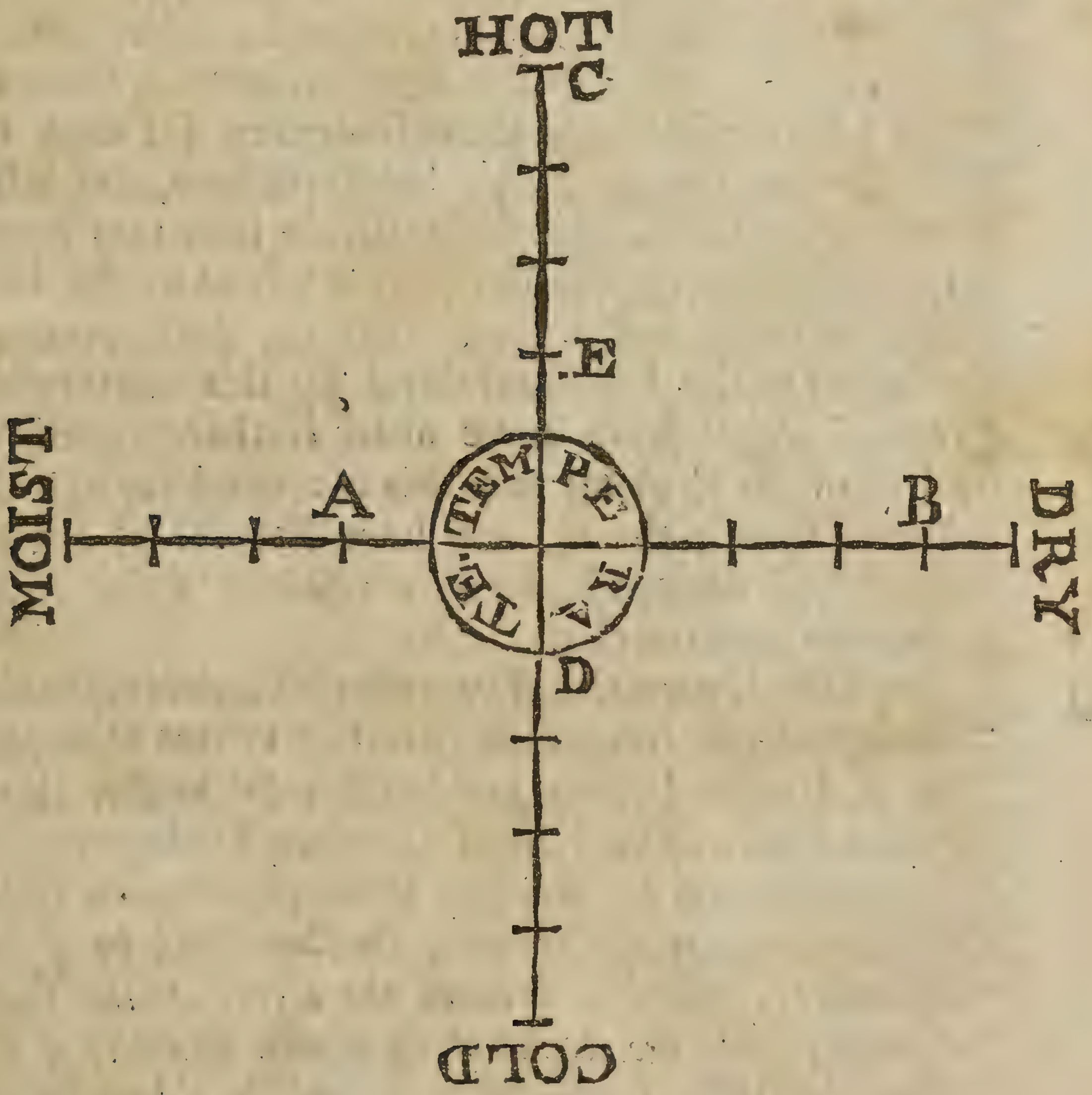
Firſt, deſcribe a circle: whoſe diameter let be an inch. Divide the Circumference into four equal parts. From the Center, by thoſe 4 ſections, extend 4 right lines: each of 4 inches and a half long: or of as many as you liſt, about 4 without the circumference of the circle: So that they ſhall be of 4 inches long (at the leaſt) without the Circle. Make good evident marks at every inches end. If you liſt, you may ſubdivide the inches again into 10 or 12 ſmaller parts, equal. At the ends of the lines, write the names of the 4 principal elemental Qualities. *Hot*, and *Cold*, one againſt the other. And likewiſe *Moiſt* and *Dry*, one againſt the other. And in the Circle write *Temperate*. Which *Temperature* hath a good Latitude: as appeareth by the Complexion of man. And therefore we have allowed unto it, the foreſaid Circle: and not a point Mathematical or Phyſical.

Now, when you have two things Miſcible, whoſe degrees are * truly known: Of neceſſity, either they are of one Quantity and waight, or of diverſe. If they be of one Quantity and waight: whether their forms,

* Take ſome part of *Lullus* counſell in his Booke *de Q. Eſſentia.*

be Contrary Qualities, or of one kind (but of diverſe, intentions and degrees) or a *Temperate*, and a Contrary, *The form reſulting of their Mixture, is in the Middle between the degrees of the forms mixt.* As for example, let *A*, be *Moiſt* in the firſt degree: and *B*, *Dry* in the third degree. Adde 1 and 3, that maketh 4: the half or middle of 4, is 2. This 2 is the middle, equally diſtant from *A* and *B*, (for the * *Temperament* is counted none. And for it you muſt put a Cipher, if at any time it be in mixture.) Counting then from *B*, 2 degrees, toward *A*: you finde it to be *Dry* in the firſt degree: So is the *Form reſulting* of the Mixture of *A*, and *B*, in our example. I wil give you another example.

* *Note.*



Suppoſe, you have two things as *C*, and *D*; and of *C*, the Heat to be in the 4 degree: and of *D*, the Cold, to be remiſſe, even unto the *Temperament*. Now, for *C*, you take 4; and for *D*, you take a Cipher: which added unto 4, yieldeth onely 4. The middle, or half, whereof, is 2. Wherefore the *Form reſulting* of *C* and *D*, is hot in the ſecond degree: for, 2. degrees, accounted from *C*, toward *D*, end juſt in the 2 degree of heat. Of the third manner, I will give alſo an example, which let be this: I have a liquid Medicine, whoſe Quality of heat is in the 4 degree exalted: as was *C*, in the example foregoing: and another liquid Medicine I have: whoſe Quality, is heat in the firſt degree. Of each of theſe, I mixt a like quantity: Subſtract here, the leſſe from the more; and the reſidue divide into two equal parts: whereof, the one part, either added to the leſs, or ſubſtracted from the higher degree, doth produce the degree of the Form reſulting by this mixture of *C*, and *E*. As, if from 4, ye abate 1, there reſteth 3, the half of 3 is $1\frac{1}{2}$. Adde to 1, this $1\frac{1}{2}$: you have $2\frac{1}{2}$. Or Subſtract from 4 this $1\frac{1}{2}$, you have likewiſe $2\frac{1}{2}$ remaining. Which declareth, the *Form reſulting*, to be *Heat*, in the middle of the third degree.

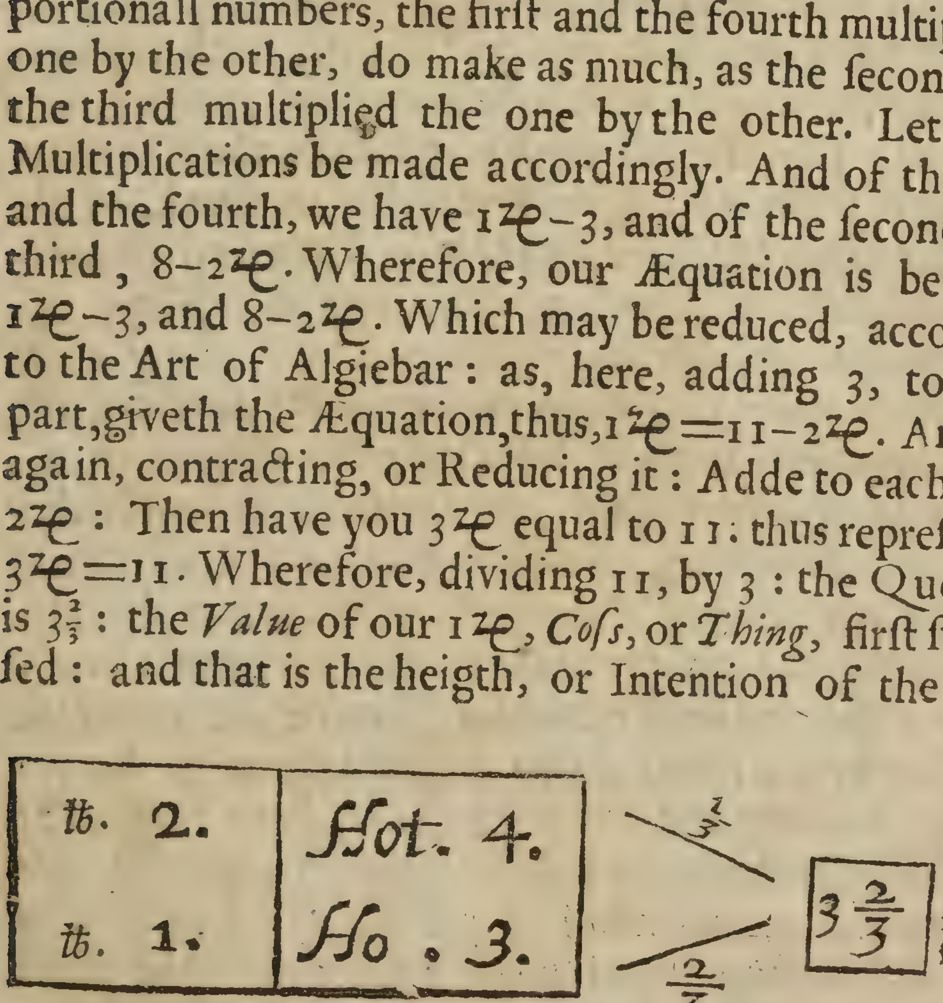
Note.

The ſecond Rule.

But if the Quantities of two things Commixt, be di-
“ verſe, and the Intentions (of their Forms Miſcible)
“ be in diverſe degrees, and heights. (Whether thoſe
“ Forms be of one kind, or of Contrary kinds, or of a
“ Temperate and a Contrary. *What proportion is of the*
“ *leſſe quantity to the greater, the ſame ſhall be of the*
“ *difference, which is between the degree of the Form*
“ *reſulting, and the degree of the greater quantity of the*
“ *thing Miſcible, to the difference, which is between the*
“ *ſame degree of the Form reſulting, and the degree of*
“ *the leſſe quantity.* As for example. Let two pound of
“ Liquor be given, hot in the 4 degree: and one pound
“ of Liquor be given, hot in the third degree. I would
“ gladly know the Form reſulting, in the Mixture of
theſe

℔. 2.	*Hot.* 4.
℔. 1.	*Hot.* 3.

theſe two Liquors. Set down your numbers in order, thus. Now by the rule of Algiebar, have I deviſed a very eaſie, brief, and general manner of working in this caſe. Let us firſt ſuppoſe that *MiddleForm reſulting* to be 1℞: as that Rule teacheth. And becauſe (by our Rule, here given) as the weight of 1. is to 2: So is the difference between 4. (the degree of the greater quantity) and 1℞: to the difference between 1℞ and 3: (the degree of the thing, in leſſe quantity. And withall, 1℞, being always in a certain middle, between the two heigths or degrees.) For the firſt difference, I ſet 4-1℞: and for the ſecond, I ſet 1℞-3. And now again I ſay, as 1 is to 2. ſo is 4-1℞ to 1℞-3. Wherefore, of theſe four proportionall numbers, the firſt and the fourth multiplyed, one by the other, do make as much, as the ſecond and the third multiplied the one by the other. Let theſe Multiplications be made accordingly. And of the firſt and the fourth, we have 1℞-3, and of the ſecond and third, 8-2℞. Wherefore, our Æquation is between 1℞-3, and 8-2℞. Which may be reduced, according to the Art of Algiebar: as, here, adding 3, to each part, giveth the Æquation, thus, 1℞=11-2℞. And yet again, contracting, or Reducing it: Adde to each part, 2℞: Then have you 3℞ equal to 11: thus repreſented 3℞=11. Wherefore, dividing 11, by 3: the Quotient is $3\frac{2}{3}$: the *Value* of our 1℞, *Coſs*, or *Thing*, firſt ſuppoſed: and that is the heigth, or Intention of the *Form*



The form reſulting.

reſulting: which is, *Heat*, in two thirds of the fourth degree: and here I ſet the ſhew of the work in concluſion thus. The proof hereof is eaſie, by ſubſtracting 3, from $3\frac{2}{3}$, reſteth $\frac{2}{3}$. Subſtract the ſame height of the Form reſulting, (which is $3\frac{2}{3}$) from 4: then reſteth $\frac{1}{3}$: You ſee that $\frac{2}{3}$ is double to $\frac{1}{3}$: as 2. ℔. is double to 1. ℔. So ſhould it be: by the rule here given. Note. As you added to each part of the Æquation, 3: ſo if you firſt added to each part 2℞, it would ſtand, $3\text{℞}-3=8$: and now adding to each part 3: you have (as before) $3\text{℞}=11$.

And though, I, here, ſpeak onely of two things Miſcible: and moſt commonly more then three, four, five or ſix, &c. are to be mixed: (and in one Compound to be reduced: and the Form reſulting of the ſame, to ſerve the turn) yet theſe Rules are ſufficient, duly repeated and iterated. In proceeding firſt, with any two: and then, with the Form Reſulting, and another, and ſo forth: For, the laſt work concludeth the Form reſulting of them all: I need nothing to ſpeak, of the Mixture (here ſuppoſed) what it is. Common Philoſophy hath defined it, ſaying, *Mixtio eſt miſcibilium, alteratorum, per minima conjunctorum Unio.* Every word in the definition, is of great importance. I need not alſo ſpend any time, to ſhew, how, the other manner of diſtributing of degrees, doth agree to theſe Rules. Neither need I of the farther uſe belonging to the Croſſe of Graduation (before deſcribed) in this place declare, unto ſuch as are capable of that, which I have already ſaid. Neither yet with examples, ſpecifie the manifold varieties, by the foreſaid two generall Rules to be ordered. The witty and Studious, here, have ſufficient: And they which are not able to attein to this, without lively teaching, and more in perticular: would have larger diſcourſing, then is meet in this place to be dealt withall: And other (perchance) with a proud ſnuffe will diſdain this little: and would be unthankfull for

Note.

much more. I,therefore conclude;and wiſh ſuch as have modeſt and earneſt Philoſophicall minds, to laud God highly for this : and marvail that the profoundeſt and ſubtileſt point, concerning *Mixture of Forms, and Qualities Naturall* , is ſo matcht and married with the moſt ſimple, eaſie, and ſhort way of the noble Rule of *Algiebar*. Who can remain, therefore,unperſwaded, to love, allow, and honour the excellent Science of *Arithmetick?* For, here, you may perceive that the little finger of *Arithmetick* is of more might and contriving, then a hundred thouſand mens wits, of the middle ſort, are able to perform, or truly to conclude, without help thereof.

Now, will we farther, by the wiſe and valiant Captain, be certified, what help he hath, by the Rules of *Arithmetick:* in one of the arts to him appertaining : “ and of the Greeks named Τακτικὴ. That is, the Skill “ of ordering Souldiers in Battle ray, after the beſt “ manner to all purpoſes. This art ſo much dependeth upon Numbers uſe, and the Mathematicals, that *Ælianus* (the beſt writer thereof,) in his works to the *Emperour Hadrianus*, by his perfection, in the Mathematicals, (being greater than other before him had,) thinketh his book to paſſe all other the excellent works, written of that art, unto his dayes. For, of it, had written *Æneas* : *Cyneas* of *Theſſaly* : *Pyrrhus Epirota*, and *Alexander* his ſonne : *Clearchus* : *Pauſanias* : *Evangelus* : *Polybius*, familiar friend to *Scipio*, *Eupolemus*, *Iphicrates*, *Poſſidonius*, and very many other worthy Captains, Philoſophers and Princes of Immortall fame and memory. Whoſe faireſt flower of their garland (in this feat) was *Arithmetick:* and a little perſeverance, in *Geometricall* Figures. But in many other caſes doth *Arithmetick* ſtand the Captain in great ſtead. As in proportioning of victuals for the army, either remaining at a ſtay ; or ſuddenly to be encreaſed with a certain number of Souldiers : and for a certain time. Or by good

Τακτικὴ.

art to diminiſh his company, to make the victuals, longer to ſerve the remanent, and for a certain determined time: if need ſo require. And ſo in ſundry his other accounts, Reckonings, Meaſurings, and proportionings, the wiſe, expert, and Circumſpect Captain wil affirm the Science of *Arithmetick*, to be one of his chief Counſailors, directers and aiders. Which thing (by good means) was evident to the Noble, the Couragious, ☞ the loyall, and Courteous *John*, late Earl of *Warwick*. Who was a young Gentleman, throughly known to very few. Albeit his luſty valiantneſſe, force, and Skill in Chivalrous feats and exerciſes: his humbleneſs, and friendlineſs to all men, were things, openly, of the world perceived. But what roots (otherwiſe,) vertue had faſtened in his breſt, what Rules of godly and honourable life he had framed to himſelf: what vices, (in ſome then living) notable, he took great care to eſchew, what manly vertues, in other noble men, (flouriſhing before his eyes,) he Sythingly aſpired after: what proweſſes he purpoſed and meant to atchieve: with what feats and arts, he began to furniſh and fraught himſelf, for the better ſervice of his King and Countrey, both in peace and war. Theſe (I ſay) his Heroicall Meditations, forecaſtings, and determinations, no twain, (I think) beſide my ſelf, can ſo perfectly, and truly report. And therefore, in Conſcience, I count it my part, for the honour, preferment, and procuring of vertue, (thus briefly) to have put his Name, in the Regiſter of *Fame Immortall*.

To our purpoſe. This *John* by one of his acts (beſides many other: both in *England* and *France*, by me, in him noted) did diſcloſe his hearty love, to vertuous Sciences: and his noble intent, to excell in Martiall proweſſe: When he with humble requeſt, and inſtant Solliciting: got the beſt Rules (either in time paſt, by Greek or Roman, or in our time uſed: and new Stratagems therein deviſed) for ordering of all Companies, ſummes, and Numbers of men, (Many, or few) with one kind of weapon, or more, appointed: with Artillery, or without: on horsback or on foot: to give or take on-ſet: to ſeem many, being few: to ſeem few, being many. To march in battail or journey: with many ſuch feats, to fight in field, Skirmiſh or Ambuſh appertaining: and of all theſe lively deſignments (moſt curiouſly) to be in velame parchment deſcribed: with Notes and peculiar marks, as the art requireth: and all theſe Rules and deſcriptions Arithmetical, incloſed in a rich Caſe of Gold, he uſed to wear about his neck, as his Jewel moſt precious, and Counſaylour moſt truſty. Thus, *Arithmetick*, of him, was ſhrined in gold: Of *Numbers* fruit, he had good hope. Now Numbers therefore innumerable, in *Numbers* praiſe, his ſhrine ſhal finde.

This noble Earle, died *Anno* 1554. ſcarce of 24 years of age, having no iſſue by his wife. Daughter to the Duke of *Somerſet*.

What need I, (for farther proof to you) of the Schoolmaſters of Juſtice, to require teſtimony: how needful, how fruitful, how skilful a thing *Arithmetick* is? I mean the Lawyers of all ſorts. Undoubtedly, the Civilians can marveilouſly declare: how, neither the Ancient Romane laws, without good knowledg of *Numbers art*, can be perceived: Nor (Juſtice in infinite Caſes) without due proportion, (narrowly conſidered) is able to be executed. How juſtly, and with great knowledg of Art, did *Papinianus* inſtitute a law of partition, and allowance, between man and wife after a divorce? But how *Accurſius*, *Baldus Bartolus*, *Jaſon*, *Alexander*, and finally *Alciatus*, (being otherwiſe, notably well learned) do jumble, gueſſe and erre, from the æquity, art and intent of the Law-maker: *Arithmetick* can detect, and convince: and cleerly, make the truth to ſhine. Good *Bartolus* tyred in the examining and proportioning of the matter, and with *Accurſius* Gloſſe much cumbred: burſt out, and ſaid: *Nulla eſt in toto libro, hac gloſſa difficilior: Cujus computationem nec Scholaſtici, nec Doctores intelligunt, &c.* That is:

In the whole book, there is no gloſſe harder than this: Whoſe account or reckoning, neither the Scholers nor the Doctours underſtand, &c. What can they ſay of *Julianus* law, *Si ita Scriptum, &c.* Of the Teſtators will, juſtly performing, between the wife, Sonne, and daughter? How can they perceive the æquity of *Aphricanus, Arithmeticall* Reckoning, where he treateth of *Lex Falcidia?* How can they deliver him, from his Reprovers: and their maintainers: as *Joannes, Accurſius Hypolitus*, and *Alciatus?* How juſtly and artificially, was *Africanus* reckoning made? Proportionating to the Summes bequeathed, the Contributions of each part? Namely, for the hundred preſently received, $17\frac{1}{7}$. And for the hundred, received after ten moneths, $12\frac{6}{7}$: which make the 30: which were to be contributed by the legataries to the heir. For, what proportion 100 hath to 75: the ſame hath $17\frac{1}{7}$ to $12\frac{6}{7}$: Which is *Seſquitertia*, that is, as 4, to 3, which make 7. Wonderfull many places in the Civill law, require an expert *Arithmetician*, to underſtand the deep Judgment and juſt determination of the Ancient Roman Law-makers. But much more expert ought he to be, who ſhould be able to decide with æquity, the infinite variety of Caſes, which do, or may happen, under every one of thoſe laws and ordinances Civill. Hereby, eaſily, ye may now conjecture: that in the Canon law: and in the laws of the Realm (which with us, bear the chief authority) Juſtice and equity might be greatly preferred, and ſkilfully executed through due ſkill of Arithmetick, and proportions appertaining. The worthy Philoſophers and prudentLaw-makers(who have written many books *De Republica*: How the beſt ſtate of Common wealths might be procured and mainteined,) have very well determined of Juſtice: (which, not onely is the Baſe and foundation of Common weales: but alſo the total perfection of all our works, words, and thoughts:)
Juſtice. " defining it, to be that vertue, by which, to every one,

“ one, is rendered, that to him appertaineth. God challengeth this at our hands to be honoured as God: to be loved, as a father, and to be feared, as a Lord and Maſter. Our neighbours proportion is alſo preſcribed of the Almighty Law-maker: which is, to do to others, even as we would be done unto. Theſe proportions, are in Juſtice neceſſary: in duty, commendable: and of Common wealths, the life, ſtrength, ſtay, and flouriſhing. *Ariſtotle* in his *Æthicks* (to fetch the ſeed of Juſtice, and light of direction, to uſe, and execute the ſame) was fain to fly to the perfection, and power of Numbers: for proportions Arithmetical and Geometrical. *Plato* in his book called *Epinomis*, (which book is the Treaſury of all his doctrine) where, his purpoſe is to ſeek a Science, which, when a man had it, perfectly: he might ſeem, and ſo be, indeed, *Wiſe*. He briefly of other Sciences diſcourſing, findeth them, not able to bring it to paſſe: But of the Science of Numbers, he ſaith. *Illa, quæ numerum mortalium generi dedit, id profectò efficiet. Deum autem aliquem, magis quàm fortunam, ad ſalutem noſtram, hoc munus nobis arbitror contuliſſe, &c. Nam ipſum bonorum omnium Authorem, cur non maximi boni, Prudentiæ dico, cauſam arbitramur?* That Science, verily, which hath taught mankind number, ſhall be able to bring it to paſſe. And, I think, a certain God, rather than Fortune, to have given us this gift, for our bliſſe. For, why ſhould we not Judg him who is the Authour of all good things, to be alſo the cauſe of the greateſt good thing, namely, Wiſedome? There, at length, he proveth *Wiſedome* to be atteined by good Skill of *Numbers*. With which great Teſtimony, and the manifold proofs, and reaſons, before expreſſed, you may be ſufficiently and fully perſwaded; of the perfect Science of *Arithmetick*, to make this account: That of all Sciences, next to *Theologie*, it is ☞ moſt divine, moſt pure, moſt ample, and generall, moſt profound, moſt ſubtile, moſt commodious, and moſt neceſſary. Whoſe next Siſter, is the Abſolute Science of *Magnitudes*: of which (by the Direction and aid of him, whoſe *Magnitude* is Infinite, and of us Incomprehenſible) I now intend ſo to write, that both with the *Multitude*, and alſo with the *Magnitude* of marvelous and fruitfull verities, you (my friends and Countreymen) may be ſtir'd up and awaked, to behold what certain Arts and Sciences (to our unſpeakable behoof) our heavenly Father, hath for us prepared, and revealed, by ſundry *Philoſophers* and *Mathematicians*.

BOth, *Number* and *Magnitude*, have a certain Originall ſeed, (as it were) of an incredible property: and of man, never able, fully to be declared. Of *Number*, an Unit, and of *Magnitude*, a Point, do ſeem to be much like Origininall cauſes: But the diverſity neverthele ſs, is great. We defined an *Unit* to be a thing Mathematicall Indiviſible: A Point, likewiſe, we ſaid to be a Mathematicall thing Indiviſible. And farther, that a Point may have a certain determined Situation: that is, that we may aſſign, and preſcribe a Point, to be here, there, yonder, &c. Herein, (behold) our Unit is free, and can abide no bondage, or to be tyed to any place, or ſeat: diviſible or indiviſible. Again, by reaſon, a Point may have a Situation limited to him: a certain motion, therefore, (to a place, and from a place) is to a Point incident, and appertaining. But an *Unit* cannot be imagined, to have any motion. A Point, by his motion, produceth Mathematically, a line, (as we ſaid before) which is the firſt kind of Magnitudes, and moſt ſimple: An *Unit*, cannot produce any number. A Line, though it be produced of a Point moved, yet it doth not conſiſt of points: Number, though it be not produced of an *Unit*, yet doth it conſiſt of Units, as a materiall cauſe. But formally, Number, is the Union and Unity of Units. Which uniting and knitting, is the workmanſhip of our mind: which, of diſtinct and diſcreet Units maketh a Number: by uniformity, reſulting

Number.

ting of a certain multitude of Units. And ſo,every number may have his leaſt part given: namely, an Unit: But not of a Magnitude, (no, not of a Line,) the leaſt part can be given: becauſe, infinitely, diviſion thereof, may be conceived. All Magnitude, is either a Line, a Plain, or a Solid. Which Line, Plain, or Solid, of no Senſe, can be perceived,nor exactly by hand (any way) repreſented: nor of Nature produced: But, as (by degrees) Number did come to our perceiverance: So by viſible forms, we are holpen to imagine, what our Line Mathematicall is. What our Point, is. So preciſe, are our Magnitudes, that one Line is no broader than an other: for they have no breadth: Nor our Plains have any thickneſſe. Nor yet our Bodies, any weight: be they never ſo large of dimenſion. Our Bodies, we can have ſmaller, than either Art or Nature can produce any: and greater alſo, then all the world can comprehend. Our leaſt Magnitudes, can be divided into ſo many parts, as the greateſt. As, a Liue of an inch long, (with us) may be divided into as many parts, as may the diameter of whole world, from Eaſt to Weſt: or any way extended: What priviledges, above all manuall Art, and Natures might, have our two Sciences Mathematicall? to exhibite, and to deal with things of ſuch power, liberty, ſimplicicity, purity, and perfection? And in them, ſo certainly, ſo orderly, ſo preciſely, to proceed: as, excellent, is that workman Mechanicall Judged, who neereſt can approach to the repreſenting of works, Mathematically demonſtrated? And our two Sciences, remaining pure, and abſolute in their proper terms, and in their own Matter, to have, and allow, onely ſuch Demonſtrations, as are plain, certain, univerſall, and of an æternall verity? This Science of *Magnitude*, his properties, conditions, and appurtenances: commonly, now is, and from the beginning, hath of all Philoſophers, been called *Geometry*. But verily, with a name too baſe and ſcant, for a Sci-

Geometry.

ence of ſuch dignity and ampleneſſe. And, perchance, that name, by common and ſecret conſent, of all wiſe men, hitherto hath been ſuffered to remain: that it might carry with it a perpetuall memory of the firſt and notableſt benefit, by that Science, to common people ſhewed: Which was, when Bounds and Meres of land and ground, were loſt, and confounded, (as in *Egypt*, yearly, with the overflowing of *Nilus*, the greateſt and longeſt river in the world) or, that ground bequeathed, were to be aſſigned: or ground ſold, were to be laid out: or (when diſorder prevailed) that Commons were diſtributed into ſeveralties. For, where, upon theſe and ſuch like occaſions, Some by ignorance, ſome by negligence; Some by fraud, and ſome by violence, did wrongfully limit, meaſure, encroach, or challenge (by pretence of juſt content and meaſure) thoſe lands and grounds: great loſſe, diſquietneſſe, murther, and war did (full oft) enſue: Till, by Gods mercy, aud mans induſtry, The perfect Science of Lines, Plains, and Solids (like a divine Juſticier,) gave unto every man his own. The people then, by this art pleaſured, and greatly relieved, in their lands juſt meaſuring: and other Philoſophers, writing rules for land meaſuring: between them both, thus, confirmed the name of *Geometria*, that is, (according to the very Etymologie of the word) Land meaſuring. Where the people knew no farther of Magnitudes uſe, but in Plains: and the Philoſophers of them had no fit hearers, or Scholers: farther to diſcloſe unto, then of flat, plain *Geometry*. And though, theſe Philoſophers, knew of farther uſe, and beſt underſtood the Etymologie of the word, yet this name *Geometria*, was of them applyed generally to all ſorts of Magnitudes: unleſſe otherwhile, of *Plato* and *Pythagoras*. When they would preciſely declare their own doctrine. Then was* *Geometria*, vvith them, *Studium quod circa planum verſatur*. But, vvell you may perceive by *Euclid's Elements*, that more ample is our Science, then

Plato 7 de Rep.

then to meaſure Plains: and nothing leſſe therein is taught (of purpoſe) then how to meaſure Land. Another name, therefore, muſt needs be had, for our Mathematicall Science of Magnitudes: which regardeth neither clod nor turff: neither hill, nor dale: neither earth nor heaven: but is abſolute *Megethelogia*, not creeping on ground, and daſling the eye, with pole, “ perch, rod or line: but lifting the heart above the “ heavens, by inviſible lines, & immortal beams: “ meeteth with the reflexions, of the light “ incomprehenſible: and ſo procureth joy, and perfection “ unſpeakable. Of which true uſe of our *Megethica*, or *Megethologia*, Divine *Plato* ſeemed to have good taſt and judgment: and (by the name of *Geometry*) ſo noted it: and warned his Scholers thereof: as, in his ſeventh *Dialog*, of the Common wealth may evidently be ſeen. Where (in Latin) thus it is; right well tranſlated: *Profectò, nobis hoc non negabunt, Quicunque vel paululum quid Geometriæ guſtârunt, quin hæc Scientia, contrà, omnino ſe habeat, quàm de ea loquuntur, qui in ipſa verſantur.* In Engliſh, thus. **Verily** (ſaith *Plato,*) **whoſoever hath (but even very little) taſted of Geometry, will not deny unto us, this: but that this Science, is of another condition, quite contrary to that, which they that are exerciſed in it, do ſpeak of it.** And there it followeth of our *Geometry. Quòd quæritur cognoſcendi illius gratia, quod ſemper eſt, non & ejus quod oritur quandoque & interit. Geometria, ejus quod eſt ſemper, Cognitio eſt. At tollet igitur (ô Generoſe vir) ad Veritatem, animum: atque ita, ad Philoſophandum preparabit cogitationem, ut ad ſupera convertamus: quæ, nunc, contra quàm decet, ad inferiora dejicimus, &c. Quàm maximè igitur præcipiendum eſt, ut qui præclariſſimam hanc habitant Civitatem, nullo modo, Geometriam ſpernant. Nam & quæ præter ipſius propoſitum, quodam modo eſſe videntur, haud exigua ſunt, &c.* It muſt needs be confeſſed (ſaith *Plato.*) **That** [Geometrie] **is learned, for the knowing of that which is ever: and** not of that, which in time, both is bred and is brought to an end, &c. Geometry is the knowledg of that which is everlaſting. It will lift up therefore (O Gentle Sir) our mind to the verity: and by that means, it will prepare the Thought, to the Philoſophicall love of wiſdome, that we may turn or convert, toward heavenly things, [both mind and thought] which now, otherwiſe than becommeth us, we caſt down on baſe or inferiour things, &c. chiefly, therefore Commandement muſt be given, that ſuch as do inhabite this moſt honourable City, by no means, deſpiſe Geometry. For even thoſe things [done by it] which, in manner, ſeem to be, beſide the purpoſe of Geometry: are of no ſmall importance, &c. And beſides the manifold uſes of *Geometry*, in matters appertaining to war, he addeth more, of ſecond unpurpoſed fruit, and commodity, ariſing by *Geometry*, ſaying: *Scimus quin etiam ad Diſciplinas omnes facilius per diſcendas intereſſe omnino, attigerit, ne Geometriam aliquis, an non, &c. Hanc ergo Doctrinam, ſecundo loco diſcendam Juvenibus ſtatuamus.* That is. But alſo, we know, that for the more eaſie learning of all Arts, it importeth much, whether one have any knowledg in Geometry, or no, &c. Let us therefore make an ordinance or decree, that this Science of young men ſhall be learned in the ſecond place. This was Divine *Plato* his Judgment, both of the purpoſed, chief, and perfect uſe of *Geometry*: and of his ſecond, depending, derivative commodities. And for us Chriſtian men, a thouſand thouſand more occaſions are to have need of the help of **Megethological* Contemplations: whereby to train our Imaginations and Minds by little and little to forſake and abandon, the groſſe and corruptible Objects, of our outward ſenſes: and to apprehend by ſure doctrine demonſtrative, Things Mathematicall. And by them readily to be holpen and conducted, to conceive, diſcourſe, and conclude of things Intellectuall, Spirituall, æternall, & ſuch as concern our

J. D.
Herein I would gladly ſhake of, the earthly name of Geometry.

Bliſſe everlaſting. : which, otherwiſe (without Speciall privilegd of Illumination, or Revelation from heaven) No mortall mans wit (naturally) is able to reach unto, or to Compaſſe. And, verily, by my ſmall Talent (from above) I am able to prove and teſtifie that the literall Text, and order of our divine Law, Oracles, and Myſteries, require more skill in Numbers, and Magnitudes: then (commonly) the Expoſitors have uttered: but rather onely (at the moſt) ſo warned: and ſhewed their own want therein. (To name any, is needleſſe: and to note the places, is, here, no place: But if I be duly askked, my anſwer is ready.) And without the literall, Gramaticall, Mathematicall, or Naturall verities of ſuch places, by good and certain Art, perceived no Spirituall ſenſe, proper to thoſe places, by Abſolute *Theologie*) will thereon depend. ☞ “ No man, therefore, can doubt, “ but toward the atteining of knowledg incomparable, “ and Heavenly Wiſdome: Mathematical Speculations “ both of Numbers and Magnitudes: are means, aids, “ and guides: ready, certain, and neceſſary. From henceforth, in this my Preface, will I frame my talk, to *Plato* his fugitive Scholers: or, rather, to ſuch, who well can, (and alſo will) uſe their outward ſenſes, to the glory of God, the benefit of their Countrey, and their own ſecret contentation, or honeſt preferment on this earthly Scaffold. To them, I will orderly recite, deſcribe and declare a great Number of Arts, from our two Mathematicall fountains, derived into the fields of *Nature*. Whereby ſuch ſeeds, and roots, as lie deep hid in the ground of *Nature*, are refreſhed, quickened, and provoked to grow, ſhoot up, flour, and give fruit, infinite and incredible. And theſe Arts ſhall be ſuch, as upon Magnitudes properties do depend more than upon Number. And by good reaſon we may call them Arts, and Arts Mathematicall Derivative, for (at this time) I Define **An art, to be a Methodicall compleat Doctrine, having abundancy of ſufficient and peculiar matter**

An Art.

to

deal with, by the allowance of the Metaphyſicall Philoſopher : the knowledg whereof, to humane ſtate is neceſſary. And that I account an Art Mathematicall derivative, which by Mathematicall demonſtrative Method, in Numbers or Magnitudes, ordereth and confirmeth his doctrine, as much and as perfectly as the matter ſubject will admit. And for that I intend, to uſe the name and propertie of a *Mechanician,* otherwiſe, then (hitherto it hath been uſed, I think it good (for diſtinction ſake) to give you alſo a brief deſcription, what I mean thereby. A Mechanician or a Mechanicall workman, is he whoſe ſkil is without knowledg of Mathematicall demonſtration, perfectly to work and finiſh any ſenſible work, by the Mathematicall principall or derivative, demonſtrated or demonſtrable. Full well I know, that he which inventeth or maketh theſe demomonſtrations, is generally called *A ſpeculative Mechanician*, which differeth nothing from a *Mechanicall Mathematician.* So, in reſpect of divers actions, one man may have the name of ſundry arts, as ſometime of a Logician, ſometimes, (in the ſame manner otherwiſe handled) of a Rhetorician. Of theſe trifles, I make (as now, in reſpect of my Preface,) ſmall account : to file them for the fine handling of ſubtile curious diſputers. In other places, they may command me, to give good reaſon : and yet, here, I will not be unreaſonable.

Art Mathematicall Derivative,

A Mechanician.

I. Firſt, then, from the purity, abſoluteneſſe, and Immateriality of Principall *Geometrie,* is that kind of *Geometrie* derived, which vulgarly is counted *Geometrie,* and is the Art of meaſuring ſenſible magnitudes, their juſt quantities and contents. This, teacheth to meaſure either at hand : and the Practiſer to be by the thing Meaſured : and ſo, by due applying of Compaſſe, Rule Square, Yard, Ell, Perch, Pole, Line, Gauging rod, (or ſuch like inſtrument) to the Length, Plain, or Solid meaſured, * to be certified, either of the length,

Geometry Vulgar

1.

perimetrie, or diſtance lineall : and this is called: *Me-*
cometry, Or * to be certified of the content of any 2.
plain Superficies : whether it be in ground Surveyed,
Board or Glaſſe meaſured, or ſuch like thing : which
meaſuring, is named, *Embadometrie*. * Or elſe to un- 3.
derſtand the Solidity and content of any bodily thing :
as of Timber and Stone, or the content of Pits, Ponds,
Wels, Veſſels, ſmall and great, of all faſhions. Where
of Wine, Oyl, Beer, or Ale Veſſels, &c. the Meaſuring
commonly hath a peculiar name, and is called *Gauging*.
And the generall name of theſe Solid meaſures is *Ste-*
reometry. Or elſe, this *Vulgar Geometry*, hath conſidera- 2.
on to teach the practiſer, how to meaſure things, with
good diſtance between him and the thing meaſured :
and to underſtand thereby, either * how farre, a 1.
thing ſeen (on land or water) is from the meaſurer :
and this may be called *Apomecometrie* : Or, how High, 2.
or deep, above or under the levell of the meaſurers
ſtanding, any thing is, which is ſeen on land or water,
called *Hypſometry*. * Or, it informeth the meaſurer, 3.
how broad any thing is, which is in the meaſurers view:
ſo it be on land or water ſituated: & may be called *Pla-*
tometry. Though I uſe here to condition the thing mea-
ſured, to be on land or water ſituated: yet, know for cer-
tain, that the ſundry height of Clouds, blazing Stars, *Note.*
and of the Moon, may (by theſe means) have their di-
ſtances from the earth : and, of the blazing Stars and
Moon, the ſolidity (as wel as diſtances) to be meaſured:
but becauſe, neither theſe things are vulgarly taught, nor
of a common practiſer ſo ready to be executed. I, ra-
ther let ſuch meaſures be reckoned incident to ſome of
our other Arts, dealing with things on high, more pur-
poſely, than this vulgar Land meaſuring Geometry
doth : as in *Perſpective* and *Aſtronomy*.

OF theſe Feats (farther applyed, is ſprung the Feat
of *Geodeſie*, or Land Meaſuring : more cunningly to
meaſure and ſurveigh Land, Woods, and Waters, a-
far

far off. More cunningly, I ſay: But God knoweth (hitherto) in theſe Realms of *England* and *Ireland*, (whether through ignorance or fraud, I cannot tell in every particular) how great wrong and injury hath (in my time) been committed by untrue meaſuring and ſurveying of Land or Woods, any way. And, this I am ſure: that the Value of the difference, between the truth and ſuch Surveys, would have been able to have found (for ever) in each of our two Univerſities, an excellent Mathematicall Reader: to each allowing (yearly) a hundred Marks of lawfull money of this Realm: which indeed, would ſeem requiſite, here to be had (though by other ways provided for) as well as the famous Univerſity of *Paris* hath two Mathematicall Readers, and each two hundred French Crowns yearly, of the French Kings magnificent liberality onely. Now, again, to our purpoſe, returning: Moreover, of the former knowledg Geometricall, are grown the skils of *Geographie*, *Chorographie*, *Hydrographie*, and *Stratarithmetry*.

Note.

GEOGRAPHIE teacheth ways, by which, in ſundry forms, (as *Sphærike*, *Plain*, or other) the Situation of Cities, Towns, Villages, Forts, Caſtles, Mountains, Woods, Havens, Rivers, Creeks, and ſuch other things, upon the out-face of the earthly Globe (either in the whole, or in ſome principall member and portion thereof conteined) may be deſcribed and deſigned, in commenſurations Analogicall to Nature and verity: and moſt aptly to our view, may be repreſented. Of this Art how great pleaſure, and hovv manifold commodities do come unto us daily and hourly: of moſt men is perceived. While ſome, to beautifie their Hals, Parlers, Chambers, Galeries, Studies or Libraries with: other ſome, for things paſt, as battles fought, earthquakes, heavenly firings, and ſuch occurents, in hiſtories mentioned: thereby lively, as it were, to view the place, the Region adjoyning, the diſtance from us: and ſuch other circumſtances. Some other preſently to view the large dominion of the Turk: the wide Empire of the Moſcovite: and the little morſell of ground, where Chriſtendome (by profeſſion) is certainly known. Little, I ſay, in reſpect of the reſt, &c. Some either, for their own journeys directing into far lands: or to underſtand of other mens travails. To conclude, ſome for one purpoſe, and ſome for another, liketh, loveth, getteth, and uſeth Maps, Charts, and Geographicall Globes. Of whoſe uſe, to ſpeak ſufficiently, would require a book peculiar.

CHOROGRAPHIE, ſeemeth to be an “underling, and a twig of *Geographie*: and yet neverthe“leſſe, is in practiſe manifold, and in uſe very ample. “This teacheth Analogically to deſcribe a ſmal portion “or circuit of ground, with the contents: not regardiug “what commenſuration it hath to the whole, or any “parcell, without it, conteined. But in the territory or “parcell of ground, which it taketh in hand to make “deſcription of, it leaveth out (or undeſcribed) no no“table, or odde thing, above the ground viſible. Yea, “and ſometimes, of things under ground, giveth ſome “peculiar mark, or warning: as of Metal-mines, Cole“pits, Stone Quaries, &c. Thus a Dukedome, a Shire, “a Lordſhip, or leſſe, may be deſcribed diſtinctly. But marvellous pleaſant and profitable it is, in the exhibiting to our eye and commenſuration, the plat of a City, Town, Fort, or Palace, in true Symmetry: not approaching to any of them: and out of Gun-ſhot, &c. Hereby the *Architect* may furniſh himſelf, with ſtore of what paterns he liketh: to his great inſtruction: even in thoſe things which outwardly are proportioned: either ſimply in themſelves: or reſpectively to Hils, Rivers, Havens, and Woods adjoyning. Some alſo, term this particular deſcription of places, *Topographie*.

HYDOGRAPHIE, delivereth to our know“ledg, on Globe or in Plain, the perfect Analogicall

“ deſcription, of the Ocean, Sea-coaſts, through the
“ whole world: or in the chief & principal parts thereof:
“ with the Iſles & chief perticular places of dangers, conteined within the bounds, and Sea-coaſts deſcribed: as of Quick-ſands, Banks, Pits, Rocks, Races, Countertides, Whirle-pools, &c. This dealeth with the Element of the water chiefly: as *Geographie* did principally take the Element of the Earths deſcription (with his appurtenances) to task. And beſides this, *Hydrographie*, requireth a perticular Regiſter of certain Land-marks (where marks may be had) from the ſea, well able to be skried, in what point of the Sea compaſſe they appear, and what apparent form, ſituation, and bigneſſe they have, in reſpect of any dangerous place in the ſea, or neer unto it, aſſigned: And in all Coaſts, what Moon maketh full Sea, and what way, the Tides and Ebbes, come and go, the *Hydrographer* ought to record. The ſoundings likewiſe: and the Chanels wayes: their number, aud dephts ordinarily, at ebbe and floud, ought the *Hydrographer*, by obſervation and diligence of *Meaſuring*, to have certainly known, And many other points, are belonging to perfect *Hydrographie*, and for to make a *Rutter*, by: of which, I need not here ſpeak: as of the deſcribing, in any place, upon Globe or Plain, the 32 points of the Compaſſe, truly: (whereof, ſcarcely four, in *England*, have right knowledg: becauſe, the lines thereof, are no ſtraight lines, nor Circles.) Of making due projection of a Sphære in plain. Of the Variation of the Compaſs, from true North. And ſuch like matters (of great importance, all) I leave to ſpeak of in this place, becauſe, I may ſeem (already) to have enlarged the bounds and duty of an Hydrographer, much more than any man (to this day) hath noted or preſcribed: yet am I well able to prove, all theſe things, to appertain, and alſo to be proper to the Hydrographer. The chief uſe and end of this Art, is the Art of Navigation, but it hath other diverſe uſes, even by them to be enjoyed, that never lack ſight of land.

STRATARITHMETRIE, is the skill (appertaining to the war,) by which a man can ſet in figure analogicall to any *Geometricall* figure appointed, any certain number or ſumme of men: of ſuch a figure capable: (by reaſon of the uſuall ſpaces between Souldiers allowed: and for that, of men, can be made no Fractions: yet nevertheleſſe, he can order the given ſumme of men, for the greateſt ſuch figure, that of them can be ordered) and certifie of the overplus (if any be) and of the next certain ſumme, which, with the overplus, will admit a figure exactly proportionall to the figure aſſigned. By which skill alſo, of any army or company of men: (the figure and ſides of whoſe orderly ſtanding, or array is known,) he is able to expreſſe the juſt number of men, within that figure conteined: or (orderly) able to be conteined. * And this figure, and ſides thereof, he is able to know: either being by, and at hand: or a farre off. Thus farre ſtretcheth the deſcription and property of *Stratarithmetrie*: ſufficient for this time and place. “ It differeth from the “ feat *Tacticall, De aciebus inſtruendis*, becauſe, there is “ neceſſary the wiſedome and foreſight, to what pur- “ poſe he ſo ordereth the men, and skilfull ability, alſo, “ for any occaſion, or purpoſe, to deviſe and uſe the “ apteſt & moſt neceſſary order,array and figure of his “ Company and Sum of men.By figure,I mean,as either “ of a *Perfect Square*,*Triangle*,*Circle*, *Ovale*, *long ſquare*, (of the Greeks it is called *Eteromekes*) *Rhombe*, *Rhomboid Lunular*, *Ring*, *Serpentine*, and ſuch other Geometricall figures: Which in wars, have been, and are to be uſed for commodiouſneſſe, neceſſity and advantage, &c. And no ſmall ſkill ought he to have, that ſhould make true report, or neer the truth of the numbers and ſummes, of footmen or horſemen, in the Enemies ordering. A farre off, to make an eſtimate, between neer terms of More and Leſſe, is not a thing very rife, among thoſe that gladly would do it. Great pol-

* *Note.*

The difference between Stratarithmetry, and Tacticie.

J. D.
Friend, You will finde it hard, to performe my deſcription of this feat. But by Chorography, you may help your ſelf ſomwhat: where the figures known (in Sides and Angles) are not regular: And where, Reſolution, into Triangles can ſerve, &c. And yet you will finde it ſtrange to deale thus Generally with Arithmeticall figures: and, that for Battail ray. Their Contents, differ ſomuch from like Geometricall figures.

licy may be uſed of the Captains, (at times fit, and in places convenient) as to uſe figures, which make greateſt ſhew, of ſo many as he hath: & uſing the advantage of the three kinds of uſuall ſpaces: (between footmen or horſemen) to take the largeſt: or when he would ſeem to have few, (being many:) contrariwiſe in Figure and ſpace. The Herald, Purſuivant, Serjeant Royall, Captain, or whoſoever is carefull to come neer the truth herein, beſides the Judgment of his expert eye, his skill of Ordering *Tacticall*, the help of his Geometricall inſtrument: Ring or Staffe Aſtronomicall: commodiouſly framed for carriage and uſe.) He may wonderfully help himſelf by perſpective Glaſſes. In which, (I truſt) our poſterity will prove more skilfull and expert, and to greater purpoſes, than in theſe dayes, can (almoſt) be credited to be poſſible.

Thus have I lightly paſſed over the Artificiall Feats, chiefly depending upon vulgar *Geometry*: and commonly, and generally reckoned under the name of *Geometry*. But there are other (very many) *Methodicall Arts*, which, declining from the purity, ſimplicity, and Immateriality, of our Principall Science of *Magnitudes*: do yet nevertheleſſe uſe the great ayd, direction, and Method of the ſaid principall Science, and have proper names, and diſtinct: both from the Science of *Geometry*, (from which they are derived) and one from the other. As PERSPECTIVE, ASTRONOMY, MUSICK, COSMOGRAPHIE, ASTROLOGY, STATIKE, ANTHROPOGRAPHIE, TROCHILIKE, HELICOSOPHIE, PNEUMATITHMIE, MENADRIE, HYPOGEIODIE, HYDRAGOGIE, HOROMETRIE, ZOGRAPHIE, ARCHITECTURE, NAVIGATION, THAUMATURGIKE, *and* ARCHEMASTRIE. I think it neceſſary, orderly, of theſe to give ſome peculiar deſcriptions: and withall, to touch ſome of their commodious uſes, and ſo to make this Preface, to be a little ſweet, pleaſant Noſegay for you, to comfort your Spirits, being almoſt out of courage, and in deſpair, (through brutiſh brute) Weening that *Geometry*, had but ſerved for building of an houſe, or a curious bridg, or the roof of Weſtminſter hall, or ſome witty pretty deviſe, or engin, appropriate to a Carpenter, or a Joyner, &c. That the thing is far otherwiſe, than the world, (commonly) to this day, hath deemed, by word and work, good proof will be made.

Among theſe Arts, by good reaſon, PERSPECTIVE ought to be had, e're of *Aſtronomicall Apparences*, perfect knowledg can be atteined. And becauſe of the prerogative of *Light*, being the firſt of *Gods Creatures*: and the eye, the light of our body, and his ſenſe moſt mighty, and his organ moſt artificiall, and *Geometricall*. At *Perſpective*, we will begin therefore. **Perſpective is an Art Mathematicall, which demonſtrateth the manner and properties of all Radiations, Direct, Broken, and Reflected.** This Deſcription, or Notation, is brief: but it reacheth ſo far, as the world is wide. It concerneth all Creatures, all Actions, and paſſions, by Emanation of beams performed. Beams or naturall lines, (here) I mean, not of light onely, or of colour (though they, to eye, give ſhew, witneſſe, and proof, whereby to ground the art upon) but alſo of other *Forms*, both *Subſtantiall* and *Accidentall*, the certain and determined Radiall Emanations. By this art (omitting to ſpeak of the higheſt points) we may uſe our eyes, and the light, with greater pleaſure, and perfecter Judgment: both of things, in light ſeen, and of other: which by like order of Lights Radiations, work and produce their effects. We may be aſhamed to be ignorant of the cauſe, why ſo ſundry ways our eye is deceived, and abuſed: as, while the eye weeneth a round Globe or Sphære (being far off) to be a flat and plain Circle, and ſo likewiſe judgeth a plain Square to be a round: ſuppoſeth walls parallels, to approach, a far off: roof and floure parallels, the one to bend downward, the other to riſe upward, at a little diſtance from you. Again, of things being in like ſwiftneſſe of moving to think the neerer to move faſter,and the farther much ſlower. Nay, of two things, whereof the one (incomparably) doth move ſwifter than the other, to deem the ſlower to move very ſwift, and the other to ſtand: What an errour is this, of our eye? Of the Rainbow, both of his Colours, of the order of the colours, of the bigneſſe of it, the place and height of it, (&c.) to know the cauſes demonſtrative, is it not pleaſant, is it not neceſſary? of two or three Sunnes appearing: of blazing Stars: and ſuch like things: by naturall cauſes, brought to paſſe, (and yet nevertheleſſe,of farther matter Significative) it is not commodious for man to know the very true cauſe and occaſion Naturall? Yea, rather is it not, greatly, againſt the Soveraignty of Mans nature,to be ſo overſhot and abuſed with things (at hand) before his eyes? as with a Peacocks tail, and a Doves neck: or a whole ore, in water holden, to ſeem broken. Things far off to ſeem neer, and neer, to ſeem far off. Small things to ſeem great, and great to ſeem ſmall. One man to ſeem an Army. Or a man to be curſtly afraid of his own ſhadow. Yea, ſo much, to fear, that if you being (alone, neer a certain glaſſe, and proffer with dagger or ſword, to foyne at the glaſſe, you ſhall ſuddenly be moved to give back (in manner) by reaſon of an Image appearing in the air, between you and the glaſſe, with like hand, ſword or dagger, and with like quickneſſe foyning at your eye, likewiſe as you do at the Glaſſe. Strange, this is to hear off, but more mervailous to behold, than theſe my words can ſignifie. And nevertheleſſe, by demonſtration Opticall, the order, and cauſe thereof, is certified: even ſo as the effect is conſequent. Yea, thus much more, dare I take upon me, toward the ſatisfying of the noble courage, that longeth ardently for the wiſedome of Cauſes Naturall:

A marveilous glaſſe ☞.

as to let him underſtand, that, in *London,* he may with his own eyes, have proof of that, which I have ſaid herein. A Gentleman, (which, for his good ſervice, *S.W.P.* done to his Countrey, is famous and honourable: and for skill in the Mathematicall Sciences, and Languages, is the Odde man of this land, &c.) even he is able: and (I am ſure) will, very willingly, let the Glaſſe, and proof be ſeen: and ſo I (here) requeſt him: for the encreaſe of wiſedome, in the honourable: and for the ſtopping of the mouths malicious: and repreſſing the arrogancy of the ignorant: ye may eaſily gueſſe, what I mean. This Art of *Perſpective* is of that excellency, and may be led, to the certifying, and executing of ſuch things, as no man would eaſily believe: without Actuall proof perceived. I ſpeak nothing of *Naturall Philoſophie,* which, without *Perſpective,* cannot be fully underſtood, nor perfectly atteined unto. Nor of *Aſtronomy:* which without *Perſpective,* cannot wel be grounded. Nor *Aſtrology,* naturally verified, and avouched. That part hereof, which dealeth with Glaſſes (which name Glaſſe, is a generall name, in this Art, for any thing, from which a Beam reboundeth) is called *Catoptrike* and hath ſo many uſes, both marvellous, and profitale: that, both, it would hold me too long, to note therein the principal concluſions, already known: And alſo (perchance) ſome things might lack due credit with you: And I, thereby to loſe my labour: and you, to ſlip into light Judgment. * Before you have learned ☞ ſufficiently the power of Nature and Art.

NOw to proceed: ASTRONOMIE, is an Art Mathematicall, which demonſtrateth the diſtance, magnitudes, and all naturall motions, apparences, and paſſions proper to the Planets and fixed Stars, for any time paſt, preſent, and to come: in reſpect of a certain Horizon, or without reſpect of any Horizon. By this art we are certified of the diſtance of the ſtarry skie, and of each *Planet,* from the Center of the earth, and of the greatneſſe of any Fixed Star ſeen, or *Planet*, in reſpect of the earths greatneſſe. As, we are ſure (by this Art) that the Solidity, Maſſineſſe, and Body of the *Sun*, conteineth the quantity of the whole Earth, and Sea, a hundred threeſcore and two times, leſſe by $\frac{1}{8}$ one eight part of the earth, but the body of the whole earthly Globe and Sea, is bigger than the body of the Moon, three and forty times leſſe by $\frac{1}{8}$ of the Moon. Wherefore the *Sun* is bigger than the *Moon*, 7000 times, leſſe by $59\frac{39}{64}$ that is, preciſely $6940\frac{25}{64}$ bigger than the *Moon*. And yet the unskilful man, would judg them a like big. Wherefore, of neceſſity, the one is much farther from us than the other. The *Sun* when he is fartheſt from the earth (which, now, in our age, is, when he is in the 8 degree, of Cancer) is 1179 Semidiameters of the Earth, diſtant. And the *Moon* when ſhe is fartheſt from the earth, is 68 Semidiameters of the earth and $\frac{1}{3}$. The neereſt, that the *Moon* commeth to the Earth, is Semidiameters $52\frac{1}{4}$. The diſtance of the ſtarry skie is from us, in Semidiameters of the earth $20081\frac{1}{2}$. Twenty thouſand fourſcore, one, and almoſt a half. Subſtract from this, the *Moons* neereſt diſtance, from the Earth: and thereof remaineth Semidiameters of the earth $20029\frac{1}{4}$. Twenty thouſand nine and twenty and a quarter. So thick is the heavenly Palace, that the *Planets* have all their exerciſe in, and moſt mervailouſly perform the Commandement and Charge to them given by the omnipotent Majeſty of the King of kings. This is that, which in *Geneſis* is called *Ha Rakia*. Conſider it wel. The Semidiameter of the earth, conteineth of our common miles $3436\frac{4}{11}$ three thouſand, four hundred thirty ſix and four eleventh parts of one mile. Such as the whole earth and Sea, round about, is 21600. One and twenty thouſand ſix hundred of our miles. Allowing for every degree of the greateſt circle, threeſcore miles. Now if you weigh well with your ſelf but this little parcell of fruit *Aſtronomicall*, as concerning the bigneſſe, Diſtances of *Sun*,

Note.

Moon, Starry Skie, and the huge maſſineſſe of *Ha Rakia* will you not finde your conſciences moved, with the Kingly Prophet, to ſing the confeſſion of Gods Glory, and ſay, The Heavens declare the glory of God, & the Firmament [Ha Rakia] ſheweth forth the works of his hands. And ſo forth, for thoſe five firſt ſtaves of that kingly Pſalm. Well, well, It is time for ſome, to lay hold on wiſedome, and to Judg truly of things: and not ſo to expound the Holy Word, all by Allegories: as to Neglect the Wiſdom, Power, and Goodneſſe of God, in, and by his Creatures, and Creation to be ſeen and learned. By Parables and Analogies of whoſe natures and properties, the courſe of the Holy Scripture, alſo, declareth to us very many Myſteries. The whole Frame of Gods Creatures, (which is the whole world) is to us, a bright glaſſe: from which, by reflexion, reboundeth to onr knowledg and perceiverance, Beams, and Radiations: repreſenting the Image of his Infinite Goodneſſe, Omnipotency, and wiſdome. And we thereby are taught and perſwaded to Glorifie our Creator, as God: and be thankfull therefore. Could the Heathens finde theſe uſes, of theſe moſt pure, beautifull and Mighty Corporall Creatures: and ſhall we, after that the true *Sonne* of righteouſneſſe is riſen above the *Horizon* of our temporall *Hemiſphære*, and hath ſo abundantly ſtreamed into our hearts the direct beams of his goodneſſe, mercy, and grace: Whoſe heat All Creatures feel: Spirituall and Corporall: Viſible and Inviſible: Shall we (I ſay) look upon the *Heaven, Stars* and *Planets*, as an Oxe, and an Aſſe doth: no further, careful or inquiſitive, what they are: why were they created: how do they execute that they were created for? Seeing, all Creatures were for our ſake created: & both we, and they, created chiefly to glorifie the Almighty Creator: & that by all means to us poſſible. *Nolite ignorare* (saith *Plato in Epinomis*,) *ſapientiſſimum quiddam eſſe.* Be ye not ignorant Aſtronomie to be a thing of excellent Wiſedome. *Aſtronomy* was to us from the beginning commended, and in manner commanded by God himſelf. In aſmuch as he made the *Sun, Moon* and *Stars.* to be to us for *Signes,* and knowledg of ſeaſons, and for Diſtinctions of days and years. Many words need not. But I wiſh,every man ſhould weigh this word *Signes.* And beſides that, conferre it alſo with the tenth Chapter of *Jeremie.* And though ſome think, that there they have found a rod: Yet modeſt reaſon, will be indifferent Judg, who ought to be beaten therewith, in reſpect of our purpoſe. Leaving that: I pray you underſtand this: that without great diligence of Obſervation, Examination and Calculation,their periods and courſes (whereby *Diſtinction* of ſeaſons, years and New Moons might preciſely be known) could not exactly be certified. Which thing to perform, is that Art, which we here have defined to be *Aſtronomie.* Whereby we may have the diſtinct courſe of times, dayes, years and ages: aſwell for Conſideration of ſacred Propheſies, accompliſhed in due time, foretold: as for high Myſticall Solemnities holding: And for all other humane affairs, Conditions, and Covenants upon certain time, between man and man, with many other great uſes: Wherein, (verily,) would be great incertainty, Confuſion, untruth, and brutiſh Barbarouſneſſe: without the wonderfull diligence and skill of this Art: continually learning and determining Times, and periods of Time, by the Record of the heavenly book, wherein all times are written, and to be read with an *Aſtronomicall ſtaffe,* inſtead of a feſtue,

MUSICK, of Motion, hath his Originall cauſe. Therefore, after the Motions moſt ſwift, and moſt ſlow, which are in the Firmament, of Nature performed: and under the *Aſtronomers Conſideration:* now I will ſpeak of another kind of *Motion,* producing ſound, audible, and of Man numberable. *Muſick* I call here that *Science,*which of the Greeks is called *Harmonice.* Not medling

ſing with the Controverſie between the ancient *Harmoniſts*
and *Canoniſts*. Muſick is a Mathematicall Science,
which teacheth by ſenſe and reaſon, perfectly to
judg, and order the diverſities of ſounds high and
low. *Aſtronomy* and *Muſick* are Siſters, ſaith *Plato*. As
for *Aſtronomy*, the eyes: ſo for *Harmonious Motion*,
the ears vvere made. But as *Aſtronomy* hath a more divine
Contemplation and commodity, than mortall eye
can perceive: ſo is *Muſick* to be conſidered, that the
* Mind may be preferred before the ear. And from 1.
audible ſound, we ought to aſcend, to the examination:
vvhich numbers are *Harmonious*, and vvhich not. And
vvhy, either, the one are: or the other are not. I could
at large, in the heavenly * motions and diſtances, de- 2.
ſcribe a marvellous Harmony of *Pythagoras* Harp, vvith
eight ſtrings. Alſo ſomewhat might be ſaid * *Mercurius* 4. 3.
two Harps, each of four ſtrings Elementall. And very
ſtrange matter, might be alledged of the *Harmony*, to
our * ſpirituall part appropriate. As in *Ptolomæus* third 5.
book, in the fourth and ſixt Chapters may appear.
* And vvhat is the cauſe of the apt bond, and friendly 6.
fellowſhip, of the Intellectuall and Mentall part of us,
vvith our groſſe and corruptible body, but a certain
Mean, and *Harmonious Spirituality, with both participating,
and of both (in a manner) reſulting? In the * Tune* 7.
*of Mans voyce, and alſo * the ſound of Inſtrument*, vvhat 8.
might be ſaid of *Harmonie*: No common Muſician
vvould lightly believe. But of the ſundry Mixture (as I
may term it) and concurſe, diverſe collation and application
of theſe *Harmonies*: as of three, four, five or
more: Marvellous have the effects been: and yet may
be found and produced the like: vvith ſome proportionall
conſideration for our time and being: in reſpect
of the ſtate, of the things then: in vvhich, and by
vvhich, the vvondrous effects vvere vvrought. *Democritus*
and *Theophraſtus* affirmed, that by *Muſick*, griefs
and diſeaſes of the mind and body might be cured or

J. D.
Read in *Ariſtotle* his 8 Book of Politicks the 5. 6. and 7. Chapters. Where you ſhall have ſome occaſion farther to think of Muſick, then commonly is thought.

inferred. And we finde in Record, that *Terpander, Arion, Iſmenias, Orpheus, Amphion, David, Pythagoras, Empedocles, Aſclepiades,* and *Timothæus* by *Harmonicall* Conſonancy, have done, and brought to paſſe, things more than marvellous to hear of. Of them then, making no farther diſcourſe in this place. Sure I am, that Common *Muſick*, commonly uſed, is found to the *Mnſicians* and Hearers,to be ſo Commodious and pleaſant, That if I would ſay and diſpute but thus much: That it were to be otherwiſe uſed, then it is, I ſhould finde more reprievers, then I could finde privy or skilfull of my meaning. In things therefore evident, and better known, than I can expreſſe: and ſo allowed and liked of, (as I would wiſh ſome other things had the like hap) I will ſpare to enlarge my lines any farther, but conſequently follow my purpoſe.

Of COSMOGRAPHIE, I appointed briefly in this place, to give you ſome intelligence. **Coſmographie is the whole and perfect deſcription of the heavenly and alſo elementall part of the world, and their homologall application, and mutuall collation neceſſary.** This Art requireth *Aſtronomy, Geographie Hydrographie* and *Muſick*. Therfore, it is no ſmall Art, nor ſo ſimple, as in common practiſe, it is (ſlightly) conſidered. This matcheth Heaven, and the Earth in one frame, and aptly applyeth parts correſpondent: So, as, the Heavenly Globe, may (in practiſe) be duly deſcribed upon the Geographicall and Hydrographicall Globe, And there, for us to conſider an *Æquinoctiall Circle, an Ecliptique line, Colours Poles, Stars* in their true Longituds, Latitudes, Declinations and Verticality: alſo Climes and Parallels: and by an *Horizon* annexed, and revolution of the earthly Globe (as the Heaven, is by the *Primovant*, carried about in 24 æquall Hours) to learn the Riſings and Settings of Stars (of *Virgil* in his *Georgicks*, of *Heſiod*: of *Hippocrates* in his *Medicinall Sphære*, to *Perdicca* King of the *Macedonians* : of *Diocles*, to King *Antigonus*, and of other famous *Philoſophers* preſcribed) a thing neceſſary, for due manuring of the earth, for *Navigation*, for the Alteration of mans body : being whole, ſick, wounded, or bruiſed, By the Revolution, alſo, or moving of the Globe Coſmographicall, the Riſing and Setting of the Sun : the Lengths of days and nights : the Hours and times (both night and day) are known : with very many other pleaſant and neceſſary uſes : Whereof, ſom are known : but better remain, for ſuch to know and uſe : who of a ſpark of true fire, can make a wonderful bonfire, by applying of due matter, duly. ☞

Of ASTROLOGIE, here I make an Art, ſeverall from *Aſtronomy* : not by new deviſe, but by good reaſon and authoritie : for, **Aſtrologie is an Art Mathematicall, which demonſtrateth reaſonably the operations and effects, of the naturall beams, of light, and ſecret influence : of the Stars and Planets : in every element and elementall body : at all times in any Horizon aſſigned.** This art is furniſhed with many other great arts and experiences : As with perfect *Perſpective*, *Aſtronomy*, *Coſmographie*, *Naturall Philoſophie*, of the 4 Elements, the Art of Graduation, and ſome good underſtanding in *Muſick*, & yet moreover, with another great Art, hereafter following, though I, here, ſet this before, for ſome conſiderations me moving. Sufficient (you ſee) is the ſtuffe, to make this rare and ſecret art, of: & hard enough to frame to the concluſion Syllogiſticall : yet both the manifold and continuall travails of the moſt ancient and wiſe Philoſophers, for the atteining of this art : and by examples of effects, to confirm the ſame : hath left unto us ſufficient proof and witneſs, and we alſo daily may perceive, That mans body, and all other Elementall bodies are altered, diſpoſed, ordered, pleaſured and diſpleaſured, by the Influential working of the *Sun*, *Moon*, and other Stars and Planets. And herefore, ſaith *Ariſtotle*, in the firſt of his *Meteorologi-*

call books, in the ſecond Chapter : *Eſt autem neceſſario Mundus iſte ſupernis lationibus ferè continuus. Ut, inde, vis ejus Univerſa regatur. Ea ſiquidem cauſa prima putanda omnibus eſt, unde motus principium exiſtit.* That is : This [Elemental] World is of neceſſity, almoſt next adjoyning, to the heavenly motions : That from thence all his vertue or force may be governed. For that is to be thought the firſt Cauſe unto all : from which, the beginning of motion is. And again, in the tenth Chapter. *Oportet igtur, & horum principia ſumamus, & cauſas omnium ſimiliter. Principium igitur ut movens præcipuumq; & omnium primum, Circulus ille eſt, in quo manifeſte Solis latio, &c.* And ſo forth. His *Meteorologicall* books, are full of arguments, and effectuall demonſtrations, of the vertue, operation, and power of the heavenly bodies, in and upon the four Elements, and other bodies of them (either perfectly or unperfectly) compoſed. And in his ſecond book, *De generatione & Corruptione.* in the tenth Chapter. *Quo circa & prima latio, Ortus & Interitus cauſa non eſt : Sed obliqui Circuli latio : ea namque & continua eſt ., & duobus motibus fit.* In Engliſh thus. Wherefore the uppermoſt motion is not the cauſe of Generation and corruption, but the motion of the Zodiack, for that both is continuall, and is cauſed of two movings. And in his ſecond book, and ſecond Chapter of his *Phiſicks. Homo namque generat hominem, atq; Sol..* For Man (ſaith he) and the Sun, are cauſe of mans generation. Authorities may be brought very many : both of 1000. 2000. yea and 3000 years Antiquity : of great *Philoſophers, Expert, Wiſe,* and godly men, for that Concluſion : which, daily and hourly, we men, may diſcern and perceive by ſenſe and reaſon. All beaſts do feel, and ſimply ſhew by their actions and paſſions, outward and inward. All Plants, Herbs, Trees, Flowers and Fruits, And finally, the Elements, and all things of the Elements compoſed, do give Teſtimony (as *Ariſtotle* ſaid) that their Whole diſpoſitions, vertues, and naturall motions, depend of the Activity of the heavenly Motions and Influences. Wherby, beſides the ſpecificall order and form, due to every ſeed: and beſide the Nature, proper to the Individuall Matrix, of the thing produced: What ſhall be the heavenly Impreſſion, the perfect and circumſpect Aſtrologian hath to conclude. Not onely (by *Apoteleſmes*) τὸ ὅτι, but by Naturall and Mathematicall demonſtration τὸ διότι. Whereunto, what Sciences are requiſite (without exception) I partly have here warned: And in my *Propædeumes* (beſide other matter there diſcloſed) I have Mathematically furniſhed up the whole Method. To this our age, not ſo carefully handled by any, that ever I ſaw, or heard of, I was (for * 21 years ago) by certain earneſt diſputations, of the Learned *Gerardus Mercator*, & *Antonius Gogava*, (& other) thereto ſo provoked: and (by my conſtant and invincible zeal to the verity) in obſervations of heavenly Influences (to the Minute of time,) than ſo diligent: And chiefly by the Supernaturall influence, from the Star of *Jacob*, ſo directed: That any modeſt and ſober Student, carefully and diligently ſeeking for the Truth, will both finde and confeſſe, therein, to be the Verity, of theſe my words: and alſo become a Reaſonable Reformer, of three ſorts of people: about theſe Influentiall operations, greatly erring from the truth. Whereof the one is Light Believers, the other Light Deſpiſers, and the third Light Practiſers. The firſt, and moſt common ſort, think the Heaven and Stars, to be anſwerable to any their doubts or deſires: which is not ſo: and, indeed, they, too much, over-reach. The ſecond ſort think no Influentiall vertue (from the heavenly Bodies) to bear any ſway in Generation and Corruption, in this Elementall World. And to the *Sun*, *Moon* and *Stars* (being ſo many, ſo pure, ſo bright, ſo wonderfull big, ſo far in diſtance, ſo manifold in their motions, ſo conſtant in their periods, &c.) they aſſign a ſlight, ſimple office or two, and

* *Anno* 1548 and 1549 in *Louain*.

Note.

1.

2.

and ſo allow unto them (according to their capacities) as much vertue, and power Influentiall, as to the Signe of the *Sun*, *Moon*, and ſeven Stars, hanged up (for Signs) in *London*, for diſtinction of houſes, and ſuch groſſe helps, in our worldly affairs: And they underſtand not (or will not underſtand) of the other workings, and vertues of the Heavenly *Sun*, *Moon* and *Stars*: not ſo much, as the Mariner or Husbandman: no, not ſo much, as the *Elephant* doth, as the *Cynocephalus*, as the *Porpentine* doth: nor will allow theſe perfect and incorruptible mighty bodies, ſo much vertuall Radiation and Force, as they ſee in a little piece of a *Magnes ſtone*: which, at great diſtance, ſheweth his operation. And perchance they think, the Sea and Rivers (as the Thames) to be ſome quick thing, and ſo to ebbe and flow, run in and out, of themſelves, at their own fantaſies. God help, God help. Surely theſe men come too ſhort: and either are too dull: or wilfully blind: or, perhaps too malicious, 3. The third man is the common and vulgar *Aſtrologian*, or Practiſer, who being not duly, artificially and perfectly furniſhed: yet, either for vain glory, or gain: or like a ſimple Dolt, and blind Bayard, both in matter and manner erreth: to the diſcredit of the *Wary* and modeſt *Aſtrologian*: and to the robbing of thoſe moſt noble corporall Creatures, of their Naturall Vertue: being moſt mighty: moſt beneficiall to all elementall Generation, Corruption and the appurtenances: and moſt Harmonious in their Monarchie: For which things being known, and modeſtly uſed: we might highly and continually glorifie God, with the princely Prophet, ſaying. The Heavens declare the Glory of God: who made the Heavens in his wiſedome: who made the Sun for to have dominion of the day: the Moon and Stars to have dominion of the night: whereby Day to day uttereth talk and night to night declareth knowledg. Praiſe him, all ye Stars and Light. Amen.

IN order, now followeth, of STATIKE, ſomewhat to ſay what we mean by that name: and what commodity, doth, on ſuch Art, depend. **Statike is an Art Mathematicall, which demonſtrateth the cauſes of heavineſſe and lightneſſe of all things: and of motions and properties, to heavineſſe and lightneſſe belonging.** And foraſmuch as, by the Bilanx, or Ballance (as the chief ſenſible Inſtrument,) Experience of theſe demonſtrations may be had: we call this Art *Statike*: that is, *the Experiments of the Ballance.* Oh, that men wiſt, what profit (all manner of ways) by this Art might grow, to the able examiner, & diligent practiſer. “ Thou onely, “ knoweſt all things preciſely (O God) who haſt made “ weight & Ballance, thy Judgment: who haſt created all “ things in *Number*, *Weight and Meaſure*: and haſt “ weighed the mountains and hils in a Ballance: who “ haſt peyſed in thy hand, both heaven and earth. We “ therefore warned by the Sacred Word, to conſider thy “ Creatures: and by that conſideration, to win a glimps “ (as it were,) or ſhadow of perceiverance, that thy “ wiſdome might, and goodneſſe is infinite, and unſpea“ kable, in thy Creatures declared: And being farther “ advertiſed, by thy mercifull goodneſſe, that, three “ principall ways, were of thee, uſed in Creation of all “ thy Creatures, namely *Number*, *Weight*, and *Meaſure*, “ And for as much as, of *Number* and *Meaſure*, the two “ Arts (ancient, famous, and to humane uſes moſt ne“ ceſſary) are, all ready, ſufficiently known and extant: “ This third key, we beſeech thee (through thy accuſto“ med goodneſſe) that it may come to the needfull and “ ſufficient knowledg, of ſuch thy Servants, as in thy “ workmanſhip, would gladly finde, thy true occaſions “ (purpoſely of thee uſed) whereby we ſhould glorifie “ thy name, and ſhew forth (to the weaklings in faith) “ thy wondrous Wiſedome and Goodneſſe. Amen.

Marvell nothing at this pang (godly friend, you gentle and zealous Student.) Another day, perchance you will perceive, what occaſion moved me. Here, as now, I will give you ſome ground, and withall ſome ſhew, of certain commodities, by this Art ariſing. And becauſe this Art is rare, my words and practiſes might be too dark: unleaſt you had ſome light holden before the matter: and that, beſt will be, in giving you, out of *Archimedes* demonſtrations, a few principall Concluſions, as followeth.

1

The Superficies of every Liquor, by it ſelf conſiſting, and in quiet, is Sphæricall: the center whereof, is the ſame, which is the center of the Earth.

2

If Solid Magnitudes, being of the ſame bigneſſe, or quantity, that any Liquor is, and having alſo the ſame Weight: be let down into the ſame Liquor, they will ſettle downward, ſo, that no part of them, ſhall be above the Superficies of the Liquor: and yet neverthele ſſe, they will not ſink utterly down, or drown.

3

If any Solid Magnitude being Lighter than a Liquor, be let down into the ſame Liquor, it will ſettle down, ſo far into the ſame Liquor, that ſo great a quantity of that Liquor, as is the part of the Solid Magnitude, ſetled down into the ſame Liquor: is in Weight, equall, to the Weight of the whole Solid Magnitude.

4

Any Solid Magnitude, Lighter than a Liquor, forced down into the ſame Liquor, will move upward, with ſo great a power, by how much, the Liquor having equall quantity to the whole Magnitude, is heavier than the ſame Magnitude.

5

Any Solid Magnitude, heavier than a Liquor, being let down into the ſame Liquor, will ſinke down utterly : And will be in that Liquor, Lighter by ſo much, as is the weight or heavines of the Liquor, having bigneſſe or quantity equall to the Solid Magnitude.

6

If any Solid Magnitude, Lighter than a Liquor, bee let down into the ſame Liquor, the weight of the ſame Magnitude, will be, to the Weight of the Liquor. (Which is equall in quantity to the whole Magnitude,) in that proportion, that the part, of the Magnitude ſetled down is to the whole Magnitude.

J.D. The cutting of a Sphære according to any proportion aſſigned, may by this Propoſition be done Mechanically by tempering Liquor to a certain weight in reſpect of the weight of the Sphære therein ſwimming.

BY theſe verities, great errours may be reformed in opinion of the Naturall Motion of things Light, and Heavy. Which errours are in Naturall Philoſophy (almoſt) of all men allowed : to much truſting to Authority, and falſe Suppoſitions. As, Of any two bodies, the heavier to moue downward faſter than the lighter This errour is not firſt by me, Noted : but by one *John*

A common errour noted.

A Paradox.

Baptiſt de Benedictis. The chief of his Propoſitions, is this: which ſeemeth a Paradox.

If there be two bodies of one form, and of one kind, equall in quantity or unequall, they will move by equall ſpace, in equall time: So that both their movings be in ayr, or both in water: or in any one Middle.

N.T. The wonderfull uſe of theſe Propoſitions.

Hereupon, in the feat of GUNNING, certain good diſcourſes (otherwiſe) may receive great amendment and furtherance, In the intended purpoſe, alſo, allowing ſomewhat to the imperfection of Nature: not anſwerable to the preciſeneſſe of demonſtration. Moreover, by the foreſaid propoſitions (wiſely uſed.) The Ayr, the Water, the Earth, the Fire, may be neerly known, how light, how heavy they are (Naturally) in their aſſigned parts: or in the whole. And then to things Elementall, turning your practiſe: you may deal for the proportion of the Elements, in the things Compouuded. Then to the proportions of the Humours in Man: their weights, and the weight of his bones, and fleſh, &c. Then, by weight, to have conſideration of the Force of man, any manner of way: in whole or in part. Then may you of Ships water drawing, diverſly in the Sea and in freſh water, have pleaſant conſideration: and of weighing up of any thing, ſunken in Sea, or in freſh water &c. And (to lift up your head aloft:) by weight, you may as preciſely, as by any Inſtrument elſe, meaſure the Diameters of *Sun* and *Moon*, &c. Friend, I pray you weigh theſe things, with the juſt Ballance of Reaſon. And you will finde marvels upon marvels: And eſteem one Drop of Truth (yea in Naturall Philoſophy) more worth than whole Libraries of Opinions undemonſtrated: or not anſwering to Natures Law, and your experience. Leaving theſe things, thus: I will give you two or three light practiſes to great purpoſe: and ſo finiſh my Annotation *Staticall*. In Mathematicall matters, by the Mechanicians aid, we will behold here the Commodity of weight. Make a Cube of any one Uniform: and through like heavy ſtuffe: of the ſame ſtuffe make a Sphær or Globe preciſely, of a Diameter equall to the Radicall ſide of the Cube. Your ſtuffe, may be Wood, Copper, Tinne, Lead, Silver, &c. (being as I ſaid of like nature, condition, and like weight throughout.) And you may by Say Ballance have prepared a great number of the ſmalleſt weights: which by thoſe Ballance can be diſcerned or tried: and ſo, have proceeded to make you a perfect Pyle, company and Number of weights: to the weight of ſix, eight, or twelve pound weight, moſt diligently tried, all and of every one, the Content known, in your leaſt weight that is weighable. [They that cannot have theſe weights of preciſeneſſe: may by Sand, uniform, and well duſted, make them a number of weights, ſomewhat neer preciſeneſſe: by halving ever the Sand: they ſhall, at length, come to a leaſt common weight. Therein, I leave the farther matter, to their diſcretion, whom need ſhall pinch.] The *Venetians* conſideration of weight may ſeem preciſe enough: by eight deſcents progreſſionall * halfing, from a grain: your Cube, Sphære, apt Ballance, and convenient weights being ready: fall to work. ✣. Firſt, weigh your Cube. Note the Number of the weight. Weigh, after that, your Sphære. Note likewiſe, the Number of the weight. If you now finde the weight of your Cube, to be to the weight of the Sphær, as 21 is to 11: Then you ſee, how the Mechanician and *Experimenter*, without Geometry and Demonſtration, are (as neerly in effect) taught the proportion of the Cube to the Sphær: as I have demonſtrated it in the end of the twelfth book of *Euclid*. Often try with the Cube and Sphær. Then, change your Sphær and Cube to another matter: or to another bigneſſe: till you have made a perfect univerſall Experience of it. Poſſible it is, that you ſhall winne to neerer terms, in the proportion.

The practiſe Staticall, to know the proportion between the Cube and the Sphær.

J. D.

* For, ſo, have you 256 parts of a grain.

When you have found this one certain Drop of Naturall verity, proceed on, to inferre, and duly to make aſſay, of matter depending. As, becauſe it is well demonſtrated, that a Cylinder, whoſe height, and Diameter of his baſe, is equall to the Diameter of the Sphær, is *Seſquialter* to the ſame Sphær, (that is, as 3. to 2.) To the number of the weight of the Sphær, adde half ſo much, as it is: and ſo have you the number of the weight of that Cylinder. Which is alſo comprehended of our former Cube: So, that the baſe of that Cylinder, is a Circle deſcribed in the Square, which is the baſe of our Cube. But the Cube and the Cylinder, being both of one height, have their Baſes in the ſame proportion, in the which, they are one to another in their Maſſineſſe or Solidity. But before, we have two numbers, expreſſing their Maſſineſſe, Solidities, and Quantities, by weight: wherefore, we have * the proportion of the Square, to the Circle, inſcribed in the ſame Square. And ſo are we fallen into the knowledg ſenſible and Experimentall of *Archimedes* great ſecret, of him, by great travail of mind, ſought and found. Wherefore to any Circle given, you can give a Square equall: * as I have taught in my Annotation, upon the firſt propoſition of the twelfth book: And likewiſe to any Square given, you may give a Circle equall: * If you deſcribe a Circle, which ſhall be in that proportion to your Circle inſcribed, as the Square is to the ſame Circle: This, you may do, by my Annotations, upon the ſecond propoſition of the twelfth book of *Euclid*, in my third Probleme there. Your diligence may come to a proportion of the Square to the Circle inſcribed neerer the truth, than is the proportion of 14 to 11: and conſider, that you may begin at the Circle and Square, and ſo come to conclude of the Sphær and the Cube, what their proportion is: as now, you came from the Sphær to the Circle. For of Silver or Gold, or Latton Lamyns or plates, (through one hole drawn, as the

* The proportion of the *S*quare, to the Circle inſcribed.

*The Squaring of the Circle Mechanically.

* To any Square given, to give a Circle equall.

manner is) if you make a ſquare figure, and weigh it: and then, deſcribing thereon, the Circle inſcribed: and cut off, and file away, preciſely (to the Circle) the overplus of the Square: you ſhall then, weighing your Circle, ſee, whether the weight of the Square, be to your Circle, as 14 to 11, As I have noted, in the beginning of *Euclids* twelfth book, &c. after this reſort to my laſt propoſition, upon the laſt of the twelfth: and there, help your ſelf, to the end, and, here, Note this, by the way. That we may ſquare the Circle, without having knowledg of the proportion of the Circumference to the Diameter: as you have here perceived. And otherwiſe alſo, I can demonſtrate it. So that many have cumbered themſelves ſuperfluouſly, by travailing in that point firſt, which was not of neceſſity, firſt: and alſo very intricate. And eaſily you may, (and that diverſly) come to the knowledg of the circumference: the Circles quantity, being firſt known. Which thing I leave to your conſideration: making haſt to diſpatch another Magiſtral Problem: and to bring it neerer to your knowledg, and readier dealing with, than the world (before this day,) had it for you, that I can tell off. And that is, *A Mechanicall doubling of the Cube, &c.* Which may thus be done: Make of Copper plates, or Tyn plates a four ſquare upright Pyramis or a Cone: perfectly faſhioned in the hollow, within. Wherein, let great diligence be uſed, to approach as neer as may be) to the Mathematicall perfection of thoſe figures. At their baſes, let them be all open: every where, elſe, moſt cloſe, and iuſt too. From the vertex, to the Circumference of the baſe of the Cone: and to the ſides of the baſe of the Pyramis: Let 4 ſtreight lines be drawn, in the inſide of the Cone and Pyramis: making at their fall, on the perimeters of the baſes, equall angles, on both ſides themſelues, with the ſaid perimeters. Theſe 4 lines (in the Pyramis, and as many in the Cone) diuide one, in 12 equall parts; and another

Note.
Squaring of the Circle, without knowing of the proportion between Circumference and Diameter.

To double the Cube readily, by Art Mechanicall, depending upon Demonſtration Mathematicall.

J. D.
The four ſides of this Piramis muſt be four Iſoſcheles Triangles, alike and equall.

J. D.

* In all workings with this Pyramis or Cone, Let their Situations be in all Points & Conditions, alike, or all one: while you are about one. Elſe you wil erre.

ther in 24. another in 60. and another, in 100. (reckoning up from the uertex) Or uſe other numbers of diuiſion, as experience ſhall teach you. Then * ſet your Cone or Pyramis with the uertex downward, perpendicularly, in reſpect of the Baſe. (though it be otherwiſe it hindereth nothing.) So let them moſt ſtedily be ſtayed. Now if there be a Cube, which you would have doubled. Make you a pretty Cube of Copper, Silver, Lead, Tinne, Wood, Stone, or Bone. Or elſe make a hollow Cube or Cubick coffen, of Copper, Silver, Tinne, or Wood, &c. Theſe you may ſo proportion in reſpect of your Pyramis or Cone, that the Pyramis or Cone, will be able to contein the weight of them, in water 3 or 4 times at the leaſt: what ſtuffe ſo ever they be made of. Let not your ſolid angle at the vertex, be too ſharp: but that the water may come with eaſe, to the very vertex, of your hollow Cone or Pyramis. Put one of your ſolid Cubes in a Ballance apt: take the weight thereof exactly in water. Poure that water, (without loſſe) into the hollow Pyramis or Cone quietly. Mark in your lines, what numbers the water cutteh: Take the weight of the ſame Cube again: in the ſame kind of water, which you had before: put that * alſo, into the Pyramis or Cone, where you did put the firſt. Mark now again, in what number or place of the lines, the water cutteth them. Two ways you may cōclude your purpoſe: it is to wit, either by numbers or lines. By numbers: as, if you divide the ſide of your Fundamentall Cubes into ſo many equall parts, as it is capable of, conveniently, with your eaſe, and preciſeneſſe of the diviſion. For, as the number of your firſt and leſs line (in your hollow Pyramis or Cone) is to the ſecond or greater (both being counted from the vertex) ſo ſhall the number of the ſide of your Fundamentall Cube, be to the number belonging to the Radicall ſide of the Cube, double to your Fundamentall Cube: Which being multiplyed Cubick wiſe, will ſoon ſhew it ſelf, whether

* Conſider well when you muſt put your waters together: and when, you muſt firſt empty your firſt water, out of your Pyramis, or Cone. Elſe you will erre.

ther it be double or no, to the Cubick number of your Fundamentall Cube. By lines, thus: As your leſſe and firſt line (in your hollow Pyramis or Cone) is to the ſecond or greater, ſo let the Radicall ſide of your Fundamentall Cube, be to a fourth proportionall line, by the 12 propoſition of the ſixth book of *Euclid.* Which fourth line, ſhall be the Root Cubick, or Radicall ſide of the Cube, double to your Fundamentall Cube: which is the thing we deſired. For this, may I (with joy) ſay, ΕΥΡΗΚΑ, ΕΥΡΗΚΑ, ΕΥΡΗΚΑ: thanking the holy and glorious Trinity, having greater cauſe thereto, then * *Archimedes* had (for finding the fraud uſed in the Kings Crown of Gold:) as all men may eaſily judg: by the diverſity of the fruit following of the one and of the other. Where I ſpeake before of a hollow Cubick Coffen: the like uſe is of it, and without weight. Thus. Fill it with vvater preciſely ful, and pour that water into your Pyramis or Cone: and here note the lines cutting in your Pyramis or Cone. Again, fil your Coffen, like as you did before. Put that Water, alſo to the firſt. Mark the ſecond cutting of your lines. Novv, as you proceeded before, ſo muſt you here proceed. * And if the Cube, which you ſhould double, be never ſo great: you have, thus, the proportion (in ſmal) between your tvvo little Cubes: And then, the ſide of that great Cube (to be doubled) being the third, vvil have the fourth, found, to it proportional: by the 12 of the ſixt of *Euclid.*

* *Vitruvius lib. 9. cap. 3.*

☞ God be thanked for this invention, and the fruit enſuing.

Note.

Note, that all this vvhile, I forget not my firſt Propoſition Statical. here rehearſed: that, the Superficies of the vvater, is Sphæricall. Wherein uſe your diſcretion to the firſt line, adding a ſmal hair breadth, more: & to the ſecond half a hair breadth more, to his length. For you will eaſily perceive, that the difference can be no greater, in any Pyramis or Cone, of you to be handled. Which you ſhall thus trye. *For finding the ſwelling of the water above levell.* " Square the Semidiameter, from

* Note, as concerning the Sphæricall Superficies of the water.

“ the Center of the earth, to your firſt Waters Su- “ perficies. Square then, half the Subtendent of that “ watry Superficies: (which Subtendent muſt have “ the equal parts of his meaſure, all one, with thoſe “ of the Semidiameter of the earth, to your watry “ Superficies:) ſubſtract this Square from the firſt: “ Of the reſidue, take the Root Square. That Root, “ Subſtract from your firſt Semidiameter of the earth “ to your watry Superficies: that which remaineth, is “ the height of the water, in the middle, above the Le- “ vell. Which you wil finde, to be a thing inſenſible: and though it were greatly ſenſible,* yet, by help of my ſixt Theoreme upon the laſt Propoſition of *Euclids* twelfth book, noted: you may reduce all to a true Levell. But farther diligence of you is to be uſed, againſt accidentall cauſes of the waters ſwelling: as by having (ſomewhat) with a moiſt ſponge, before, made moiſt your hollow Pyramis or Cone, will prevent an accidentall cauſe of ſwelling, &c. Experience wil teach you abundantly: with great eaſe, pleaſure, and commodity.

* *Note.*

Note this Abridgment of doubling the Cube, &c.

Thus, may you double the Cube Mechanically, Treble it, and ſo forth, in any proportion. Now wil I abridg your pain, coſt, and care herein. Without all preparing of your Fundamental Cubes: you may (alike) work this Concluſion. For, that was rather a kind of Experimental demonſtration, than the ſhorteſt way; and all upon one Mathematical Demonſtration depending. Take water (as much as conveniently will ſerve your turn, as I vvarned before of the Fundamental Cubes bigneſſe.) Weigh it preciſely. Put that vvater into your Pyramis or Cone. Of the ſame kind of vvater, then take again, the ſame weight you had before: put that likevviſe into the Pyramis or Cone; for in each time your markiug of the lines hovv the vvater doth cut them, ſhal give you the proportion between the Radicall ſides, of any two Cubes, whereof the one is double to the other, working as before I have taught you: * ſaving that for your Fundamental Cube his Radical ſide: here, you may take a right line, at pleaſure.

Note * ☞

Yet farther proceeding with our drop of Natural truth: **You may now give Cubes one to the other in any proportion given, Rationall or Irrationall.** On this manner. Make a hollow Parallelipipedon of Copper or Tinne: with one Baſe wanting, or open, as in our Cubick coffen. From the bottome of that Parallelipipedon, raiſe up, many perpendiculars, in every of his four ſides. Now if any proportion be aſſigned you, in right lines:
“ Cut one of your perpendiculars (or a
“ line equal to it, or leſſe than it) likewiſe: by the 10 of
“ the ſixt of *Euclid*, And thoſe two parts, ſet in two
“ ſundry lines of thoſe perpendiculars (or you may ſet
“ them both, in one line) making their beginnings to
“ be, at the baſe: and ſo their lengths to extend upward.
“ Now, ſet your hollow Parallelipipedon, upright,
“ perpendicularly, ſteady. Pour in water, handſomely,
“ to the height of your ſhorter line. Pour that
“ water, into the hollow Pyramis or Cone. Marke the
“ place of the riſing. Settle your hollow Parallelipipedon
“ again. Pour water into it: unto the height of
“ the ſecond line, exactly. Pour that water * duly into
“ the hollow Pyramis or Cone: Marke novv again,
“ vvhere the vvater cutteth the ſame line, vvhich you
“ marked before. For, there, as the firſt marked line, is
“ to the ſecond: So ſhall the tvvo Radical ſides be,
“ one to the other, of any tvvo Cubes: which in their
“ ſolidity, ſhall have the ſame proportion vvhich vvas
“ at the firſt aſſigned: vvere it Rational or Irrational.

To give Cubes one to the other in any Proportion Rationall or Irrationall.

* Emptying the firſt.

Thus, in ſundry vvays you may furniſh your ſelf vvith ſuch ſtrange and profitable matter: vvhich long hath been vviſhed for. And though it be Naturally done, and Mechanically: yet hath it a good Demonſtration Mathematicall. Which is this: Alvvays, you

The Demonſtration of this doubling of the Cube, and of the reſt.

have two like Pyramids: or two like Cones, in the proportions aſſigned: and like Pyramids or Cones, are in proportion, one to the other, in the proportion of their Homologall ſides (or lines) tripled. Wherefore, if to the firſt, and ſecond lines, found in your hollow Pyramis or Cone, you joyn a third and a fourth in continuall proportion: that fourth line ſhall be to the firſt, as the greater Pyramis or Cone is to the leſſe: by the 33 of the eleventh of *Euclid*. If Pyramis to Pyramis, or Cone to Cone, be double, then ſhall * Line to Line, be alſo double, &c. But as our firſt line, is to the ſecond, ſo is the Radicall ſide of our Fundamentall Cube, to the Radicall ſide of the Cube to be made, or to be doubled: and therefore, to thoſe twain alſo, a third and a fourth line, in continuall proportion, joyned: will give the fourth line in that proportion to the firſt, as our fourth Pyramidall, or Conick line was to his firſt: but that was double or treble, &c. as the Pyramids or Cones were, one to another (as we have proved) therefore, this fourth, ſhall be alſo double or treble to the firſt, as the Pyramids or Cones were one to another: But our made Cube, is deſcribed of the ſecond in proportion of the four proportionall lines: therefore * as the fourth line is to the firſt, ſo is that Cube to the firſt Cube: and we have proved the fourth line, to be to the firſt, as the Pyramis or Cone is to the Pyramis or Cone: Wherefore the Cube is to the Cube, as Pyramis is to Pyramis, or Cone is to Cone. But we * ſuppoſe Pyramis to Pyramis, or Cone to Cone, to be double or treble, &c. Therefore Cube is to Cube, double or treble, &c. Which was to be demonſtrated. And of the Parallelipipedon, it is evident, that the water ſolid Parallelipipedons, are one to the other, as their heighths are, ſeeing they have one baſe. Wherefore the Pyramids or Cones made of thoſe water Parallelipipedons, are one to the other, as the lines are (one to the other) between which, our proportion was aſſigned. But the Cubes made

J. D.

* Hereby help your ſelf to become a preciſe practiſer. And ſo conſider how nothing at all, you are hindred (ſenſibly) by the convexity of the Water.

* By the 33 of the eleventh Book of *Euclide*.

J. D.

* And our diligence in practiſe can ſo (in weight of water) perform it: Therefore, now, you are able to give good reaſon of your whole doing.

made of lines, after the proportion of the Pyramidal or Conick *Homologall* lines are one to the other, as the Pyramids or Cones are, one to the other (as we before did prove) therefore, the Cubes made, ſhal be one to the other, as the lines aſſigned, are one the other: which was to be demonſtrated. Note. * This my demonſtration is more general, than onely in Square Pyramis or Cone: Conſider well. Thus, have I, both Mathematically and Mechanically, been very long in words, yet (I truſt) nothing tedious to them, who, to theſe things, are well affected. And verily I am forced (avoiding prolixity) to omit ſundry ſuch things, eaſie to be practiſed: which to the Mathematician, would be a great Treaſure: and to the Mechanician no ſmall gain. * Now may you, **Between two lines given, finde two middle proportionals in continuall proportion: by the hollow Parallelipipedon, and the hollow Pyramis or Cone.** Now any Parallelipipedon rectangle being given: three right lines may be found proportionall in any proportion aſſigned, of vvhich ſhall be produced a Parallelipipedon, equal to the Parallelipipedon given. Hereof, I noted ſomwhat, upon the 36 propoſition, of the 11 book of *Euclid.* Now, all thoſe things, which *Vitruvius* in his Architecture, ſpecified, able to be done, by doubling of the Cube. Or, by finding of two middle proportionall lines, between two lines given, may eaſily be performed. Now, that Probleme, which I noted unto you, in the end of my Addition, upon the 34 of the 11 book of *Euclid*, is proved poſſible. Now may any regular body be transformed into another, &c. Now, any regular body: any Sphær, yea any mixt Solid: and (that more is) Irregular Solids, may be made (in any proportion aſſigned) like unto the body firſt given. Thus, of a *Manneken*, (as the *Dutch* Painters term it) in the ſame *Symmetry*, may a Giant be made: and that, with any geſture, by the Manneken uſed: and contrariwiſe. Now, may you, of any Mould or Model of a

* Note this Corollary.

* The great Commodities following of theſe new Inventrons.

Ship, make one, of the ſame Mould (in any aſſigned proportion) bigger or leſſer. Now, may you, of any * Gunne, or little Peece of Ordnance, make another, with the ſame *Symmetrie* (in all points) as great, and as little, as you will. Mark that, and think on it. Infinitely, may you apply this, ſo long ſought for, and now ſo eaſily concluded: and withall, ſo willingly and frankly communicated to ſuch, as faithfully deal with vertuous ſtudies. Thus can the Mathematicall mind, deal Speculatively in his own Art: and by good means, mount above the clouds and ſtars: And thirdly, he can, by order, Deſcend, to frame Naturall things, to wonderfull uſes: and when he liſt, retire home into his own Center: and there, prepare more means, to Aſcend or Deſcend by: and all, to the glory of God, and our honeſt delectation in earth.

Such is the fruit of the Mathematicall Sciences and Arts.

Although, the Printer, hath looked for this Preface, a day or two, yet could I not bring my pen from the paper, before I had given you comfortable warning and brief inſtructions, of ſome of the Commodities by *Statick*, able to be reaped. In the reſt, I will therefore, be as brief, as it is poſſible: and withall, deſcribing them ſomewhat accordingly. And that, you ſhall perceive, by this, which in order commeth next. For whereas, it is ſo ample and wonderfull, that an whole yeare long, one might finde fruitfull matter therein, to ſpeak of: and alſo in practiſe, is a Treaſure endleſſe: yet will I glanſe over it, with words very few.

This do I call ANTHROPOGRAPHIE. Which is an Art reſtored, and of my preferment to your ſervice. I pray you think of it, as of one of the chief points of Humane knowledg. Although it be but now firſt confirmed, with this new name: yet the matter, hath from the beginning, been in conſideration of all perfect Philoſophers. Anthropographie is the deſcription of the Number, Weight, Figure, Situation and colour of every diverſe thing, conteined in the perfect be-body of MAN with certain knowledg of the Symmetrie, figure, weight, Characterization, and due locall motion of any parcell of the ſaid body, aſsigned: and of numbers, to the ſaid parcel appertaining. This is the one part of the Definition, meet for this place: Sufficient to notifie, the perticularity, and excellency of the Art: and why it is, here, aſcribed to the Mathematicals. If the deſcription of the heavenly part of the world had a peculiar Art, called *Aſtronomy*. If the deſcription of the earthly Globe, hath his peculiar art, called *Geographie*. If the Matching of both hath his peculiar Art, called *Coſmographie*. Which is the deſcription of the whole and univerſall frame of the world: Why ſhould not the deſcription of him, who is the leſſe world and from the beginning called *Microcoſmus*, (that is, The leſſe world) And for whoſe ſake and ſervice, all bodily creatures elſe, were created: Who alſo participateth with Spirits and Angels, and is made to the Image and ſimilitude of *God*, have his peculiar Art? and be called the *Art of Arts*: rather than either to want a name, or to have too baſe and improper a name? You muſt of ſundry profeſſions, borrow or challenge home, peculiar parts hereof: and farther proceed: as God, Nature, Reaſon and Experience ſhall inform you. The Anatomiſts wil reſtore to you, ſome part: The Phyſiognomiſts, ſome: The Chyromantiſts ſome. The Metapoſcopiſts ſome. The excellent *Albert Durer*, a good part: the Art of Perſpective, wil ſomevvhat, for the eye help forvvard: *Pythagoras*, *Hypocrates*, *Plato*, *Galenus*, *Meletius*, and many other (in certain things) vvil be Contributaries. And farther, the Heaven, the Earth, and all other Creatures, vvil each ſhevv, and offer their Harmonious ſervice, to fil up, that, vvhich vvanteth hereof: and vvith your ovvn Experience, concluding: you may Methodically regiſter the vvhole, for the poſterity: Whereby, good proof vvil be had, of our Harmonious. and Microcoſmical conſtitution. The out-

MAN is the leſſe World.

* ☞

Microcoſmus.

* Lib 3. Cap. 1.

ward Image and view hereof, to the Art of *Zographie*, and Painting, to Sculpture, and Architecture, (for Church, Houſe, Fort, or Ship) is moſt neceſſary and profitable: for that, it is the chief baſe and foundation of them. Look in * *Vitruvius*, whether I deal ſincerely, for your behoof, or no. Look in *Albertus Durerus de Symmetria humani Corporis*. Look in the 27 and 28 Chapters, of the 2 Book *De occulta Philoſophia*. Conſider the *Arke* of *Noe*. And by that, wade farther. Remember the *Delphicall Oracle*, *NOSCE TE IPSVM* (Know thy ſelf) ſo long ago pronounced: of ſo many a Philoſopher repeated: and of the *Wiſeſt* attempted: And then you wil perceive how long ago, you have been called to the School, where this Art might be learned. Well. I am nothing affraid, of the diſdain of ſome ſuch, as think Sciences and Arts, to be but ſeven, Perhaps, thoſe ſuch, may, with ignorance, and ſhame enough, come ſhort of them ſeven alſo: and yet nevertheleſſe, they can not preſcribe a certain number of Arts: and in each certain unpaſſible bounds to God, Nature, and mans Induſtry. ☞ New Arts, daily riſe up: and there was no ſuch order taken, that, all Arts, ſhould in one age or in one land, or of one man, be made known to the world. Let us embrace the gifts of God, and ways to wiſdome, in this time of grace, from above, continually beſtowed on them, who thankfully will receive them: *Et bonis omnia Co-operabantur in bonum.*

TROCHILIKE, is that Art Mathematicall, which demonſtrateth the properties of all Circular motions, Simple & Compound. And becauſe the fruit hereof vulgarly received, is in Wheels, it hath the name of *Trochilike*: as a man would ſay *Wheel Art*. By this art, a Wheel may be given, which ſhal move once about, in any time aſſigned. Two Wheels may be given, whoſe turnings about in one and the ſame time (or equal times) ſhal have one to the other, any proportion appointed. By Wheels, may a ſtreight line be deſcribed : Likewiſe, a Spiral line in plain, Conicall Section lines, and other Irregular lines, at pleaſure, may be drawn. Theſe and ſuch like are principal Concluſions in this Art : and help forward many pleaſant and profitable Mechanical works : As Mils, to ſaw great and very long Deal boards, no man being by. Such have I ſeen in *Germany*, and in the City of *Prague*, in the Kingdome of *Bohemia :* Coyning Mils, Hand Mils for Corn grinding : And all manner of Mils and Wheel work : By Wind, Smoak, Water, Weight, Spring, Man or Beaſt moved. Take in your hand *Agricola de re Metallica:* and then ſhal you (in all Mines) perceive how great need is, of Wheel work. By Wheels, ſtrange works and incredible are done : as wil, in other Arts hereafter appear. A wonderful example of farther poſſibility, and preſent Commodity was ſeen in my time, in a certain Inſtrument : which by the Inventer and Artificer (before) was ſold for twenty Talents of Gold : and then had (by misfortune) received ſome injury and hurt. And one *Janellus* of *Cremona* did mend the ſame, and preſented it unto the Emperour *Charls* the Fift. *Hieronymus Cardanus*, can be my witneſſe, that therein, was one Wheel, which moved,& that in ſuch rate,that,in 7000 years only,his own period ſhouldbe finiſhed. A thing almoſt incredible : But how farre, I keep me within my bounds : very many men (yet alive) can tel.

Saw Mils.

HELICOSOPHIE, is neer Siſter to *Trochilike*, and is An Art Mathematicall, which demonſtrateth the deſigning of all Spirall lines in Plain, on Cylinder, Cone, Sphear, Conoid, and Sphearoid, and their properties appertaining. The uſe hereof in *Architecture*, and diverſe Inſtruments and Engines, is moſt neceſſary. For, in many things, the Scrue worketh the feat, which elſe, could not be performed. By help hereof, it is * recorded, that, where all the power of the City of *Syracuſa*, was not able to move a certain Ship (being on ground) mighty *Archimedes*, ſetting to his

* *Athenæus Lib. 5. Cap. 8.*

Scruiſh Engine, cauſed *Hiero* the King, by him ſelf, at eaſe, to remove her, as he would. Whereat, the King wondring: Ἀπὸ ταύτης τῆς ἡμήρας, περὶ παντὸς, Ἀρχιμήδει λέγοντι πιστευτέον. *From this day, forward,* (ſaid the King) *Credit ought to be given to* Archimedes, *whatſoever he ſaith.*

Proclus pag. 18.

PNEUMATITHMIE, **demonſtrateth by cloſe hollow Geometricall Figures (regular and irregular) the ſtrange properties (in motion or ſtay) of the water, Ayr, Smoak, and Fire, in their continuity, and as they are joyned to the Elements next them.** This Art, to the Natural Philoſopher, is very profitable, to prove that *Vacuum*, or *Emptineſſe*, is not in the world. And that, all Nature, abhorreth it ſo much, that, contrary to ordinary law, the Elements wil move, or ſtand. As, Water to aſcend rather: than between him and Air, Space or place ſhould be left, more than (naturally) that quantity of Air requireth, or can fill. Again, water to hang, and not deſcend: rather than by deſcending, to leave emptineſſe at his back. The like is of Fire and Air, they will deſcend: when, either their Continuity ſhould be diſſolved, or their next Element forced from them. And as they wil not be extended to diſcontinuity: So will they not, nor yet of mans force, can be preſt or pent in ſpace, not ſufficient and anſwerable to their bodily ſubſtance. Great force and violence will they uſe, to enjoy their natural right and liberty. Hereupon, two or three men together, by keeping Air under a great Cauldron, and forcing the ſame down, orderly, may without harm deſcend to the Sea bottome: and continue there a time, &c. Where, Note, how the thicker Element (as the Water) giveth place to the thinner (as is the Air:) & receiveth violence of the thinner, in manner &c. Pumps & all manner of Bellows, have their ground of this art: and many other ſtrange deviſes: as *Hydraulica*, Organes going by water, &c. Of this Feat, called commonly *Pneumatica*) goodly works are extant, both

To go to the bottom of the Sea without danger.

in Greek and Latine. With old and learned Schoolmen, it is called *Scientia de pleno & vacuo*.

MENADRIE, is an Art Mathematicall, which demonſtrateth, how, above Natures vertue, and power ſimple: vertue and force may be multiplyed: and ſo, to direct, to lift, to pul to, and to put or caſt fro, any multiplied or ſimple, determined vertue, Weight or Force, naturally, not, ſo, directible or moveable. Very much is this Art furdred, by other Arts, as in ſome points by *Perſpective*, in ſome, by *Statick*, in ſome by *Trochilike*, and in other by *Helicoſophie*, and *Pneumatithmie*. By this Art, all Cranes, Gibbets, and Engines to lift up, or to force any thing any manner of way, are ordered, and the certain cauſe of their force is known. As, the force, which one man hath with the Dutch waggen Rack: therewith, to ſet up again, a mighty waggen laden, being overthrown. The force of the Croſſebow Rack, is certainly here demonſtrated. The reaſon, why one man doth with a leaver, lift that, which ſix men, with their hands onely, could not ſo eaſily do. By this Art, in our common Cranes in *London*, where power is to crane up the weight of 2000 pound: by two Wheels more (by good order added) Art concludeth, that there may be craned up with his devices and engines 200000 pound weight, &c. So wel knew *Archimedes* this Art: that he alone (twice or thrice) ſpoyled and diſcomfited, the whole Army and Hoſt of the Romans, beſieging *Syracuſa*, *Marcus Marcellus the Conſul*, being their General Captain. Such huge ſtones, ſo many with ſuch force, and ſo far, did he with his engines hail among them out of the City. And by Sea likewiſe: though there Ships might come to the wals of *Syracuſa*, yet he utterly confounded the Romane Navy: what with his mighty ſtones hurling: what with Pikes * of 18 foot long, made like ſhafts: which he forced almoſt a quarter of a mile, what, with his catching hold of their ſhips, and hoyſing them up above the water, and ſuddenly letting them fal into the Sea again: what

Plutarchus in Marco Marcello.
Synesius in Epistol.
Polybius.
Plinius.
Quintilianus.
T. Livius.

* *Atheneus.*

* *Galenus. Anthemius.*

with his * Burning Glaſſes: by which he fired their other ſhips a far off: what, with his other policies, devices, and engines, he ſo manfully acquit himſelf: that all the force, courage, and policy of the Romanes (for a great ſeaſon) could nothing prevail, for the winning of *Syracuſa*: whereupon, the Romans named *Archimedes*, *Briareus*, and *Centimanus*. *Zonaras* maketh mention of one *Proclus*, who ſo wel had perceived *Archimedes* Art of *Menadrie*, and had ſo well invented of his own, that with his Burning Glaſſes, being placed upon the wals of *Byſance*, he multiplyed ſo the heat of the Sun and directed the beams of the ſame againſt his enemies Navy with ſuch force, and ſo ſuddenly (like lightening) that he burned & deſtroyed both man & ſhip. And *Dion* ſpecifieth of *Priſcus* a *Geometrician* in *Byſance*, who invented & uſed ſundry Engines, of force multiplied: w^ch^ was cauſe, that the *Emperour Severus* pardoned him, his life after he had wonne *Byſance*. Becauſe he honoured the Art, wit, and rare induſtry of *Priſcus*. But nothing inferiour to the invention of theſe Engines of Force, was the invention of Gunnes. Which, from an Engliſh man had the occaſion and order of firſt inventing: though in another land, and by other men, it was firſt executed. And they that ſhould ſee the record, where the occaſi-
“ on and order general of Gunning, is firſt diſcourſed
“ of, would think: that ſmal things ſlight and common:
“ comming to wiſe mens conſideration, and induſtrious
“ mens handling, may grow to be of force incredible.

Burning Glaſſes.

Guns.

HYPOGEIODIE, is an Art Mathematical, demonſtrating, how under the Sphericall Superficies of the earth, at any depth, to any perpendicular line aſsigned (whoſe diſtance from the perpendicular of the entrance: and the Azimuth likewiſe, in reſpect of the ſaid entrance is known) certain way may be preſcribed and gone: And how any way above the ſuperficies of the earth deſigned, may under earth, at any depth limited, be kept: going always perpendicularly,

cularly, under the way, on earth deſigned: and contrariwiſe, any way, (ſtraight or crooked,) under the earth, being given: upon the outface, or ſuperficies of the earth, to Line out the ſame: ſo, as, from the Center of the earth, Perpendiculars drawn to the ſphericall ſuperficies of the earth, ſhall preciſely fall in the correſpondent points of thoſe two wayes. This, with all other caſes and circumſtances herein, and appurtenances, this art demonſtrateth. This art is very ample in variety of Concluſions, and very profitable ſundry ways to the Common-wealth. The occaſion of my inventing this art, was at the requeſt of two Gentlemen, who had a certain work (of gain) under ground and their grounds did joyn over the work: and by reaſon of the crookedneſſe, divers depths, and heights of the way under ground, they were in doubt, and at controverſie, under whoſe ground, as then, the vvork vvas. The name onely (before this) vvas of me publiſhed, *De Itinere Subterraneo.* The reſt be at Gods vvil. For Pioners, Miners, Diggers for Metals, Stone, Cole, and for ſecret paſſages under ground, betvveen place and place (as this land hath diverſe) and for other purpoſes, any man may eaſily perceive, both the great fruit of this art, and alſo in this art, the great aid of Geometry.

HYDRAGOGIE, demonſtrateth the poſſible leading of Water by natures law, and by artificiall help, from any head (being a ſpring ſtanding or running Water) to any other place aſſigned. Long hath this art been in uſe: and much thereof vvritten: and very marvellous vvorks therein performed: as may yet appear in *Italy*, by the Ruines remaining, of the *Aqueducts.* In other places of Rivers, leading through the Main land Navigable many a Mile: and in other places, of the marvellous forcings of vvater to aſcend: which all declare the great skil to be required of him, vvho ſhould in this art be perfect, for all occaſions of vvaters poſſible leading. To ſpeak of the allovvance of the

the Fal, for every hundred foot: or of the Ventils (if the waters labour be far and great) I need not: ſeeing, at hand (about us) many expert men can ſufficiently teſtifie, in effect, the order: though the Demonſtration of the Neceſſity thereof, they know not: Nor yet, if they ſhould, be led, up and down, and about Mountains, from the head of the Spring: and then a place being aſſigned: and of them, to be demanded, how low or high, that laſt place is, in reſpect of the head, from which (ſo crookedly) and up and down) they be come: Perhaps, they would not, or could not) very readily or neerly aſſoil that Queſtion. *Geometry* therefore, is neceſſary to *Hydragogie*. Of the ſundry ways to force water to aſcend, either by *Tympane*, *Kettell mils*, *Scrue*, *Cteſibike*, or ſuch like, in *Vitruvius*, *Agricola*, (and other,) fully, the manner may appear. And ſo, thereby, alſo be moſt evident, how the arts of *Pneumatithmie*, *Helicoſophie*, *Statike*, *Trochilike*, and *Menadrie*, come to the furniture of this in ſpeculation, and to the Commodity of the Common-wealth in practiſe.

HOROMETRIE, **is an art Mathematicall, which demonſtrateth how at all times appointed, the preciſe uſuall denomination of time, may be known, for any place aſſigned.** Theſe words are ſmooth and and plain eaſie Engliſh, but the reach of their meaning is farther then you would lightly imagine. Some part of this art, was called in old time *Gnomonice:* and of late, *Horologiographia:* and in Engliſh, may be termed *Dialling*. Ancient is the uſe, and more ancient is the Invention. The uſe, doth wel appear to have been (at the leaſt) above two thouſand and three hundred years ago: in * King *Achaz* Dial, then, by the Sun, ſhewing the diſtinction of time. By Sun, Moon, and Stars, this Dialling may be performed, and the preciſe time of day, or night known. But the demonſtrative delineation of theſe Dials, of all ſorts, requireth good skil both of *Aſtronomie* and *Geometry* Elemental, Sphærical, Phænomenal;

2 Kings 20.

menal and Conical. Then to uſe the grounds of the art, for any regular Superficies, in any place offered: and (in any poſſible apt poſition thereof) thereon to deſcribe (all manner of ways) how, uſual hours, may be (by the *Suns* ſhadow) truly determined: wil be found no ſleight Painters work. So to paint and preſcribe the Suns Motion, to the breadth of a hair. In this Feat (in my youth) I invented a way, How in any Horizontall, Murall or Æquinoctiall Diall, &c. at all houres (the Sun ſhining) the ſign and degree aſcendent, may be known. Which is a thing very neceſſary, for the riſing of thoſe fixed ſtars: whoſe Operation in the air, is of great might evidently. I ſpeak no further, of the uſe hereof. But foraſmuch as, Mans affairs, require knowledg of Times and Moments, when neither Sun, Moon, or Star, can be ſeen: Therefore, by Induſtrie Mechanicall, was invented firſt, how by Water, running orderly, the Time and Hours might be known: whereof, the famous *Cteſibius*, was Inventor: a man of *Vitruvius*, to the skie (juſtly) extolled. Then, after that, by ſand running, were hours meaſured: Then, by *Trochilike*, with weight: and of late time, by *Trochilike* with Spring: without weight. All theſe by Sun, or Stars direction, (in certain time) require overſight and reformatiou, according to the heavenly Æquinoctial Motion: beſides the inæquality of their own Operation. There remaineth (without parabolical meaning herein), among the Philoſophers, a more excellent, more commodious, and more marvellous way, than all theſe: of having the motion of the Primovant (or firſt Æquinoctial motion) by Nature and Art, imitated: which you ſhal (by further ſearch in weightier ſtudies) hereafter, underſtand more of. And ſo it is time to finiſh this Annotation, of Times diſtinction, uſed in our common, and private affairs: The commodity whereof no man vvould vvant, that can tel, hovv to beſtovv his time.

A perpetuall Motion.

ZOGRAPHIE, is an art Mathematicall,

which teacheth & demonſtrateth, how, yͤ interſection of all viſuall Pyramids,made by any plain aſſigned, (the Center, diſtance, and lights, being determined) may be, by lines, and due proper colours repreſented. A notable art, is this, and would require a whole Volume, to declare the property thereof: and the Commodities enſuing. Great skil of *Geometrie*, *Arithmetick*, *Perſpective*, and *Anthropographie*, with many other perticular Arts, hath the Zographer, need of, for his perfection. For, the moſt excellent Painter, (who is but the proper Mechanician, and Imitator ſenſible, of the Zographer) hath atteined, to ſuch perfection, that ſenſe of man and beaſt, have judged things painted, to be things natural & not artificial: alive & not dead. This Mechanical Zographer (commonly called the Painter) is marvellous in his skil: and ſeemeth to have a certain divine power: as of friends abſent, to make a friendly,preſent comfort, yea, and of friends dead, to give a continual, ſilent preſence: not onely with us, but with our poſterity, for many ages. And ſo proceeding, conſider, how in Winter, he can ſhew you, the lively view of Sommers joy, and riches: and in Sommer, exhibite the countenance of Winters doleful ſtate and nakedneſſe.Cities, Towns, Forts, Woods, Armies,yea whol Kingdoms(be they never ſo far or great)can he with eaſe,bring wth him,home (to any mans judgment) as patterns lively of the things rehearſed. In one little houſe, can he, encloſe (with great pleaſure of the beholders) the portraiture lively, of all viſible creatures, either on earth or in the earth, living: or in the waters lying, creeping,ſliding or ſwimming: or of any foul or fly, in the air flying. Nay, in reſpect of theStars,the Skie,the Clouds: yea, in the ſhew of the very light it ſelf,that divine creature,can he match our eyes Judgment, moſt neerly. What a thing is this? things not yet being, he can repreſent ſo, as, at their being: the picture ſhal ſeeme (in manner) to have created them. To what Artificer, is not Picture,a great pleaſure and commodity? Which of them all, wil refuſe the direction and aid of Picture? The Architect, the Goldſmith, and the Arras Weaver: of Picture, make great account. Our lively Herbals, our portraitures of birds, beaſts, and fiſhes: and our curious Anatomies, which way, are they moſt perſectly made, or with moſt pleaſure, of us beholden? Is it not by Picture onely? and if Picture, by the induſtry of the Painter, be thus commodious and marvellous: what ſhal be thought of *Zographie*, the Schoolmaſter of Picture, and chief Governour? Though I mention not *Sculpture*, in my Table of arts Mathematicall: yet may all men perceive, How, that *Picture* and *Sculpture*, are ſiſters Germane: and both, right profitable, in a Common wealth and of *Sculpture*, aſwel as of Picture, excellent artificers have written great books in commendation. Witneſſe I take, of *Georgio Vaſari, Pittore Aretino*: of *Pomponius Gauricus*: and others. To theſe two arts, (with other,) is a certain odde art, called *Althalmaſat*, much beholding: more, than the common *Sculptor*, *Entayler*, *Carver*, *Cutter*, *Graver*, *Founder*, or *Painter*, (*&c.*) know their art to be commodious.

An Objection.

ARCHITECTURE, to many may ſeem not worthy, or not meet, to be reckoned among the *Arts Mathematicall*. To whom, I think good, to give ſome account of my ſo doing. Not worthy, (wil they ſay,) becauſe it is but for building of a Houſe, Palace, Church Fort, or ſuch like groſſe works: and you alſo, defined the *Arts Mathematicall*, to be ſuch as dealed with no Material or corruptible thing: and alſo did demonſtratively proceed in their Faculty, by Number or Magnitude. Firſt, you ſee, that I count here, *Architecture*, among thoſe *Arts Mathematicall*, which are derived from the principals: and you know, that ſuch may deal with naturall things, and ſenſible matter. Of which ſome draw neerer, to the ſimple and abſolute Mathematicall Speculation, then other do. And though the

The Anſwer.

“ *Architect* procureth,informeth,and directeth the *Me-*
“ *chanician*, to handwork, and the building actual, of
“ houſe, Caſtle, or Palace, and is chief Judge of the
“ ſame : yet, with himſelf (as chief *Maſter* and *Archi-*
“ *tect*)remaineth the Demonſtrative reaſon and cauſe of
“ the Mechanicians work, in Line, Plain, and Solid : by
“ *Geometricall, Arithmeticall, Opticall, Muſicall, Aſtrono-*
“ *micall,Coſmographicall* (and to be brief) by all the for-
“ mer derived *Arts Mathematicall*,and other Natural arts
“ able to be confirmed and ſtabliſhed.

If this be ſo,then, may you think, that *Architecture* hath good and due allowance, in this honeſt company of *Arts Mathematicall* Derivative. I wil herein crave Judgment of two moſt perfect *Architects*, the one being *Vitruvius* the Roman : who did write ten books thereof, to the Emperour *Auguſtus*, (in whoſe daies our Heavenly Archmaſter was born :) and the other *Leo Baptiſta Albertus*, a Florentine, who alſo publiſhed ten books thereof. *Architectura* (ſaith *Vitruvius*) *eſt Scientia pluribus diſciplinis, & variis eruditionibus ornata : cujus Judicio probantur omnia, quæ ab cæteris Artificibus perficiuntur opera.* That is : Architecture is a ſcience garniſhed with many doctrines and divers inſtructions : by whoſe Judgment, all works, by other Workmen finiſhed, are judged. It followeth. *Ea naſcitur ex Fabrica, & Ratiocinatione, &c. Ratiocinatio autem eſt, quæ, res fabricatas, ſolertia ac ratione proportionis, demonſtrare atque explicare poteſt.* Architecture groweth of Framing, and Reaſoning,&c. Reaſoning is that, which of things framed with forecaſt, and proportion : can make demonſtration, and manifeſt declaration. again. *Cùm, in omnibus enim rebus, tùm maximè etiam in Architectura, hæc duo inſunt : quod ſignificatur, & quod ſignificat. Significatur propoſita res, de qua dicitur : hanc autem ſignificat Demonſtratio, rationibus doctrinarum explicata.* Foraſmuch, as in all things: therfore chiefly in Architecture, theſe two things are : the thing ſignified : and that which ſignifieth. The thing propounded,

pounded, whereof we ſpeak, is the thing ſignified. But Demonſtration, expreſſed with the reaſons of divers doctrines, doth ſignifie the ſame thing. After that. *Ut literatus ſit, peritus Graphidos, eruditus Geometriæ, & Optices non ignarus : inſtructus Arithmetica : hiſtorias complures noverit, Philoſophos diligenter audiverit : Muſicam ſciverit : Medicinæ non ſit ignarus, reſponſa Juriſperitorum noverit : Aſtrologiam, Cœlique rationes cognitas habeat.* An Architect (ſaith he) ought to underſtand Languages, to be skilful of Painting, wel inſtructed in Geometry, not ignorant of Perſpective, furniſhed with Arithmetick, have knowledg of many hiſtories, and diligently have heard Philoſophers, have skil of Muſick, not ignorant of Phyſick, know the anſwers of Lawyers, and have Aſtronomie, and the courſes Cœleſtial, in good knowledg. He giveth reaſon, orderly, wherefore, all theſe arts, doctrines, and inſtructions, are requiſite in an excellent Architect. And (for brevity) omitting the Latin text, thus he hath. Secondly, it is behooful for an Architect to have the knowledg of Painting: that he may the more eaſily faſhion out, in patterns painted, the form of what work he liketh: and Geometry giveth to Architecture many helps: and firſt teacheth the uſe of the Rule, and the Compaſſe: whereby (chiefly and eaſily) the deſcriptions of Buildings, are diſpatched in Ground-plats: and the directions of Squares, Levels, and Lines. Likewiſe, by Perſpective, the Lights of the Heaven, are wel led, in the buildings: from certain quarters of the world. By Arithmetick, the charges of Buildings are ſummed together: the meaſures are expreſſed, and the hard queſtions of Symmetries, are by Geometrical means and methods diſcourſed on, &c. Beſides this, of the Nature of things (which in Greek is called φυσιολογία) Philoſophy doth make declaration. Which it is neceſſary for an Architect, with diligence to have learned: becauſe it hath many and divers natural queſtions: as ſpecially in Aqueducts. For in their courſes, leadings about, in the level ground, and in the mountings, the natural ſpirits and breaths are ingendred divers wayes: The hindrances, which they cauſe, no man can help, but he, which out of Philoſophy, hath learned the original cauſes of things. Likewiſe, whoſoever ſhal read *Cteſibius* or *Archimedes* books, (and of others, who have written ſuch Rules) cannot think, as they do: unleſſe he ſhal have received of Philoſophers, inſtructions in theſe things: and Muſick he muſt needs know: that he may have underſtanding, both of Regular and Mathematical Muſick: that he may temper wel his Baliſts, Catapults, and Scorpions, &c. Moreover, the Braſen Veſſels, which in Theatres, are placed by Mathematical order, in ambries, under the ſteps: and the diverſities of the ſounds (which the Grecians cal ἠχεῖα) are ordered according to Muſical Symphonies and Harmonies: being diſtributed in the Circuits, by *Diateſſaron, Diapente*, and *Diapaſon*. That the convenient voyce, of the players ſound, when it came to theſe preparations, made in order, there being increaſed: with that increaſing, might come more clear and pleaſant, to the ears of the lookers on, &c. And of Aſtronomie, is known the Eaſt, Weſt, South, and North. The faſhion of the Heaven, the Æquinox, the Solſticie, and the courſe of the ſtars. Which things, unleaſt one know: he cannot perceive, any thing at all, the reaſon of Horologies. Seeing therefore, this ample ſcience, is garniſhed, beautified, and ſtored, with ſo many and ſundry skils and knowledges: I think, that none can juſtly account themſelves Architects of the ſodain. But they onely, who from their childs years aſcending by theſe degrees of knowledges, being foſtered up with the atteining of many Languages and Arts, have wonne to the high Tabernacle of Architecture, &c. And to whom Nature hath given ſuch quick Circumſpection, ſharpneſſe of Wit, and Memory, that they may be very abſolutely skilful in Geometry, Aſtronomy, Muſick, and the reſt of the Arts Mathematical : Such ſurmount and paſſe the calling, and ſtate of Architects : and are become Mathematicians, &c. And they are found ſeldom: as in times paſt, was *Ariſtarchus, Samius, Philolaus,* and *Archytas Tarentynes : Apollonius Pergeus, Eratoſthenes Cyreneus : Archimedes*, and *Scopas, Syracuſians.* Who alſo left to their poſterity, many Engines and Gnomical works : by numbers, and natural means, invented and declared.

A Mathematician

Thus much, and the ſame words (in ſenſe) in one onely Chapter of this incomparable *Architect Vitruvius*, ſhal you finde. And if you ſhould, but take his book in your hand, and ſlightly look through it, you would ſay ſtraightway : This is *Geometry, Arithmetick, Aſtronomy, Muſick, Anthropographie, Hydragogie, Horometrie, &c.* and (to conclude) the ſtorehouſe of all workmanſhip. Now, let us liſten to our other Judg, our Florentine, *Leo Baptiſta*, and narrovvly conſider, hovv he doth determine of *Architecture. Sed anteque ultra progrediar, &c.* But before I proceed any further (ſaith he) I think, that I ought to expreſſe, vvhat man I vvould have to be allovved an Architect. For, I vvil not bring in place a Carpenter : as though you might compare him to the Chief Maſters of others arts. For the hand of the Carpenter, is the Architects Inſtrument : But I vvil appoint “ the Architect to be that man, vvho hath the skil, (by a “ certain and marvellous means and vvay) both in “ mind and Imagination to determine : and alſo in “ vvork to finiſh : vvhat vvorks ſo ever, by motion of “ vveight and coupling and framing together of bodies, may moſt aptly be commodious for the vvorthieſt uſes of Man. And that he may be able to perform theſe things, he hath need of atteining and knovvledge of the beſt and moſt vvorthy things, &c. The vvhole feat of Architecture in building, conſiſteth in Lineaments and in framing. And the vvhole povver and skill of Lineaments, tendeth to this : that the right and ab-

Vitruvius.

Who is an Architect.

ſolute way may be had, of Coapting and joyning Lines and angles: by which, the face of the building, or frame may be comprehended and concluded. And it is the property of Lineaments, to preſcribe unto buildings, and every part of them, an apt place, and certain number: a worthy manner and a ſeemly order: that, ſo the whole form and figure of the building, may reſt in the very Lineaments, &c. And we may preſcribe in mind and imagination the whole forms, * all materiall ſtuffe being ſecluded. Which point we ſhal attein, by noting and fore-pointing the angles, and lines, by a ſure and certain direction and connexion. Seeing then, theſe things are thus: Lineament ſhal be the certain and conſtant preſcribing, conceived in mind: made in lines and angles: and finiſhed with a learned mind and wit. We thank you Maſter *Baptiſt*, that you have ſo aptly brought your art and phraſe, thereof, to have ſome Mathematicall perfection: by certain order, number, form, figure, and *Symmetrie* mental: all natural and ſenſible ſtuffe ſet apart. Now then it is evident, (Gentle Reader) how aptly and worthily I have preferred *Architecture*, to be bred and foſtered up in the Dominion of the peerleſſe *Princeſſe*, *Mathematica*, and to be a natural Subject of hers. And the name of *Architecture*, is of the principality, which this ſcience hath, above all other arts. And *Plato* affirmeth the *Architect* to be *Maſter* over all, that make any work. Whereupon, he is neither Smith nor Builder: nor, ſeparately, any Artificer: but the Head, the Provoſt, the Director, and Judge of all artificial works, and all Artificers. For, the true *Architect*, is able to teach, demonſtrate, diſtribute, deſcribe, and Judg all works wrought. And he, onely ſearcheth out the cauſes and reaſons of all Artificial things. Thus excellent, is *Architecture*: though few (in our days) attein thereto: yet may not the art, be otherwiſe thought on, than in very deed it is worthy. Nor we may not o ancient arts, make new and imperfect Definitions in

* The Immaterialitie of perfect Architecture.

What Lineament is.

Note.

our dayes: for ſcarcity of Artificers: No more than we may pinch in, the Definitions of *Wiſdome*, or *Honeſtie*, or of *Friendſhip* or *Juſtice*. No more wil I conſent, to diminiſh any whit of the perfection and dignity, (by juſt cauſe) allowed to abſolute *Architecture*. Under the Direction of this Art, are three principal, neceſſary *Mechanicall Arts*. Namely, *Houſing*, *Fortification*, and *Naupegie*. *Houſing*, I underſtand, both for Divine Service, and Mans common uſage: publick and private. Of *Fortification* and *Naupegie*, ſtrange matter might be told you: But perchance, ſome wil be tyred, with this Bed-roll, already rehearſed: and other ſome, wil nicely nip my groſſe and homely diſcourſing with you: made in poſt haſt: for fear you ſhould want this true and friendly warning, and taſt giving, of the *Power Mathematicall*. Life is ſhort, and uncertain: Times are perilous, &c. And ſtil the Printer awaiting, for my pen ſtaying: all theſe things, with farther matter of Ingratefulneſſe, give me occaſion to paſſe away, to the other Arts remaining, with all ſpeed poſſible.

The Art of NAVIGATION, demonſtrateth how, by the ſhorteſt good way, by the apteſt direction, and in the ſhorteſt time, a ſufficient Ship, between any two places (in paſſage Navigable) aſsigned: may be conducted: and in all ſtorms and naturall diſturbances chancing, how to uſe the beſt poſsible means, whereby to recover the place firſt aſsigned. What need the *Maſter Pilot* hath of other Arts, here before recited, it is eaſie to know: as, of *Hydrographie*, *Aſtronomy*, *Aſtrologie*, and *Horometrie*. Preſuppoſing continually, the common Baſe, and Foundation of all: namely, *Arithmetick*, and *Geometrie*. So that he be able to underſtand and judg his own neceſſary Inſtruments, and furniture neceſſary. Whether they be perfectly made or no: and alſo can, (if need be) make them himſelf. As Quadrants, the Aſtronomers Staffe, the Aſtrolabe Univerſal, An Hydrographical Globe. Charts Hydrographical

Anno 1599.

graphical, true,(not with parallel Meridians.) The common Sea Compaſſe : The Compaſſe of variation : The Proportional and Paradoxal Compaſſes (of me invented, for our two Muſcovy Maſter Pilots, at the requeſt of the Company) Clocks with ſpring : hour, half hour, and three hour Sand-glaſſes : and ſundry other Inſtruments : and alſo be able on Globe, or Plain to deſcribe the Paradoxal Compaſſe : and duly to uſe the ſame, to all manner of purpoſes, whereto it was invented. And alſo, be able to Calculate the Planets places for all times.

Moreover, with Sun, Moon or Star (or without) be able to define the Longitude and Latitude of the place, which he is in : So that the Longitude and Latitude of the place, from which he ſayled, be given : or by him, be known : whereto, appertaineth expert means, to be certified ever , of the Ships way, &c. and by foreſeeing the Riſing, Setting, Noonſteding, or Midnighting of certain tempeſtuous fixed Stars : or their Conjunctions, and Anglings with the Planets, &c. he ought to have expert conjecture of ſtorms, tempeſts and ſpouts : and ſuch like Meteorological effects, dangerous on ſea. For (as *Plato* ſaith,) *Mutationes opportunitateſque temporum preſentire, non minus rei militari, quàm Agriculturæ, Navigationique convenit.* **To foreſee the alterations and opportunities of times is convenient, no leſs to the Art of War, than to Huſbandry and Navigation.** And beſides ſuch cunning means, more evident tokens in Sun and Moon, ought of him to be known: ſuch (as the Philoſophical Poet) *Virgilius* teacheth in his *Georgicks*, where he ſaith.

Sol quoque & exoriens & quum ſe condet in undas,
Signa dabit, Solem certiſſima ſigna ſequuntur, &c.

———— Nam ſæpe videmus,
Georgic.1. *Ipſius in vultu varios errare colores.*

Caru-

Cæruleus, pluviam denunciat, igneus Euros.
Sin maculæ incipient rutilo immiſcerier igni,
Omnia tum pariter vento, nimbiſq; videbis
Fervere : non illa quiſquam me nocte per altum
Ire, neque à terra moveat convellere funem, &c.
Sol tibi ſigna dabit. Solem quis dicere falſum
Audeat ? ———— *&c.*

And ſo of Moon, Stars, Water, Ayr, Fire, Wood, Stones, Birds, and Beaſts, and of many things elſe, a certain Sympathical forewarning may be had: ſometimes to great pleaſure and profit, both on Sea and Land. Sufficiently, for my preſent purpoſe, it doth appear, by the premiſſes, how *Mathematicall* the *Art* of *Navigation* is, and how it needeth and alſo uſeth other *Mathematicall Arts:* And now if I would go about to ſpeak of the manifold Commodities, comming to this Land, and others, by ſhips and *Navigation,* you might think, that I catch at occaſions, to uſe many words, where no need is.

Yet this one thing may I, (juſtly) ſay. In *Navigation*, none ought to have greater care, to be skilful, than our Engliſh Pilots. And perchance, ſome, would more attempt: and other ſome, more willingly would be aiding, if they wiſt certainly, What priviledg, God hath endued this Iſland with, by reaſon of ſcituation, moſt commodious for *Navigation*, to places moſt Famous and Rich. And though (of * Late) a young Gentleman, a ** Anno* 1567.
couragious Captain, was in great readineſſe, with good *S.H.G.*
hope, and great cauſes of perſwaſion, to have ventured for a Diſcovery, (either *Weſterly*, by *Cape de Paramantia:* or *Eſterly*, above *Nova Zemla*, and the *Cyrimiſſes*) and was at the very neer time of attempting, called and imployed otherwiſe (both then, and ſince,) in great good ſervice to his Country, as the Iriſh Rebels * have ** Anno* 1569.
taſted: Yet, I ſay, (though the ſame Gentleman, do

not hereafter deal therewith) ſome one or other ſhould liſten to the matter: and by good adviſe, and diſcreet circumſpection, by little and little, winne to the ſufficient knowledg of that Trade and Voyage: Which now I would be ſorry, (through careleſneſs, want of skil and courage) ſhould remain unknown, or unheard of. Seeing, alſo, wee are herein, half challenged by the learned, by half requeſt publiſhed. Thereof, verily, might grow commodity, to this Land chiefly, and to the reſt of the Chriſtian Common-wealth, far paſſing all riches and worldly Treaſure.

THAUMATURGIKE, is that Art Mathematicall, which giveth certain oꝛder, to make ſtrange woꝛks, of the ſenſe to be perceived, and of men greatly to be wondꝛed at. By ſundry means, this *Wonder-Work* is wrought. Some by *Pneumatithmie*: as the works of *Cteſibius* and *Hero*: ſome by weight, whereof *Timæus* ſpeaketh: ſome by Strings ſtrained, or Springs, therewith imitating lively Motions: ſome by other means, as the Images of *Mercury*: and the brazen head, made by *Albertus Magnus* which did ſeem to ſpeak. *Boethius* was excellent in theſe feats. To whom *Caſſiodorus* writing, ſaith. Your purpoſe is to know pꝛofound things, and to ſhew marvels. By the diſpoſitiou of your art, Metals do low: Diomedes of bꝛaſſe, doth blow a Trumpet loud, a bꝛazen Serpent hiſſeth: Birds made ſing ſwéetly. Small things we rehearſe of you, who can imitate the heaven, &c. Of the ſtrange ſelf-moving, which at Saint *Denis*, by *Paris*, * I ſaw once or twice (*Orontius* being then with me, in company) it were too ſtrange to tell. But ſome have written it: and yet, (I hope) it is there, of other to be ſeen. And by *Perſpective* alſo ſtrange things, are done: as partly (before) I gave you to underſtand in *Perſpective*: as, to ſee in the air aloft, the lively image of another man, either walking to and fro: or ſtanding ſtil. Likewiſe, to

* *Anno* 1551.

come into an houſe, and there to ſee the lively ſhew of Gold, Silver, or precious ſtones: and comming to take them in your hand, to finde nought but ayr. Hereby, have ſome men (in all other matters counted wiſe) fouly over-ſhot themſelves: miſdeeming of the means. Therefore ſaid *Claudius Cæleſtinus, Hodie magnæ literaturæ viros, & magnæ reputationis videmus, opera quædam quaſi miranda, ſupra Naturam putare, de quibus in Perſpectiva doctus cauſam faciliter reddiſſet.* That is. Now a dayes, we ſee ſome men, yea of great learning and reputation, to judge certain works as marvellous above the power of nature: of which works, one yͭ were ſkilfull in Perſpective, might eaſily have given the cauſe. Of *Archimedes Sphær, Cicero* witneſſeth. Which is very ſtrange to think on. For when Archimedes (ſaith he) did faſten in a Sphear, the movings of the Sun, Moon, and of the five other Planets, he did, as the God, which (in Timæus of Plato) did make the world. That one turning ſhould rule Motions moſt unlike in ſlowneſſe and ſwiftneſſe. But a greater cauſe of marvelling we have by *Claudianus*-report hereof. Who affirmeth this, *Archimedes work*, to have been of Glaſſe, and diſcourſeth of it more at large: which I omit. The Dove of wood, which the *Mathematician Archytas* did make to flye, is by *Agellius* ſpoken of. Of *Dædalus* ſtrange Images, *Plato* reporteth. *Homer* of *Vulcans Selfmovers,* (by ſecret wheels, leaveth in writing. *Ariſtotle* of his *Politicks* of both, maketh mention. Marvellous was the workmanſhip of late days, performed by good skil of *Trochilike*, &c. For in *Noremberg*, a Flye of Iron, being let out of the Artificers hand, did (as it were) fly about by the gueſts at the Table, and at length, as though it were weary, return to his Maſters hand again. Moreover, an artificiall Eagle, was ordered, to fly out of the ſame Town, a mighty way, and that aloft in the ayr, toward the Emperour coming thither: and follow-

De his quæ Mundo mirabiliter eveniunt, cap. 8.

Tuſc. 1.

☞ *

ed him, being come to the gate of the Town.* Thus you ſee what Art Mathematical can perform, when skil, wil, induſtry, and ability, are duly applyed to proof.

A Digreſſion. Apologeticall.

AND for theſe, and ſuch like marvellous Acts and Feats, Naturally, Mathematically, and Mechanically, wrought and contrived: ought any honeſt Student, and modeſt Chriſtian Philoſopher, be counted and called a Conjurer? Shal the folly of Idiots, and the malice of the ſcornful, ſo much prevail, that He, who ſeeketh no worldly gain or glory at their hands: But onely, of God, the treaſurer of heavenly wiſdome and knowledg of pure verity: Shal he (I ſay) in the mean ſpace, be robbed and ſpoiled of his honeſt name and fame? He that ſeeketh (by Saint *Pauls* advertiſement) in the Creatures, Properties, and wonderful vertues, to finde juſt cauſe, to glorifie the Æternal and Almighty Creator by: Shal that man, be (in hugger-mugger) condemned, as a Companion of the Helhounds, and a Caller, and a Conjurer of wicked and damned ſpirits? He that bewaileth his great want of time ſufficient (to his contentation) for learning of Godly Wiſedome and Godly Verities in: and onely therein ſetteth all his delight. Wil that man loſe and abuſe his time, in dealing with the chief enemy of Chriſt our Redeemer, the deadly foe of all mankind: the ſubtle and impudent perverter of Godly Verity: the Hypocritical Crocodile: the Envious Baſilisk, continually deſirous, in the twinke of an eye, to deſtroy all mankind both in body and ſoul, æternally? Surely (for my part, ſomwhat to ſay herein) I have not learned to make ſo brutiſh and ſo wicked a bargain. Should I, for my xx or xxv years ſtudie: for two or three thouſand Marks ſpending: ſeven or eight thouſand Miles going and travailing, onely for good learnings ſake: And that, in all manner of weathers: in all manner of ways and paſſages: both early and late: in danger of violence by man: in danger of deſtruction by wild beaſts: in hunger, in thirſt, in perilous heats by day, with toil on foot: in dangerous damps of cold, by night, almoſt bereaving life: (as God knoweth) with lodgings, oft times, to ſmal eaſe: and ſometime to leſſe ſecurity. And for much more (than all this) done and ſuffered for Learning, and atteining of Wiſedome: Should I (I pray you) for all this, no otherwiſe, or more wearily: (or by Gods mercifulneſſe) no more luckily, have fiſhed, with ſo large, and coſtly a Net, ſo long time in drawing (and that with the help and adviſe of Lady Philoſophy, and Queen Theologie) but at length to have catched, and drawn up * a Frog? Nay, a Devil? For, ſo, doth the common peeviſh pratler Imagine and Jangle: and ſo doth the malicious ſcorner, ſecretly wiſh, and bravely and boldly face down, behind my back. Ah, what a miſerable thing, is this kind of Men? How great is the blindneſſe and boldneſs of the multitude, in things above their Capacity? What a Land, what a People, what Manners, what Times are theſe? Are they become Devils themſelves: and by falſe witneſſe againſt their Neighbour, would they alſo become Murderers? Doth God, ſo long give them reſpite, to reclaim themſelves in, from this horrible ſlandering of the guiltleſſe: contrary to their own Conſciences: and yet wil they not ceaſe? Doth the Innocent forbear the calling of them, Juridically to anſwer him, according to the rigour of the Laws, and wil they deſpiſe his Charitable patience? as they, againſt him, by name, do forge, fable, rage, and raiſe ſlander, by Word and Print: Wil they provoke him, by Word and Print, likewiſe, to note their names to the World: vvith their deviſes, fables, beaſtly Imaginations, and unchriſtian like ſlanders? Well: Well. O (you ſuch) my unkind Countreymen! O unnatural Countreymen! O unthankful Countreymen! O Brain-ſick, Raſh, Spiteful and Diſdainful

* A Proverb, Fair fiſht and caught a Frog.

dainfull Countreymen. Why oppreſſe you me thus violently, with your ſlandering of me: Contrary to Verity: and contrary to your own Conſciences? And I, to this hour, neither by word, deed, or thought, have been, any way, hurtful,damageable, or injurious to you or yours? Have I, ſo long, ſo dearly, ſo far, ſo carefully, ſo painfully, ſo dangerouſly ſought and travailed for the learning of Wiſdome, and atteining of Vertue: And in the end (in your judgment) am I become, worſe, than when I began? Worſe than a Mad man? A dangerous Member in the Common-wealth: and no Member of the Church of Chriſt? Call you this to be Learned? Call you this, to be a Philoſopher? and a lover of Wiſdome? To forſake the ſtraight heavenly way: and to wallow in the broad way of damnation? To forſake the light of heavenly Wiſdome: and to lurk in the dungeon of the Prince of darkneſſe? To forſake the Verity of God, and his Creatures, and to fawn upon the impudent, craftie, obſtinate Liar, and continual diſgracer of Gods Verity, to the uttermoſt of his power? To forſake the Life and Bliſſe Eternal: and to cleave unto the Author of Death everlaſting? that murderous Tyrant, moſt greedily awaiting the Prey of Mans ſoul? Wel: I thank God, and our Lord Jeſus Chriſt, for the comfort, which I have by the Examples of other men, before my time: To whom, neither in godlineſſe of life, nor in perfection of learning, I am worthy to be compared: and yet they ſuſteined the very like injuries that I do: or rather greater. Patient *Socrates* his *Apologie* wil teſtifie: *Apuleius* his *Apologies*, wil declare the brutiſhneſſe of the Multitude: *Joannes Picus*, Earl of *Mirandula*, his *Apologie* wil teach you, of the Raging ſlander of the malicious ignorant againſt him. *Joannes Trithemius* his *Apologie* wil ſpecifie, how he had occaſion to make publick Proteſtation: as wel by reaſon o the Rude Simple: as alſo in reſpect of ſuch, as were

counted to be of the wiſeſt ſort of men. “ Many could
“ I recite: But I deferre the preciſe and determined
“ handling of this matter: being loth to detect the Fol-
“ ly and Malice of my Native Countreymen. * Who, ☞ *
“ ſo hardly, can diſgeſt or like any extraordinary courſe
“ of Philoſophical ſtudies: not falling within the Compaſſe
“ of their Capacity: or where they are not made
“ privie of the true and ſecret cauſe, of ſuch wonderful
“ Philoſophical Feats. Theſe men, are of four ſorts,
chiefly. The firſt, I may name, *Vain pratling buſie-bodies.*
The ſecond, *Fond Friends.* The third, *Imperfectly Zealous:*
and the fourth *Malicious Ignorant.* To each of
theſe (briefly, and in charity) I wil ſay a word or two,
and ſo return to my Præface. *Vain pratling buſie-bodies,* 1.
uſe your idle aſſemblies, and conferences, otherwiſe,
than in talk of matter, either above your capacities, for
hardneſſe: or contrary to your conſciences in verity.
Fond Friends, leave off, ſo to commend your unacquainted
friend upon blind affection: As, becauſe he knoweth 2.
more, than the common Student: that, therefore
he muſt needs be skilful, and a doer, in ſuch matter and
manner, as you term *Conjuring.* Weening thereby, you
advance his fame: and that you make other men, great
marvels of your hap, to have ſuch a learned friend.
Ceaſe to aſcribe Impiety, where you pretend Amity.
For, if your tongues were true, then were that your
friend *Untrue,* both to God and his Sovereign. Such
Friends and *Fondlings,* I ſhake off, and renounce you:
Shake you off, your Folly. *Imperfect Zealous,* to you, 3.
do I ſay: that (perhaps) wel, do you mean: but far
you miſſe the Mark: If a Lamb you wil kil, to feed the
flock with his bloud. Sheep, with Lambs bloud, have
no natural ſuſtenance: No more, is Chriſts flock, with
horrible ſlanders, duly edified. Nor your fair pretenſe,
By ſuch raſh ragged Rhetorique, any whit, wel graced.
But ſuch, as ſo uſe me, wil finde a foul crack in their

credit. Speak that you know: And know, as you ought: Know not, by Hear-ſay, when life lyeth in danger. Search to the quick, and let charity be your guide. 4. *Malicious Ignorant*, what ſhal I ſay to thee? *Probibe tuam linguam à malo. A Detractione parcitur linguæ.* Cauſe thy tongue to refrain from evill. Refrain your tongue from ſlaunder. Though your tongues be ſharpned Serpent-like, and Adders poyſon lye in your lips: Pſal.140. yet take heed, and think, betimes, with your ſelf, *Vir linguoſus non ſtabilietur in terra. Virum violentum venabitur malum, donec præcipitetur.* For, ſure I am, *Quia faciet Dominus Judicium afflicti, & vindictam pauperum.*

Thus, I require you my aſſured friends and Countreymen (you Mathematicians, Mechanicians, and Philoſophers; charitable and diſcreet) to deal in my behalf, with the light and untrue tongued, my envious Adverſaries or Fond friends. And farther, I would wiſh, that at leaſure, you would conſider, how *Baſilius Magnus* layeth *Moſes* and *Daniel* before the eyes of thoſe, which count all ſuch ſtudies Philoſophical (as minehath been) to be ungodly or unprofitable. Weigh wel Saint *Stephen* Acts 7. his witneſſe of *Moſes*. *Eruditus eſt Moſes omni Sapientia Ægyptiorum: & erat potens in verbis & operibus ſuis.* Moſes was inſtructed in all manner of wiſdome of the Ægyptians, and he was of power both in his words and works. You ſee this Philoſophical Power and Wiſdome. which *Moſes* had, to be nothing miſliked of the Holy Ghoſt. Yet *Plinius* hath recorded *Moſes* to be a wicked *Magician*. And that (of force) muſt be, either for this Philoſophical wiſdome, learned, before his calling to the leading of the Children of *Iſrael*: or for thoſe his wonders wrought before King *Pharoah*, after he had the conducting of the *Iſraelites*. As concerning the firſt, you perceive, how Saint *Stephen*, at his Martyrdome (being ful of the Holy Ghoſt) in his Recapitulation of the Old Teſtament, hath made mention of *Mo-*

ſes Philoſophie: with good liking of it: And *Baſilius*
Magnus, alſo avoucheth it, to have been to *Moſes* profitable
(and therefore I ſay, to the Church of God neceſſary.)
But as concerning *Moſes* wonders done before
King *Pharoah*: God himſelf ſaid: *Vide ut omnia oſtenta,*
quæ poſui in manu tua, facias coram Pharaone. **See**
that thou do all thoſe wonders before Pharoah, **which**
I have put in thy hand. Thus, you evidently perceive,
how raſhly, *Plinius* hath ſlandered *Moſes*, of vain fraudulent
Magick, ſaying: *Eſt & alia Magices Factio,* *Lib.30.cap.1.*
à Moſe, Jamne, & Jotape, Judæis pendens: ſed multis millibus
annorum poſt Zoroaſtrem, &c. Let all ſuch, therefore, 1.
who, in Judgment and Skil of Philoſophie, are far inferiour
to *Plinie,* take good heed, leaſt they over-ſhoot
themſelves raſhly, in judging of *Philoſophers ſtrange acts:*
and the means how they are done. But much more
ought they to beware of forging, deviſing, and imagining 2.
monſtrous feats, and wonderful works, when and
where no ſuch were done: no, not any ſpark or likelihood,
of ſuch, as they, without all ſhame, do report.
And (to conclude) moſt of all, let them be aſhamed of
Man, and afraid of the dreadful and juſt Judg: both 3.
fooliſhly or maliciouſly to deviſe: and then, devilliſhly
to father their new fond Monſters on me: Innocent in
hand and heart: for treſpaſſing either againſt the law
of God or man, in any my Studies, or Exerciſes, Philoſophical,
or Mathematical: As in due time, I hope, wil be
more manifeſt.

NOw end I, with ARCHEMASTRIE. Which name, is not ſo new, as this Art is rare. For another Art, under this, a degree (for skil and power) hath been indued with this Engliſh name before. And yet this may ſerve for our purpoſe, ſufficiently, at this preſent. **This art, teacheth to bring to actuall experience, ſenſible, all worthy concluſions by all the Arts Mathema-**

ticall purpoſed, and by true Naturall Philoſophie concluded: and both addeth to them a farther ſcope, in the termes of the ſame Arts, and alſo by his proper Method, and in peculiar termes, proceedeth, with help of the foreſaid Arts, to the performance of compleat Experiences, which of no perticular Art, are able (Formally) to be challenged. If you remember, how we conſidered *Architecture*, in reſpect of all common handworks: ſome light may you have, thereby, to underſtand the Sovereignty and property of this Science. *Science* I may cal it, rather, than an Art: for the excellency and Maſterſhip it hath, over ſo many, and ſo mighty Arts and Sciences. And becauſe it proceedeth by *Experiences*, and ſearcheth forth the cauſes of Concluſions, by *Experiences*: and alſo putteth the Concluſions themſelves, in *Experience*, it is named of ſome *Scientia Experimentalis*. The **Experimentall Science**, *Nicolaus Cuſanus* termeth it ſo, in his *Experiments Staticall*, and another *Philoſopher* of this Land Native (the flower of whoſe worthy fame, can never die nor wither) did write thereof largely, at the requeſt of *Clement the Sixt*. The art carrieth with it, a wonderful credit: by reaſon, it certifieth, ſenſibly, fully, and compleatly to the utmoſt power of Nature and Art. This art certifieth by *Experiments* compleat and abſolute: and other arts, with their arguments and demonſtrations, perſwade: and in words prove very wel their Concluſions. * But words and arguments, are no ſenſible certifying: nor the full and final fruit of Sciences practiſable. And though ſome arts have in them *Experiences*, yet they are not compleat, and brought to the uttermoſt, they may be ſtretched unto, and applyed ſenſibly. As for example: the natural Philoſopher diſputeth and maketh goodly ſhew of reaſon: And the Aſtronomer, and the Optical Mechanician put ſome things in *Experience*: but neither, all, that they may: nor yet ſufficiently, and t

R. B.

* ☞

the utmoſt, thoſe, which they do. There, then, the *Archmaſter*, ſteppeth in, and leadeth forth on, the *Experiences*, by order of his doctrine *Experimentall*, to the chief and finall power of Natural and Mathematical arts. Of two or three men, in whom this Deſcription of *Archmaſtry* was *Experimentally*, verified, I have read and heard: and good record, is of their ſuch perfection. So that this art is no fantaſtical imagination: as ſome Sophiſter, might, *Cum ſuis Inſolubilibus*, make a flouriſh: and dazle your imagination: and daſh your honeſt deſire and courage, from believing theſe things, ſo unheard of, ſo marvellous, and of ſuch importance. Wel: as you wil. I have forewarned you. I have done the part of a friend: I have diſcharged my Duty toward God: for my ſmal Talent, at his moſt merciful hands received. To this Science, doth the *Science Alnirangiat*, great Service. Muſe nothing of this name. I change not the name, ſo uſed, and in Print, publiſhed by others: being a name, proper to the Science. Uuder this, commeth *Ars Sintrillia*, by *Artephius*, briefly written. But the chief Science, of the Archmaſter, (in this world) as yet known, is another (as it were) OPTICAL Science: whereof, the name ſhal be told (God willing) when I ſhal have ſome, (more juſt) occaſion, thereof, to diſcourſe.

Here I muſt end, thus abruptly (Gentle friend, and unfeigned lover of honeſt and neceſſary verities.) For, they, who have (for your ſake, and vertues cauſe) requeſted me, (an old forworn Mathematician) to take pen in hand: (through the confidence they repoſed in my long experience: and tryed ſincerity) for the declaring and reporting ſomewhat, of the fruit and commodity, by the Arts Mathematicall to be atteined unto: even they, ſore againſt their wils are forced, for ſundry cauſes, to ſatisfie the workmans requeſt, in ending forthwith: He, ſo feareth this, ſo new an attempt, and ſo coſtly: and in matter ſo ſlenderly (hitherto) among the Common Sort of Students, conſidered or eſteemed.

And where I was willed ſomewhat to alledg, why, in our vulgar ſpeech, this part of the principal Science of *Geometry*, called *Euclids Geometricall Elements*, is publiſhed, to your handling: being unlatined people, and not Univerſity Scholers: Verily, I think it needleſſe.

1. For, the Honour, and Eſtimation of the Univerſities and Graduates, is, hereby nothing diminiſhed. Seeing, from, and by their Nurſe Children, you receive all this Benefit, how great ſoever it be.

2. Neither are their ſtudies hereby any whit hindred. No more than the Italian *Univerſities*, as *Academia Bononienſis, Ferrarienſis, Florentina, Mediolanenſis, Patavina, Papienſis, Peruſina, Piſana, Romana, Senenſis* or any one of them find themſelves, any deal diſgraced, or their Studies any thing hindred, by *Frater Lucas de Burgo*, or by *Nicolaus Tartalea*, who in vulgar Italian language, have publiſhed, not onely *Euclids Geometry*, but of *Archimedes* ſomewhat: and in Arithmetick, and Practical Geometry, very large Volumes, all in their vulgar ſpeech. Nor in *Germany*, have the famous *Univerſities*, any thing been diſcontent with *Albertus Durerus*, his Geometrical Inſtitutions in Dutch: or with *Gulielmus Xylander*, his learned tranſlation of the firſt ſix books of *Euclid*, out of the Greek, into the High Dutch Nor with *Gualterus H. Riffius*, his Geometrical Volume: very diligently tranſlated into the High Dutch tongue, and publiſhed. Nor yet the *Univerſities* of *Spain* or *Portugall*, think their reputation to be decayed: or ſuppoſe any their ſtudies to be hindred by the Excellent *P. Nonnius*, his Mathematical works, in vulgar ſpeech by him put forth. Have you not, likewiſe in the French tongue, the whole Mathematical Quadrivie? and yet neither *Paris, Orleans*, or any of the other Univerſities of *France*

at any time, with the Tranſlators, or Publiſhers offended: or any mans ſtudie thereby hindred?

3. And ſurely, the common and vulgar ſcholer (much more the Grammarian) before his comming to the *Univerſity*, ſhal (or may) be, now (according to *Plato* his Counſel) ſufficiently inſtructed in *Arithmetick* and *Geometry*, for the better and eaſier learning of all manner of *Philoſophie Academicall* or *Peripateticall*. And by that means, go more cheerfully, more skilfully, and ſpeedily forward in his ſtudies, there to be learned. And, ſo, in leſſe time, profit more, than (otherwiſe) he ſhould or could do.

4. Alſo ~~many good and~~ pregnant Engliſh wits, of young Gentlemen, and others, who never intend to meddle with the profound ſearch and ſtudie of Philoſophie (in the *Univerſities* to be learned) may neverthe leſſe, now, with more eaſe and liberry, have good occaſion, vertuouſly to occupie the ſharpneſſe of their wits: where, elſe (perchance) otherwiſe, they would in fond exerciſes, ſpend (or rather loſe) their time: neither ſerving God: nor furthering the Weal, common or private.

5. And great Comfort, with good hope, may the *Univerſities* have, by reaſon of this *Engliſh* Geometrie and Mathematicall Præface, that they (hereafter) ſhal be the more regarded, eſteemed, and reſorted unto. For when it ſhal be known and reported, that of the *Mathematicall Sciences* onely, ſuch great Commodities are enſuing (as I have ſpecified:) and that in deed, ſome of you unlatined ſtudents, can be good witneſſe, of ſuch rare fruit by you enjoyed (thereby:) as either, before this, was not heard of: or elſe not ſo fully credited: “Wel, may all men conjecture, that far greater aid and “better furniture, to win to the perfection of all Phi-“loſophy, may in the Univerſities be had: being the “Storehouſes and Treaſury of all Sciences and all Arts, ne-

Univerſities.

“ neceſſary for the beſt and moſt noble State of Com- “mon wealths.

6. Beſides this, how many a common Artificer, is there in theſe Realms of *England* and *Ireland*, that dealeth with Numbers, Rule and Compaſſe: Who, with their own skil and experience, already had, wil be able (by theſe good helps and informations) to finde out, and deviſe, new Works, ſtrange Engines, and Inſtruments: for ſundry purpoſes in the Common Wealth? or for private pleaſure? and for the better maintaining of their own eſtate? I wil not (therefore) fight againſt mine own ſhadow. For no man (I am ſure) wil open his mouth againſt this Enterprize. No man (I ſay) who either hath charity toward his brother, (and would be glad of his furtherance in vertuous knowledg:) or that hath any care and zeal for the bettering of the common ſtate of this Realm. Neither any, that make accompt, what the wiſer ſort of men (*Sage* and *Stayed*) do think of them. To none (therefore) wil I make any *Apologie*, for a vertuous act doing: and for commending, or ſetſetting forth, profitable arts, to Engliſh men in the Engliſh tongue. “ But, unto God our Creator, let us all ☞ “ be thankful: for that, **As he, of his goodneſſe, by his** “ **power, in his wiſdome, hath created all things in** “ **number, weight and meaſure**. So, to us, of his great “ mercy, he hath revealed means, whereby to attein the “ ſufficient and neceſſary knowledg of the foreſaid his “ three principal Inſtruments: Which means, I have “ abundantly proved unto you, to be the *Sciences* and “ *Arts Mathematicall*.

And though I have been pinched with ſtreightneſſe of time: that, no way, I could ſo pen down the matter (in my mind) as I determined: hoping of convenient leaſure: Yet, if vertuous zeal, and honeſt intent, provoke and bring you to the reading and examining of this Compendious Treatiſe, I do not doubt, but as the verity thereof (according to our purpoſe) wil be evident unto you: So the pith and force thereof, wil perſwade you: and the wonderful fruit thereof, highly pleaſure you. And that you may the eaſier perceive and better remember, the principal points, whereof my Preface treateth, I wil give you the **Groundplat** of my whole diſcourſe, in a Table annexed: from the firſt to the laſt, ſomewhat Methodically contrived.

The ground plat of this Preface in a Table.

If haſte, hath cauſed my poor pen, any where to ſtumble: You wil (I am ſure) in part of recompence, (for my earneſt and ſincere good wil to pleaſure you) conſider the rockiſh huge mountains, and the perilons unbeaten ways, which (both night and day, for the while) it hath toyled and laboured through, to bring you this good News, and Comfortable proof of Vertues fruit.

So I commit you unto Gods merciful direction,
for the reſt: heartily beſeeching him, to proſ-
per your Studies, and honeſt Intents:
to his Glory, and the Commo-
dity of our Country.
Amen.

Writen at my poor Houſe,
At Mortlake.

Anno 1570. Febr. 9.

THE PROPOSITIONS OF The firſt Six Books of EUCLIDS Elements:

In the Firſt Book.

1 V*Pon a Right line given, not being infinite, to deſcribe an equilaterall* Triangle, *or with three equall ſides.*

2 *From a point given, to draw a right line, that ſhall be equall to a right line given.*

3 *Two unequall right lines being given, to cut off from the longeſt, a right line equall to the ſhorteſt.*

4 *If there be two* Triangles,*of which, the two ſides of the one, be equall to two ſides of the other* Triangle, *each to his correſpondent ſide, and the* Angles, *which are conteined under thoſe lines, be alſo equall the one to the other; then is the third ſide (called tho* Baſe) *of the one, equall to the* Baſe *of the other; and the whole* Triangle *of the one, equall to the whole* Triangle *of the other, and the other* Angles *remaining, ſhall be alſo equall to the other* Angles *remainiug, the one to the other, under which are ſubtended equall ſides.*

5 *An* Iſoſceles *or* Triangle, *of two equall ſides, hath his* Angles *at the* Baſe, *equall the one to the other, and thoſe equall ſides being produced, the* Angles *that are under the* Baſe, *are alſo equall the one to the other.*

6 *If a* Triangle *have two* Angles *equall, the one to the other, the ſides alſo, which ſubtend the equall* Angles, *ſhall be equall the one to the other.*

7 *If from the two extream points of a line, two other lines be drawn, to any point. There cannot, from the ſelf ſame points or ends of the ſaid line (on the ſame ſide) be drawn, two other lines, equall to the firſt lines, the one to the other, unto any other point.*

8 *If two* Triangles *have two ſides of the one, equall to two ſides of the other, each to his correſpondent ſide, and alſo the* Baſe *of the one, equall to the* Baſe *of the other, then have thoſe* Triangles, *alſo two* Angles, *the one equall to the other, namely, thoſe* Angles *conteined nnder the equall ſides.*

9 *To divide a right lined* Angle, *into two equall parts.*

10 *To divide a right line, being finite, into two equall parts.*

11 *Uppon a right line given, and from a point given,*

in the ſame line, how to raiſe up a Perpendicular *line.*

12 *Unto a right line given, being infinite, and from a point given, without the ſame line, to draw a* Perpendicular *line.*

13 *If a right line doth ſtand upon another right line, making any* Angles *with the ſame, then are thoſe* Angles, *on both ſides of the ſtanding line, either two right* Angles, *or equall together, to two right* Angles.

14 *If from a point in a right line, two right lines be drawn, if then thoſe two* Angles *touching one another, at the ſame point, be together equall to two right* Angles, *then do thoſe two lines come ſo together, that they both together make one right line.*

15 *If two right lines do cut through another, then are the two* Angles, *which are oppoſite, equall the one to the other.*

16 *In all* Triangles, *the one ſide being produced forth, then is the outward angle greater than one of the inward* Angles, *that ſtand oppoſite to that* Angle, *which is produced.*

17 *In all* Triangles, *two* Angles, *which ſo ever be taken together, are leſſe than two right* Angles.

18 *In all* Triangles, *the greateſt ſide ſubtendeth the greateſt angle, and the leaſt ſide the leſſe* Angle.

19 *In every* Triangle, *under the greater* Angle *is ſubtended the greater ſide.*

20 *In all* Triangles, *two ſides (which ſoever be taken) are greater than the third ſide.*

21 *If from the two uttermoſt points of the ſides of a* Triangle, *be drawn two lines to a point, within the* Triangle, *then are thoſe two lines together, ſhorter than the two other ſides of the* Triangle, *but do include a greater* Angle.

22 *How to make a* Triangle, *of three right lines, which ſhall be equall to three given lines, of which it is required, that two of them, together, ſhall be longer than the third.*

23 *Upon a point, in a line given, to make a right line* Angle, *equall to a right line* Angle *given.*

24 *If in two* Triangles *right lined, two ſides of the one, be equall to two ſides of the other, but the one hath a greater* Angle *included, of thoſe ſides, than the other: then hath the ſame alſo, a longer* Baſe.

25 *If in two* Triangles, *the two ſides of the one, be equall to two ſides of the other, but the one hath a greater* Baſe *than the other, then hath the ſame alſo a greater* Angle *conteined under or between thoſe equall ſides.*

26 *If two* Triangles, *have two* Angles *of the one, equall to two* Angles *of the other, each to his correſpondent* Angle, *and have alſo one ſide of the one, equall to one ſide of the other, either that ſide, which lyeth between the equall* Angles, *or that which is ſubtended under the one of the equall* Angles, *the other ſides alſo of the one, ſhall be equall to the other ſides of the other, each to his correſpondent ſide, and the other* Angle *of the one, ſhall be equall to the other* Angle *of the other.*

27 *If a right line be drawn through two other right lines, making the alternate* Angles *equall, the one to the other, then are thoſe two right lines parallel.*

28 *If a right line be drawn through two other right lines, making the outward* Angle *of the one line, equall to the inward* Angle *of the other line (which ſtandeth on the ſame ſide of that line, which goeth through the other lines) or doth make the two inward* Angles, *on the*

one ſide, equall to two right Angles, *then are thoſe two lines parallel.*

29 *If one right line cut through two* Parallel *right lines, then are the* Angles *oppoſite, one againſt another, equall, And alſo the outward* Angles *are equall to the inward, that ſtandeth on the ſame ſide, over againſt the ſame, and the two inward angles that ſtandeth on the one ſide of the line, that cutteth through, are equall to two right* Angles.

30 *All thoſe right lines, which are parallel to one right line, thoſe lines are alſo parallel one to another.*

31 *From a given point, to draw a right line, which ſhall be parallel to a right line given.*

32 *In all* Triangles, *if the one ſide be produced forth, then is that outward* Angle, *equall to both the inward* Angles, *oppoſite againſt the ſame, alſo all the three inward* Angles *together, are equall to two right* Angles.

23 *If two equall parallel lines be joyned together with two other right lines, then are thoſe lines, alſo equall and parallel.*

34 *In all* Parallelograms, *the* Angles *and ſides that are oppoſite, one to another, are equall, and the* Diameter *or* Diagonall *line, doth divide the ſame, into two equall parts.*

35 Parallelograms, *which do ſtand upon one, and the ſame* Baſe, *and between the ſame parallel lines, are equall the one to the other.*

36 *All* Parallelograms, *which ſtandeth upon equall* Baſes, *and between two parallel right lines, are equall.*

37 *All* Triangles, *that have one and the ſame* Baſe, *and ſtand between two* Parallel *lines, are equall.*

38 *All* Triangles, *which do ſtand upon equall* Baſes, *and between two* Parallel *lines, thoſe* Triangles, *are equall.*

39 *All* Triangles, *which are equall, and ſtand upon one* Baſe, *thoſe* Triangles *ſtand alſo between two* Parallel *lines.*

40 *All* Triangles, *that are equall, and ſtand uppon equall* Baſes, *they ſtand alſo between two* Parallel *lines.*

41 *If a* Parallelogram *and a* Triangle, *ſtand upon one and the ſame* Baſe, *and between two* Parallel *lines, then is that* Parallelogram, *twice as great (or double to) the* Triangle.

42 *How to make a* Parallelogram, *equall to a given* Triangle, *and that the ſame ſhall have an* Angle *equall to a given* Angle.

43 *In all* Parallelograms, *the* Suppliments, *(or* Fillings) *which doth ſtand upon the* Diameter, *are equall.*

44 *Upon a right line given, how to make a* Parallelogram, *that ſhall be equall to a* Triangle *given, and alſo ſhall have an* Angle, *equall to a right line* Angle *given.*

45 *How to make a* Parallelogram, *equall to a right line* Figure *given, and that the ſame ſhall have an* Angle, *equall to a given* Angle.

46 *Upon a right line given, to make a* Square.

47 *In right angled* Triangles, *the* Square, *which is made of the ſide ſubtending the right* Angle, *is equall to the* Squares, *which are made of the ſides, conteining the right* Angle.

48 *If the* Square, *which is made of one ſide of a* Triangle, *be equall to the* Squares, *which are made of the*

two other ſides, of the ſame Triangle, *the* Angle *comprehending under thoſe ſides is a right* Angle.

In the ſecond Book.

1 T*Wo right lines being given, whereof the one is divided by chance, into as many parts as you will, then is the right angle* Figure *comprehended of thoſe two whole lines, equall to all the right angle* Figures *together, that are comprehended of the one of the whole lines, and each of the parts of the other line.*

2 *If a right line, be divided by chance, into parts, then are the right angled* Figures, *included with the ſame line, and each of the parts, equall to the* Square *of the whole line.*

3 *If a right line be divided by chance, into two parts, then is the right* Angle *comprehended of the ſame line, and one part of the ſame, eqnall to the right angle* Figure, *made of the parts, with the* Square *of the ſaid part.*

4 *If a right line be divided by chance, whether it be into equall, or unequall parts, then is the* Square *of the whole line, equall to the* Squares *of the parts, with two right angle* Figures, *comprehended of the parts of the ſame.*

5 *If a right line be divided, in two equall parts, and in two unequall: the right angle figure included with the unequall parts, with the* Square *of the difference of the*

the parts, are equall to the Square, that is made of the half line.

6 If a right line be divided into two equall parts, and then there be another line added to the ſame, directly in one right line, then is the right angle Figure incloſed with the whole line, and the added line (taken together, for one line) with the line added; with the Square of the half line, equall to the Square of the half line, together with the added line, as one line.

7 If a right line be divided by chance, into two parts, then is the Square of the whole line, with the Square of one of the parts, equall to two right angle Figures, made of the whole line, and the aforeſaid part, with the Square of the other part.

8 If the right line be divided by chance, into two parts then is the right line Figure made of the whole line, and one of the parts four times, with the Square of the other part, equall to the Square of the whole line, and the firſt part, taken together as one line.

9 If a right be divided into two equall parts, and into two unequall parts, then are the two Squares of the unequall parts together, double to the Square of the half line, with the Square of the difference of the half line, and one of the unequall parts together.

10 If a right line be divided into two equall parts, and another right line be added in a right line with the ſame, then is the Square of the whole line, together with the added line (as being one line) with the Square of the added line together, double to the Square of the half line, with the Square of the half line, and added line together, as one line.

11 To divide a right line in two parts, ſo that the right angle Figure, made of the whole line, and one

part, ſhall be equall to the Square *of the other part.*

12 *In all obtuſe angled* Triangles, *the* Square *of the ſide ſubtending the obtuſe* Angle *is greater than the two* Squares *which are made of the ſides that comprehend the obtuſe* Angle, *by two right* Angle Figures, *made of the one ſide, together with the extending of the ſame ſide, till a* Perpendicular *falling from the upper point of the* Triangle, *doth cut the extended line.*

13 *In all ſharpe or acute angled* Triangles, *the* Square *which is made of the ſide ſubtendiug the ſharp* Angle *is leſſer than the* Squares, *made of the two ſides comprehending the ſame* Angle, *by two right* Angle Figures, *made of the ſide (whereupon the* Perpendicular *from the upper* Angle, *doth fall from the uttermoſt point) together with the part from the* Perpendicular, *to the utter point of the ſharp* Angle *of the ſaid* Triangle.

14 *To make a* Square, *equall to a given right line* Figure.

15 *To make a* Gnomon *about a* Square, *that ſhall be equall to a given* Square.

In the third Book.

1 T*O finde the* Center *of a* Circle.

2 *If two points be taken, in the* Circumference *of a* Circle, *by chance, and from the one poiut to the other, be drawn a right line, that line doth fall within the* Circle.

3 *If a right line be drawn through the* Center *of a*

Circle, *and within the ſame* Circle, *it doth cut through another right line* (*which doth not go through the* Center) *in two equall parts, then doth thoſe right lines, make* (*at the* Interſection) *right* Angles, *and if thoſe lines in the* Interſection, *do make right* Angles, *then is that line, which goeth not through the* Center, *cut in two equall parts, with the other line.*

4 *If in a* Circle, *two right lines do cut through one another, whereof neither of them both doth goe through the* Center, *then are neither of thoſe lines divided into equall parts.*

5 *If two* Circles *cut the one the other, they have not one and the ſame* Center.

6 *Two* Circles, *which doth inwardly touch one another, have not one and the ſame* Center.

7 *If in the* Diameter *of a* Circle, *be taken any point, which is not the* Center *of the* Circle, *and from that point, be drawn unto the* Circumference *certain right lines, the greateſt of thoſe lines, ſhall be that line wherein the* Center *is, and the leaſt ſhall be the reſidue of the ſame line, and of all the other lines, that which is neereſt to the line which paſſeth through the* Center, *is greater than that which is further diſtant, and from that point, can fall within the* Circle, *on each ſide of the leaſt line, onely two equall right lines.*

8 *If without a* Circle *be taken any point, and from that point be drawn into the* Circle, *unto the* Circumference, *certain right lines, of which, let one be drawn by the* Center, *and let the reſt fall in the Concavity, by adventure, then the greateſt of thoſe lines which fall in the hollow of the* Circumference *of the* Circle, *is that which paſſeth by the* Center, *and of all the other lines, that line which is neerer to that which paſſeth by the* Center, *is*

greater, then that which is more diſtant, but of thoſe right lines, which end at the convex ſide, or part of the Circumference, *that is the leaſt, which is drawn from the point to the* Diameter, *and of the other lines, that which is neerer to the leſſer, is leſſe then that which is more diſtant, and from that point, can be drawn unto the* Circumference, *on each ſide of the leaſt line, onely two equall right lines.*

9 *If within a* Circle, *be taken a point, and from that point be drawn unto the* Circumference, *more than two equall right lines, then is that point, the* Center *of the* Circle.

10 *One* Circle *doth not cut through another, in more places than two.*

11 *If two* Circles *do touch one another inwardly, the* Centers *being given, and from the one* Center *to the other, be drawn a right line, and produced forth, the ſame ſhall cut the* Circumferences *in the point of touching.*

12 *If two* Circles *touch one another, outwardly, then a right line drawn from one* Center, *to the other, doth paſſe through the touch point.*

13 *Two* Circles *cannot touch one another in more than one place, whether it be inwardly or outwardly.*

14 *In a* Circle, *equall right lines, are equally diſtant from the* Center, *and right lines, equally diſtant from the* Center, *are equall.*

15 *In a* Circle, *the greateſt line is the* Diameter, *and of all the other lines, that next the* Diameter *or* Center *is greater than that farther diſtant.*

16 *If from the uttermoſt point of the* Diameter *of a* Circle *be drawn a line* Perpendicular *to the ſame, that right line doth fall without the* Circle, *and between the*

ſame

ſame right line, and the Circles Circumference *cannot be drawn, another right line, and the* Angle *of the half* Circle, *and* Diameter, *is greater than any acute* Angle, *made of right lines, and the* Angle, *without the* Circumference, *is leſſer than any right line* Angle.

17 *From a point given, without a* Circle, *to draw a line, that ſhall touch a* Circle *given.*

18 *If a right line do touch a* Circle, *and from the* Center *be drawn a right line to the point of touching, then is that line* Perpendicular *to the touch line.*

19 *If a right line do touch a* Circle, *and from the point of touching, be raiſed upon the ſame line, a* Perpendicular *through the* Circle, *in the ſame* Perpendicular *line, is the* Center *of the* Circle.

20 *In a* Circle, *an* Angle *at the* Center *is double to an* Angle *at the* Circumference, *ſo that both the* Angles *have to their* Baſe *the ſame part of the* Circumferen.

21 *In a* Circle, *the* Angles *which ſtand upon one and the ſame part of the* Circumference *are equall one to another.*

22 *If within a* Circle, *be deſcribed a* Figure *of four ſides, the* Angles *thereof, which are oppoſite one to another, are together equall to two right* Angles.

23 *Upon one and the ſame right line, cannot be deſcribed two ſegments of a* Circle, *which ſhall be alike in form, and unequall in quantity.*

24 *Like ſegments of* Circles, *deſcribed upon equall right lines, are equall one to another.*

25 *A ſegment of a* Circle, *being given, to deſcribe the whole* Circle, *whereof the ſegment is a part.*

26 *Equall* Angles *in equall* Circles, *do ſtand upon equall parts of the* Circumference, *whether the* Angles *be in the* Center, *or in the* Circumference *of the* Circle.

27 *In all* Circles, *the* Angls *which ſtand upon equall parts of the* Circumference, *are equall the one to the*

other, whether thoſe Angles *come at the* Center, *or in the* Circumference.

28 *In equall* Circles, *equall right lines doe cut off equall parts of the* Circumference, *the greateſt part to the greateſt, and the leaſt to the leaſt.*

29 *In equall* Circles, *equall parts of the* Circumference, *are ſubtended with equall right lines.*

30 *To divide the part of a* Circumference *of a* Circle, *into two equall parts.*

31 *In a* Circle, *the* Angle *which ſtandeth in a half* Circle, (*that is upon the* Diameter) *is a right* Angle, *but an* Angle *that ſtandeth in a ſegment, that is greater than a half* Circle, *that* Angle *is leſſe than a right* Angle, *and an* Angle *that ſtandeth in a ſegment, that is leſſe than a half* Circle, *that* Angle *is greater then a right* Angle, *and the* Angle *of the greater ſegment is alſo greater than a right* Angle, *and the* Angle *of the leſſer ſegment, is leſſer than a right* Angle.

32 *If a right line touch a* Circle, *and from the touchpoint be drawn a right line, cutting the* Circle *through, the* Angles *which that line and the touch doth make in the touchpoint, are equall to the* Angles *which can be made in the ſegments of the* Circle, *each in his alternate ſegment.*

33 *Upon a right line given, to deſcribe a ſegment of a* Circle, *which ſhall contein an* Angle *equall to a given right line* Angle.

34 *From a given* Circle, *to cut off a ſegment, which ſhall contein an* Angle *equall to a given* Angle.

35 *If in a* Circle, *two right lines do cut through the one the other; the right angled* Parallelograms, *comprehended under the parts of the one line, is equal to the right angled* Parallelogram *comprehrnded under the parts of the other line.*

36 *If without a* Circle *be taken a point by adventure, &*

from that point be drawn two right lines, to the Circle, *ſo that the one of them do cut the* Circle, *and the other doth touch the ſame* Circle, *the right angled* Parallelogram, *which is made of the whole right line, which doth cut the* Circle, *and of that part of the ſame line, which lieth between the point and the utter* Circumference *of the* Circle, *is equall to the* Square *that is made of the line that toucheth the* Circle.

37 *If without a* Circle, *a point be taken by adventure, and from that point be drawn to the* Circle, *two right lines, of which, the one doth cut the* Circle, *and the other doth fall upon the* Circle, *and that in ſuch ſort, that the right angle* Figure *or* Parallelogram, *which is conteined under the whole line, which cutteth the* Circle, *and that part of the ſame line, that is between the point, and the utter* Circumference *of the* Circle, *is equall to the* Square *of the line falling upon the* Circle, *then that line, which ſo falleth upon the* Circle *ſhall touch the* Circle.

In the fourth Book.

1 I*N a given* Circle, *to apply a right line equall to a right line given, which doth not exceed the* Diameter *of the* Circle.

2 *In a given* Circle, *to deſcribe a* Triangle, *equiangled to a given* Triangle.

3 *About a* Circle *given, to deſcribe a* Triangle *equiangled, to a* Triangle *given.*

4 *In a* Triangle *given, to deſcribe a* Circle.

5 *About a* Triangle *given, to deſcribe a* Circle.

6 *To deſcribe a* Square *within a given* Circle.

7 *About a* Circle *given, to deſcribe a* Square.

8 *In a given* Square *to inſcribe a* Circle.

9 *About a* Square *given, to deſcribe a* Circle.

10 *To make a* Triangle *of two equall ſides, called* Iſoſceles, *which ſhall have either, or each* Angle *at the* Baſe *double to the other* Angle.

11 *In a* Circle *given, to deſcribe a* Pentagon Figure, *or an equilaterall, and equiangled five* Angle.

12 *About a given* Circle, *to deſcribe an equilaterall and equiangled* Pentagon *or five* Angle Figure.

13 *An equilaterall and equiangle* Pentagon Figure *being given, to inſcribe in it a* Circle.

14 *About a* Pentagon *or* Figure *of five* Angles *given, being equilaterall, and equiangled, to deſcribe a* Circle.

15 *In a* Circle *given, to deſcribe a* Hexagon, *or a* Figure *equilaterall and equiangle, of ſix* Angles.

16 *In a given* Circle, *to inſcribe a* Quindecagon, *or a* Figure *of fifteen* Angles, *equilaterall and equiangled.*

In the fift Book.

1 IF *there be a number of* Magnitudes, *how many ſoever, equimultiply to a like number of* Magnitudes, *each to each: how* Multiplex *one* Magnitude *is to one, ſo* Multiplex *are all the* Magnitudes *to all.*

2 *If the firſt be equimultiplex to the ſecond, as the third is to the fourth, and if the fiſt be alſo equimultiplex to the ſecond, as the ſixt is to the fourth, then ſhall the firſt and the fiſt added together, be equimultiplex to the ſecond, as the third and ſixt added together, is to the fourth.*

3 *If there be four* Magnitudes *in* Proportion, *ſo that the firſt is to the ſecond, as the third is to the fourth, and if there come two* Magnitudes *more to the ſame, in ſuch ſort, that the* Proportion *of the fiſt is to the firſt, as the ſixt is to the third, then is the fiſt alſo to the ſecond, as the ſixt is to the fourth.*

4 *If the firſt, be to the ſecond in the ſame* Proportion *that the third is to the fourth, then alſo the equimultiplices of the firſt and third, unto the equimultiplices of the ſecond and of the fourth (according to any* Multiplication) *ſhall have the ſame* Proportion, *being compared together: the fiſt to the firſt, as the ſixt to the third, and the ſeventh to the ſecond, as the eight to the fourth, then are the four laſt* Magnitudes *alſo proportioned.*

5 *If a* Magnitude *be equimultiplex to a* Magnitude, *as a part taken away of the one, to a part taken away of the other, the reſidue of the one, to the reſidue of the other ſhall be alſo equimultiplex, as the whole is to the whole.*

6 *If four* Magnitudes *be proportioned, and from the firſt and the third, a part be taken away, ſo that the firſt part be to the ſecond* Magnitude, *as the ſecond part is to the fourth* Magnitude, *then is the firſt reſidue, to the ſecond* Magnitude, *as the ſecond reſidue is to the fourth* Magnitude.

7 *Equall* Magnitudes *have to one and the ſame* Magnitude, *one and the ſame* Proportion, *and one and the*

ſame Magnitude, hath to equall Magnitudes, and the ſelf ſame proportion.

8 *Unequall Magnitudes being taken, the greater hath to one and the ſame Magnitude, a greater proportion then hath the leſſer, and that one and the ſame Magnitude hath to the leſſe, a greater proportion, then it hath to the greater.*

9 Magnitudes *which have to one and the ſame* Magnitude, *one and the ſame* Proportion, *are equall the one to the other, and theſe* Magnitudes *unto whom, one and the ſame* Magnitude *hath one and the ſame* Proportion *are alſo equall*

10 *Of* Magnitudes *compared to one and the ſame* Magnitude, *that which hath the greater* Proportion, *is the greater, and that* Magnitnde, *whereunto one and the ſame* Magnitude *hath the greater* Proportion *is the leſſer.*

11 Proportions, *which are one and the ſelf ſame to any one* Proportion, *are alſo the ſelf ſame the one to the other.*

12 *If there be a number of* Magnitudes, *how many ſoever proportionall, as one of the* Antecedents *is to one of the* Conſequents, *ſo are all the* Antecedents *to all the* Conſequents.

13 *If the firſt hath to the ſecond, the ſame proportion, that the third hath to the fourth, and if the third hath to the fourth a greater proportion than the fift hath to the ſixt: then ſhall the firſt alſo have unto the ſecond, a greater* Proportion *than hath the fift to the ſixt.*

14 *In four* Magnitudes, *if the firſt have to the ſecond the ſelf ſame* Proportion, *that the third hath to the fourth, and if the firſt be greater than the third, the ſe-*

cond is alſo greater than the fourth, and if it be equall, then equall, if leſſe, then leſſe.

15 *Like parts of* Multiplices, *and alſo their* Multiplices *compared together, have one and the ſame* Proportion.

16 *If four* Magnitudes *be proportionall, then alternatelie they are alſo proportionall.*

17 *If four* Magnitudes *be proportionall, being compoſed, then are the ſame alſo proportionall, being divided.*

18 *If* Magnitudes *divided, be proportionall, then alſo compoſed, they ſhall be proportioall.*

19 *If the whole be to the whole, as the part taken away, is to the part taken away, then ſhall the reſidue be to the reſidue, as the whole, is to the whole.*

20 *If there be three* Magnitudes *in one order, and as many other* Magnitudes *in another order, which being taken two and two, in each order, are in one and the ſame* Proportion, *and if of* Æquality, *in the firſt order, the firſt be greater than the third, then in the ſecond order, the firſt ſhall be greater than the third, and if it be equall, then equall, and if it be leſſe, then leſſe.*

21 *If ſix* Magnitudes *be proportioned without order, that is, that the firſt be to the ſecond, as the fift is to the ſixt, and the fourth be to the fift, as the ſecond is to the third, then (like as in the former* Propoſition) *ſhall the firſt be to the third, as the fourth is to the ſixt.*

22 *If there be ſix* Magnitudes *in* Proportion *one to another (according to the Contents of the* 20 Propoſition) *then hath the firſt ſuch* Proportion *to the third, as the fourth hath to the ſixt.*

23 *If there be three* Magnitudes *in one order, and as many in another order, which being taken two and two in*

each order, are in one and the ſame Proportion, *and if alſo their* Proportion *be perturbate, then of* Equality, *they ſhall be in one and the ſame* Proportion.

24 *This* Propoſition *is in effect of the ſame contents of the former ſecond of this book, and therefore needeth not to be repeated, onely there is ſpoken of* Multiplices, *and here of* Magnitudes.

25 *If four* Magnitudes *be in* Proportion, *then are the greateſt and leaſt (or the firſt and fourth) greater than the other two.*

In the ſixt Book.

TRiangles *and* Parallelograms *which are under one and the ſelf ſame height, are in* Proportion, *as the* Baſe *of the one is to the* Baſe *of the other.*

2 *If to any one ſide of a* Triangle, *be drawn a* Parallel *right line, it ſhall cut the ſides of the ſame* Triangle *proportionally, and if the ſides of a* Triangle *be cut proportionally, a right line drawn from* Section to Section, *is a* Parallel *to the other ſide of the* Triangle.

3 *If an* Angle *of a* Triangle, *be divided into two equall parts, and if the right line which divideth the* Angle, *divide alſo the* Baſe, *the ſegments of the* Baſe *ſhall be in the ſame* Proportion *the one to the other, that the other ſides of the ſaid* Triangle *are, and if the ſegments of the* Baſe *be in the ſame* Proportion *that the other ſides of the ſaid* Triangle *are, a right line drawn from the top of the* Triangle *to the* Section, *ſhall divide the* Angle of the Triangle *into two equall parts.*

4 *In equiangled* Triangles, *the ſides which contein the equall* Angles *are proportionall, and the ſides which are ſubtended under the equall* Angles *are of like* Proportion.

5 *If two* Triangles *have their ſides proportionall, the* Triangles *are equiangled, and thoſe* Angles *in them are equall, under which are ſubtended, ſides of equall* Proportion.

6 *If there be two* Triangles, *whereof the one hath one* Angle *equall to one* Angle *of the other, and the ſides including the equall* Angles, *be proportionall, the* Triangles *ſhall be equiangled, and thoſe* Angles *in them ſhall be equall, under which are ſubtended ſides of like* Proportion.

7 *If there be two* Triangles, *whereof the one hath one* Angle *equall to one* Angle *of the other, and the ſides which include the other* Angles, *be proportionall, and if either of the other* Angles *remaining, be either leſſe or not leſſe than a right* Angle, *then ſhall the* Triangles *be equiangled, and thoſe* Angles *in them ſhall be equall, which are conteined under the ſides proportionall.*

8 *If in a right angled* Triangle, *be drawn from the right* Angle, *unto the* Baſe, *a* Perpendicular *line, the* Perpendicular *line ſhall divide the* Triangle *into two* Triangles *like to the whole, and alſo like the one to the other.*

9 *A right line being given, to cut off from the ſame any part required.*

10 *To divide a right line given, not divided, like unto a right line given, being divided.*

11 *Unto two right lines given, to finde a third in* Proportion *to them.*

12 *Unto three right lines given, to finde a fourth in* Proportion *with them.*

13 *Unto two right lines given, to finde out a mean* Proportionall.

14 *In equall* Parallelograms, *which have one* Angle *of the one equall to one* Angle *of the other, the ſides ſhall be reciprocall, namely, thoſe ſides which contein the equall* Angles, *and if* Parallelograms *which have one* Angle *of the one, equall to one* Angle *of the other, have alſo their ſides reciprocall, namely, thoſe which contein the equall* Angles, *they ſhall alſo be equall.*

15 *In all equall* Triangles, *which have one* Angle *of the one equall to one* Angle *of the other, thoſe ſides are reciprocall, which include the equall* Angles, *and thoſe* Triangles, *which having one* Angle *of the one, equall to one* Angle *of the other, have alſo their ſides which doth include the equall* Angles *reciprocall, are alſo equall.*

16 *If there be four right lines in* Proportion, *the right* Angle Figure *made of the extreams, is equall to the right* Angle Figure, *which is conteined under the means, and if the right* Angle Figure *conteined under the extreams, be equall to the right* Angle Figure, *which is conteined under the means, then are thoſe four lines in* Proportion.

17 *If there be three right lines in* Proportion, *the right* Angle Figure *that is included of or under the extreams, is equall to the* Square *which is made of the mean, and if the right* Angle Figure, *which is made of the extreams, be equall to the* Square *of the mean, then are thoſe three right lines proportionall.*

18 *Upon a right line given, to deſcribe a right line* Figure, *like, and in like ſort ſituate, to a right line* Figure *given.*

19 *Like* Triangles *are one to another in duple* Proportion, *that the ſides of like* Proportion *are.*

20 *Many ſided* Figures *right lined, which are like ſituate, may be divided into like* Triangles *ſituated, equall in number, and of like* Proportion *to the whole, and the one of thoſe many ſided* Figures, *is to the other* Figure, *in duple* Proportion, *that the one of the ſides of like* Proportion, *is to one of the ſides of like* Proportion.

21 *Right lined* Figures, *which are like to one and the ſame right line* Figure, *are alſo the one to the other alike.*

22 *If there be four right lines proportionall, the right line* Figures *alſo deſcribed upon them, being like, and in like ſort ſet, ſhall be proportionall, and if the right line* Figure *deſcribed upon them, be proportionall, thoſe right lines alſo ſhall be proportionall.*

23 *Equiangled* Parallelograms, *have the one to the other, that* Proportion *which is compoſed of their ſides.*

24 *In every* Parallelogram, *the* Parallelograms *about the* Diametient *or* Diagonall, *are like unto the whole, and alſo like the one the other.*

25 *Unto a right line* Figure *given, to deſcribe another* Figure *like, which ſhall alſo be equall unto another right line* Figure *given.*

26 *If from a* Parallelogram *be taken away a* Parallelogram, *like unto the whole, and in like ſort ſet, having alſo an* Angle *common with it, then is the* Parallelogram *about one and the ſame* Diagonall *line with the whole.*

27 *If upon the half of a right line be made a* Parallelogram, *and upon a part of the ſame line, be made another* Parallelogram *in ſuch ſort, that a* Parallelogram *made upon the part remaining, like unto the firſt, and in like ſort ſituate, and ſtanding upon the ſame* Diametient, *then is the* Parallelogram *ſtanding upon the half line, greater then that which ſtandeth upon the firſt part*

28 *Upon a given right line, to make two* Parallelograms *of one and the ſame height, whereof the one ſhall be in forme and ſituation like to a given* Parallelogram, *and the other ſhall be in quantity equall to a right line* Figure *given (if the* Figure *given do not exceed a* Parallelogram, *made of the half line given, and like and in like ſort ſituate to a* Parallelogram *given) & wanting in* Figure *by the* Parallelogram *firſt named, to the filling up of the line given.*

29 *Upon a given right line, to make a* Parallelogram *equall to a right line* Figure *given, and exceeding in* Figure, *by a* Parallelogram, *like unto a given* Parallelogram

30 *To divide a right line given, by extream and mean* Proportion.

31 *In right angle* Triangles, *the* Figures *made of the ſide ſubtending the right* Angle, *is equall to the* Figures *made of the ſides comprehending the right* Angle, *ſo that the ſaid three* Figures *be like, and in like ſort ſituate or deſcribed.*

32 *If two* Triangles *be ſet together at one* Angle, *having two ſides of the one proportionall to two ſides of the other, ſo that their ſides of like* Proportion *be alſo parallels, then the other ſides remaining of theſe* Triangles *ſhall be in one right line.*

33 *In equall* Circles, *the* Angles *have one and the ſame* Proportion *that the* Circumferences *have, wherein they conſiſt, whether the* Angles *be ſet at the* Centers *or at the* Circumferences, *and in like ſort are the* Sectors, *which are deſcribed upon the* Centers.

Prop. 1 T*O make two right line* Figures *like & equall and in like ſort ſituate, unto a right line* Figure *given, which ſhall alſo have a* Proportion *given.*

2 *If two right lines cut one another Obtuſe angled wiſe and from the ends of thoſe lines, which cut one another be*

drawn lines perpendicular to either line, the lines which are between the ends, and thoſe Perpendicular *lines are cut reciprocally.*

3 *If two right lines make an acute* Angle, *and from their ends be drawn to each line,* Perpendicular *lines cutting the ſame, the two right lines given, ſhall be reciprocally cut proportionall, as the ſegments which are about the* Angle.

4 *If in a* Circle, *be drawn two right lines, cutting the one the other, the* Sections *of the one, to the* Sections *of the other, ſhall be reciprocally proportionall.*

5 *If from a point given, (upon a plain* Superficies) *be drawn two right lines to the concave* Circumference *of the* Circle, *theſe lines ſhall reciprocally be proportioned with their parts taken without the* Circle, *and moreover a right line drawn from the ſaid point, and touching the* Circle, *ſhall be a mean proportionall, between the whole line, and the utter part or ſegment.*

6 *If two right lines do ſtand between two* Parallel *lines, which are cut through with a third* Parallel *line, that is done in equall* Proportion.

EUCLIDES ELEMENTS.

The Firſt Booke.

THE ARGUMENT.

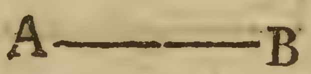
THis firſt Book treateth of the moſt ſimple, eaſie, and firſt grounds of *Geometry*; as of *Lines*, *Angles*, *Triangles* *Parallels*, *Squares*, and *Parallelograms*: firſt defining what they are, and after to draw parallel lines, and to form divers *Figures* according to the variety of their ſides and angles, and compareth them with *Triangles*, and alſo one with another. In this Book alſo is taught how a figure of one form may be reduced or changed into a figure of another form. This Book being of moſt frequent and common uſe, is put in the firſt place, as having dependance on none of thoſe which follow, though they do much depend on this. And foraſmuch as all the *Demonſtrations* and *Proofs* of all the *Propoſitions* in this Book do depend upon the grounds and principles next following, which by reaſon of their plainneſſe need no declaration, yet to remove all inconveniencies, a briefe explanation of them followeth.

DEFINITIONS.

1 A Point *is that which hath no part.*

It is called in Greeke σημεῖον, that is, a *Signe*, becauſe it is void of all Magnitude, as the *point* above noted with A, is a *ſigne* of that which is conceived in the minde: It is the ſame with *Unity* in *Number*, with *Inſtant* in *Time*, and with *Sound* in *Muſick*.

2 *A* Line *is length without breadth.*



There appertain to *quantity* three *dimenſions*, as *length*, *breadth*, and *thickneſſe*, and by theſe three are all quantities meaſured, the firſt of which is a *Line*, defined as above, to be a length without breadth or thickneſſe, as the line A B, becauſe a *point*, from whence it proceedeth, is void of all quantity.

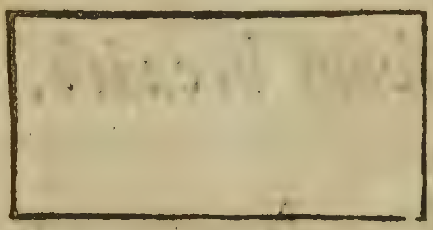
3 *The ends or limits of a* line *are points.*

A *line* hath his beginning from a *point*, and likewiſe endeth in a *point*, ſo that it is manifeſt, that *points* are neither *quantity* nor parts of *quantity*, but only the terms and ends of *quantity*, as the points A and B are only the ends of the *line* A B, and no parts thereof.

4 *A* right line *is that which lyeth equally between his points.*

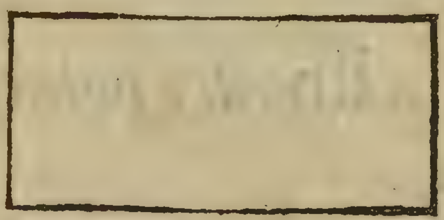
As the whol line A B, lyeth ſtreight and equally between the points A and B, without any going up or comming down on either ſide; being the ſhorteſt extent that is or may be from one point to another.

5 *A* Superficies *is that which hath only length and breadth.*



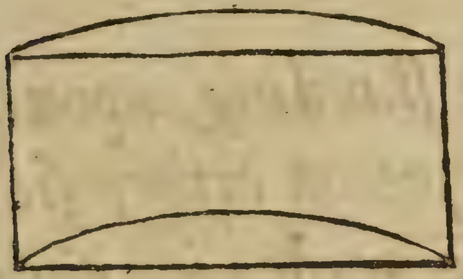
A *Superficies* is the ſecond kind of quantity, and to it are attributed two dimenſions, *length* and *breadth*, but not *thickneſſe*, for a *Superficies* is the term or end of a *Body*, as a *line* is the end and terme of a *Superficies*.

6 *The extreams of a* Superficies *are lines.*



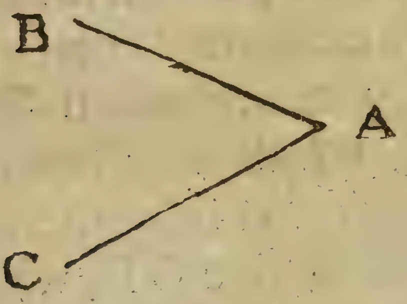
As the ends, limits, or bounds of a line are points, confining the line, ſo are lines, the limits, bounds, and ends incloſing *Superficies*: As in the figure, you may ſee the *Superficies* incloſed with four lines, which are the extreams or limits thereof.

7 *A* plain Superficies *is that which lyeth equally between his lines.*

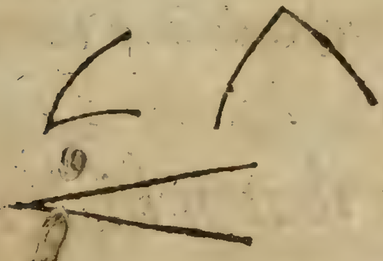


Like as a right Line is the ſhorteſt extentiõ or draught from one point to another, ſo a plain *Superficies* is the ſhorteſt extention or draught from one line to another.

8 *A* plain Angle *is the inclination or bowing of two lines the one to the other, and the one touching the other, and not being directly joyned together.*



As the two lines B A and C A, incline the one to the orher, and touch the one the other in the point A, in which point is made the angle B A C.

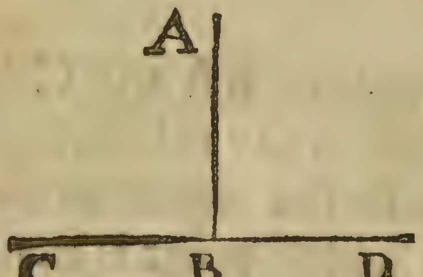


9 *And if the* Lines *which contain the* Angle *be right lines, then is it called a right lined angle.*

As the angle B A C is a right lined angle, where note, that an angle is (for the moſt part) deſcribed by three letters, of which the ſecond, or middle letter, repreſenteth the very angle, and therefore is ſet at the very angle, as in the angle B A C, A repreſenteth the angular point.

A crooked lined angle is that which is contained of crooked lines, and a mixt angle both of right and crooked lines, as in the figure.

10 *When a right line ſtanding upon a right line, maketh the* angles *on either ſide equall, then either of thoſe* angles *is a right* angle: *and the right line which ſtandeth erected, is called a* Perpendicular line *to that upon which it ſtandeth.*



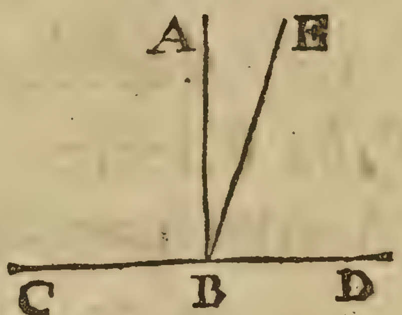
As upon the right line C D, ſuppoſe there doe ſtand another line A B, in ſuch ſort, that it maketh the angles on either ſide thereof equall, namely, the angle A B C on the one ſide, equall to the angle A B D on the other ſide, then are each of thoſe angles right angles, and the line A B, which ſtandeth erected on the line C D, is a *Perpendicular* to the ſaid line C D.

11 *An* Obtuſe Angle *is that which is greater than a* right angle.

As the angle C B E (in the following figure) is greater than the right angle A B C, by the angle A B E, and therefore is an *Obtuſe Angle*.

12 *An* Acute Angle *is that which is leſſe than a* right angle.

As the angle E B D is an acute angle, becauſe it is leſſe than the right angle A B D, by the other acute angle A B E.



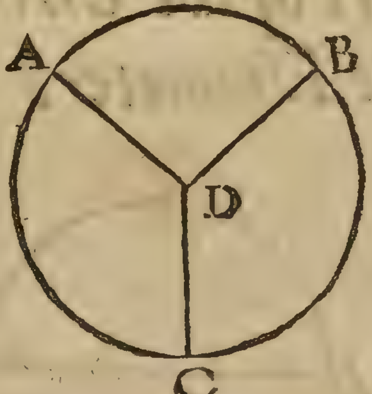
13 *A limit or term is the end of every thing.*

As a point is the limit or term of a line, a line is the limit or term of a Superficies, and a Superficies is the end or limit of a Body ſolid.

14 *A* Figure *is that which is contained under one limit or term, or many.*

15 *A* Circle *is a plain figure contained under one line, which is called a* Circumference, *unto which all lines drawn from one point within the figure, and falling upon the* Circumference *thereof, are equall the one to the other.*

As the *Circle* A B C is a figure contained under one line, namely, the crooked line A B C, which is the Circumference thereof, in the middle of which figure there is the

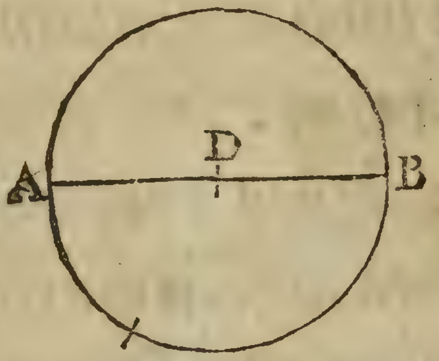


the point D, from which point all lines drawn to the Circumference are equall one to another, as the lines DA, DB, DC, are all equall.

16 *And that point is called the* Centre *of the* Circle.

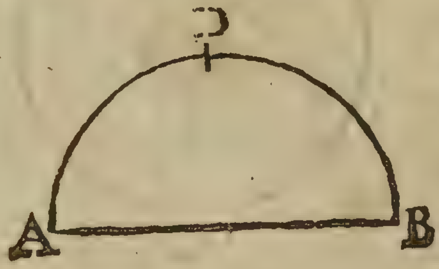
As in the former figure, the point D is the Centre.

17 *A* Diameter *of a Circle is a right line drawn by the Centre thereof, and ending at the Circumference on either ſide, dividing the Circle into two equall parts.*

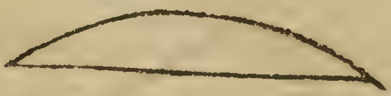


As the line A D B in this Circle, is the Diameter thereof, becauſe it paſſeth from the point A to the point B, and ſo likewiſe through the Center D, and divideth the whol Circle into two equall parts.

18 *A* Semicircle *is a figure which is contained under the Diameter, & under that part of the Circumference which is cut off by the Diameter.*



As the *Semicircle* A D B, is contained under the Diameter A B, and alſo under the part of the Circumference A D B, which is cut off by the Diameter A B.



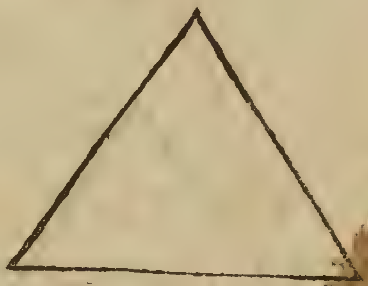
19 *A* ſection, *or portion of a* Circle *is a figure which is contained under a right line, and a part of the* circumference, *greater or leſſer than the* diameter.

20 *Right lined* Figures *are ſuch which are contained under right lines.*

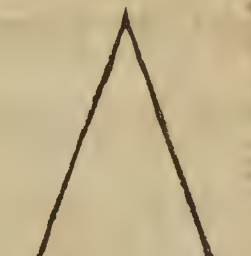
21 *Three ſided* Figures *are ſuch as are contained under three right lines.*

22 *Four ſided* Figures *are ſuch as are contained under four right lines.*

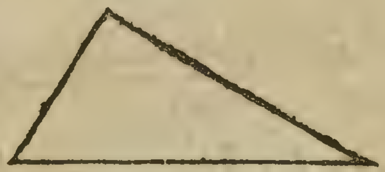
23 *Many ſided* Figures *are ſuch which have more ſides than four.*



24 *Of three ſided* Figures *or* Triangles, *an* equilaterall Triangle *is that which hath three equall ſides.*

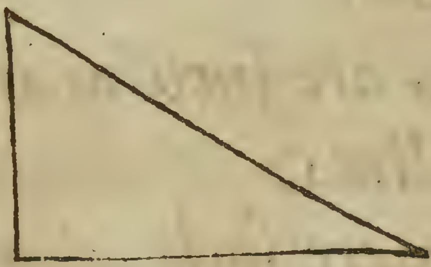


25 Iſoſceles *is a* triangle *which hath only two ſides equall.*

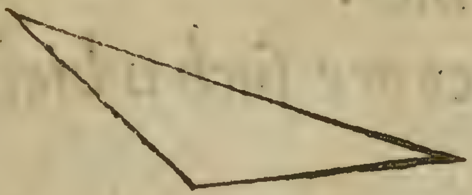


26 Scalenum *is a triangle whoſe three ſides are all unequall.*

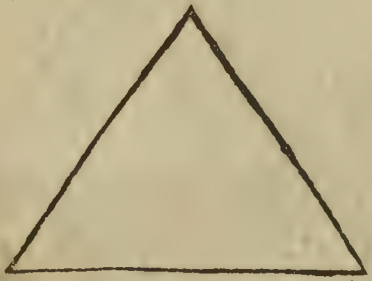
As there are three kindes of Triangles by reaſon of the diverſity of the *ſides*, ſo are there alſo three kindes of Triangles by reaſon of the variety of the *angles* : For every Triangle containeth one right angle and two acute angles, or one obtuſe angle and two acute ; or three acute angles.



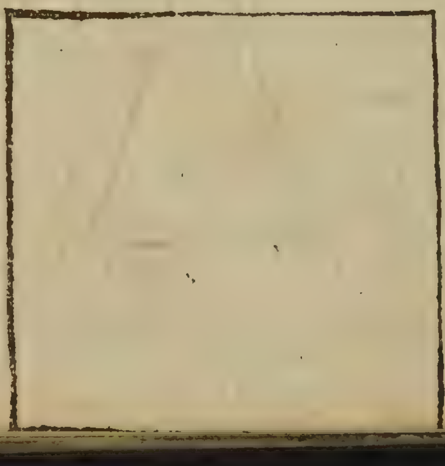
27 *An* Orthigonium *or a right angled Triangle, is a Triangle which hath a right angle.*



28 *An* Ambligonium, *or an obtuſe angled Triangle, is a Triangle which hath an obtuſe angle.*

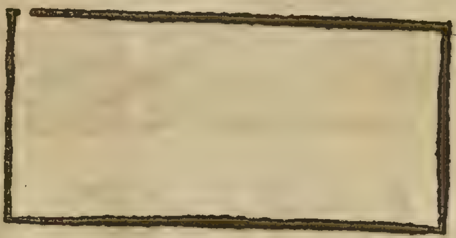


29 *An* Oxigonium *or an acute angled Triangle, is a Triangle which hath all his three angles acute.*

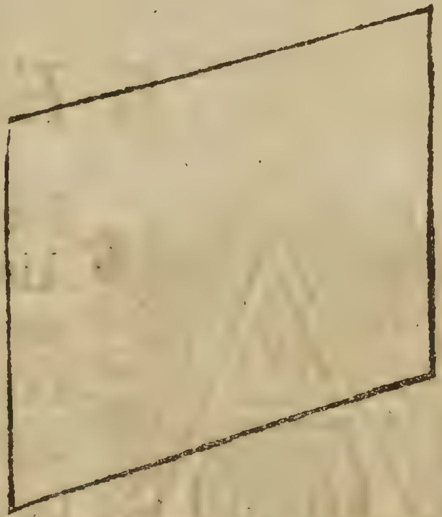


30 *A* Quadrat *or* Square *is that whoſe ſides are equall, and his angles right.*

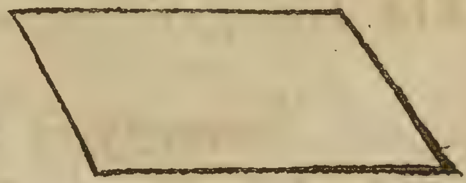
31 *A* long Square *is that whoſe angles are right, but the ſides unequall.*



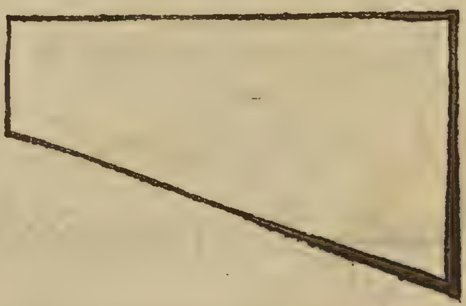
32 Rombus, *or a Diamond, is a figure having four equal ſides, but is not rightangled.*



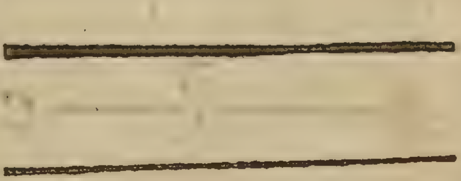
33 Rhomboides, *is a figure whoſe oppoſite ſides are equall, & whoſe oppoſite angles are alſo equall, but hath neither equal ſides nor right angles*



34 *All other figures of four ſides, beſides theſe, are called* Trapezia *or Tables.*



35 Parallel *or equidiſtant right lines are ſuch which being in one & the ſame Superficies produced, would never meet.*



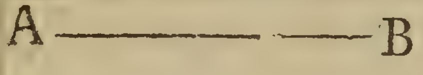
PETITIONS,

OR,

REQVESTS.

After the Definitions which are the firſt kinde of principles, now follow *Petitions*, which are certain generall ſentences, ſo plain and perſpicuous, that they are perceived to be true as ſoon as they be uttered, and no man that hath but common ſence can nor will deny them. The *Requeſts* follow.

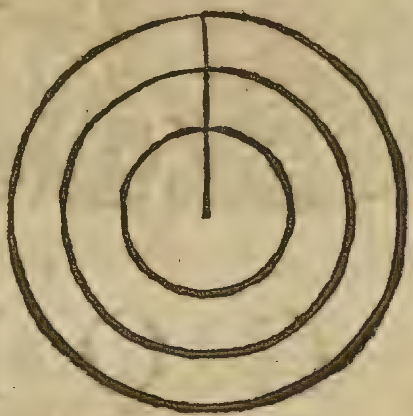
1 *From any point to any point, to draw a right line.*



As from the point A to the point B, who wil deny, but eaſily grant, that a right line may be drawn.

2 *To produce a right line, finite, ſtreight forth continually.*

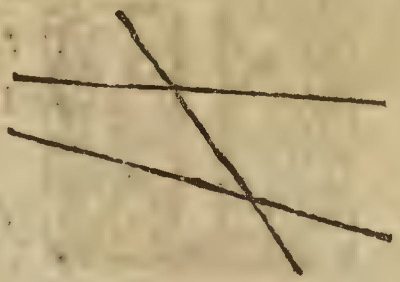
As to draw in length continually the right line A B, who will not grant? For there is no Magnitude ſo great but that there may be a greater; nor any ſo little but that there may be a leſſe: So that the line A B may be extended to the point C, and ſo on to another.



3 *Upon any* centre, *and at any diſtance, to deſcribe a* Circle.

4 *All* right angles *are equall the one to the other.*

5 *When a* right line *falling upon two* right lines, *doth make on one and the ſelf ſame ſide, the two inward* angles *les than two right* angles, *then ſhall thoſe two* right lines *being produced at length, concur on that part in which the two* angles *are leſſe than two* right angles.



6 *That two* right lines *include not a* Superficies.

COMMON SENTENCES.

1 *Things equall to one and the ſelfeſame thing, are alſo equall the one to the other.*

2 *And if ye adde equall things to equall things, the whole ſhall be equall.*

3 *And if from equall things, ye take away equall things, the things remaining ſhall be equall.*

4 *And if from unequall things, ye take away equall things, the things which remain ſhall be unequall.*

5 *And if to unequal things, ye adde equall things, the whol ſhall be unequall.*

6 *Things which are double to one and the ſelfeſame thing, are equall the one to the other.*

7 *Things which are the half of one and the ſelfeſame thing are equall one to the other.*

8 *Things which agree together, are equall the one to the other.*

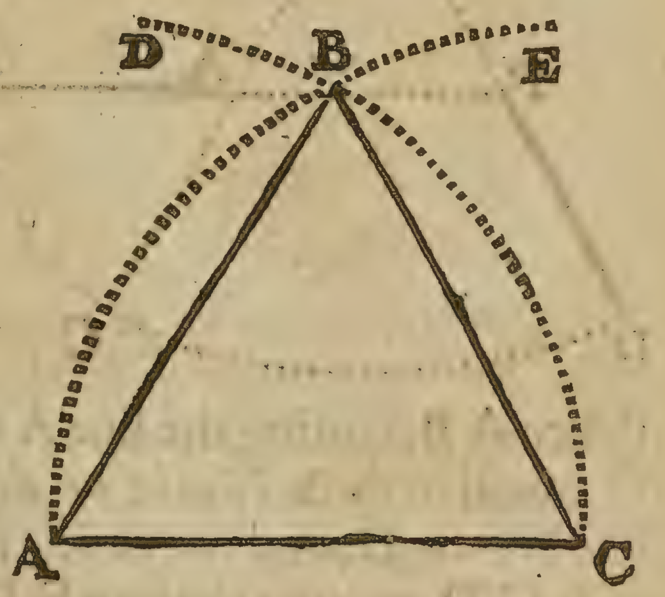
9 *Every whole is greater then his parts.*

EUCLIDES ELEMENTS.

Problem 1. Propoſition 1.

Upon a right line given, not being infinite, to deſcribe an equilaterall Triangle, or with three equall ſides.

Conſtruction.



LEt the right line given be A C, ſet your compaſſes at the diſtance of the line A C, and with one foot in A, wirh the other foot draw the arch of a Circle, as C D, and the ſame diſtance remaining, ſet one foot in C, & with the other draw the arch A E, cutting through the former arch C D in the point B, and from the point of cutting B, draw lines to A and C, then is the Triangle A B C equilaterall, or alike ſided.

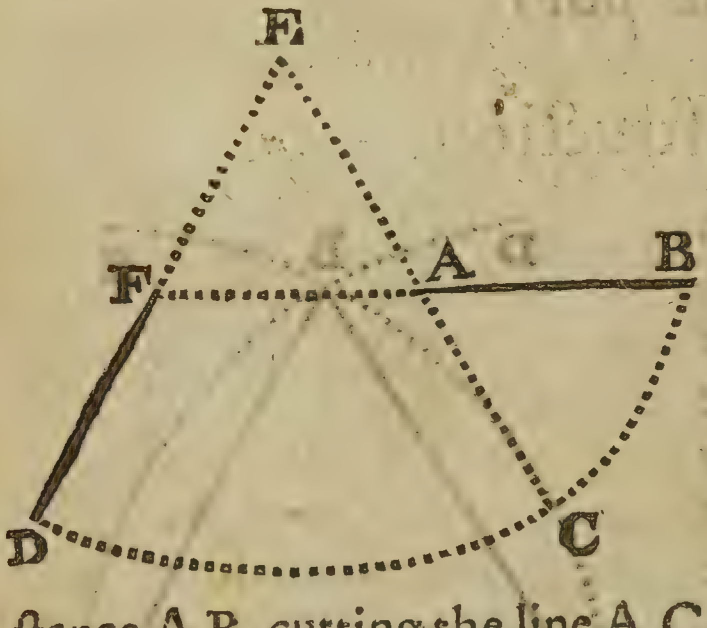
Demonſtration.

Foraſmuch as the line A B is equall to the line A C, and alſo B C equall to A C,*(by the 15 definition,)* it followeth, *(by the firſt common ſentence)* that A B and B C, are each of them equall to A C, and the Triangle equall ſided.

Probl. 1. Prop. 2.

From a point given, to draw a right line, that ſhall be equall to a right line given.

Conſtruction.



The line given is A B, and the point given is F. From the point F, draw a line to A, as A F, and upon the line A F, make an equilateral Triangle, as A F E *(by the former Propoſition,)* and extend A E to C, and from A, as from a centre, draw an arch at the diſtance A B, cutting the line A C in the point C: then is A C equall to A B *(by the 15 definition,)* prolong alſo the line E F to D, and out of E, from the point C, draw the arch C D, cutting the line E D in D: then is the line FD equall to the given line A B.

Demonſtration.

It is evident *(by the 15 definition)* that E C is equall to E D, from whence if we take A E and E F, being equall, there ſhall reſt A C equall to A B, and D F equal to AC, then is F D equall to A B alſo; *by the 1 common ſentence.*

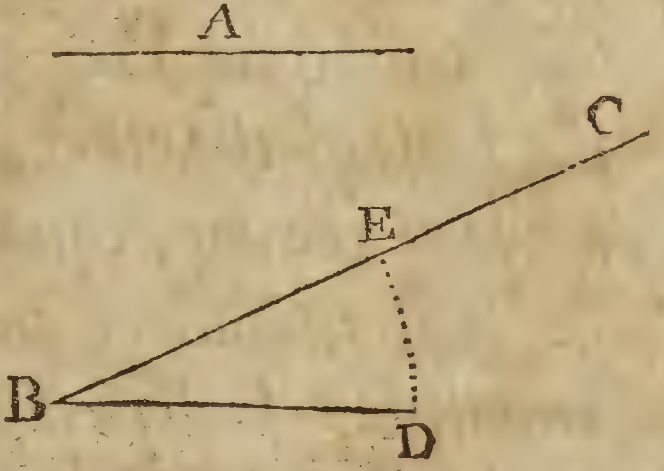
The ſame another way.

Take with your Compaſſes, the length A B, and then with the ſame diſtance, ſet one foot in the point F, and with the other foot, make the point in D, then draw the line from F to D, then is that line F D equall to A B.

Probl. 3. Prop. 3.

Two unequall right lines being given, to cut off from the longeſt, a right line equall to the ſhorteſt.

Conſtruction.



The ſhorteſt line given is A, the longeſt is B C, *(by the former Propoſition)* draw from the point B, a line equall to A, as B D: Then from the point B, with the diſtance B D, draw an arch, as D E, cutting the line B C, in the point E, then is the line B E, cut off from the line B C, equall to the given line A.

Demonſtration.

Foraſmuch as the line B D, is equall to the line A, and the line B E equall to the line B D: then is the line B E, alſo equall to A, *(by the firſt common ſentence.)*

Another way.

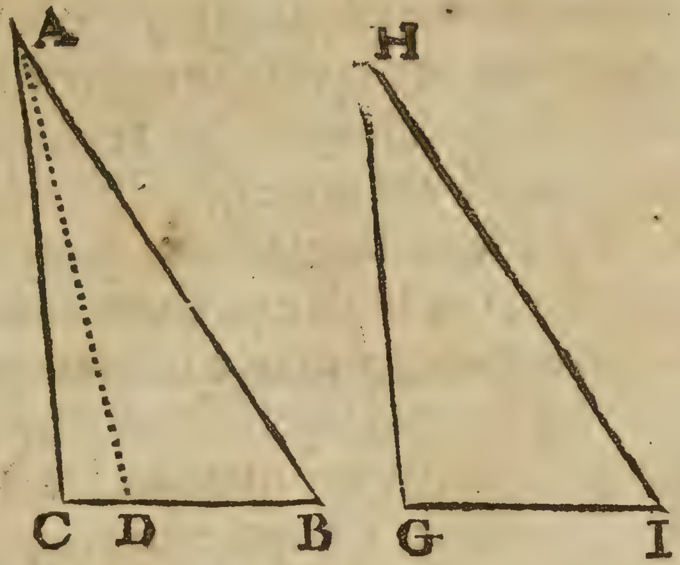
Take with your Compaſſes, the diſtance or length of the line A, and that diſtance noted from B toward E, in the line B C, comes to E, then is B E equall to A.

Theorem 1. Propoſition 4.

If there be two Triangles, of which the two ſides of the one Triangle be equall to two ſides of the other Triangle, each to his correſpondent ſide, and the angles, which are conteined under thoſe lines, be alſo equall the one to the other, then is the third ſide (called the Baſe) *of the one, equall to the Baſe of the other, and the whole Triangle of the one, equall to the whole Triangle of the other, and the other angles remaining, ſhall be alſo equall to the other angles remaining; the one to the other, under which are ſubtended equall ſides.*

Conſtruction.

Of thoſe two Triangles A B C, and H I G, the two



ſides of the one Triangle A B C, namely, the ſides A B, and A C, are equall to two ſides of the other Triangle, namely, to H G, and H I, and the angle A equall to the angle H, Then (*by the intent of this Propoſition*) ſhall the Baſe as C B, be equall to the baſe of the other, as G I, and the angles upon the baſe C B, equall to the angles upon the baſe G I, that is, the angle C equall to G, and B to I.

Demonſtration.

For if they were not equall, let us then take the baſe of the Triangle A B C, that is, the line B C, to be longer then the baſe of the Triangle H I G, that is the line I G, and let the ſame be cut off, in D, ſo that B D, be equall (by ſuppoſition) to I G, and let the line A D be drawn, and then the ſides A D, A B of the one Triangle ſuppoſed to be equall to the ſides H G, H I, of the other Triangle, namely, A D equall to H G, and A B equall to H I; then muſt the angle B A D be alſo equall to the angle I H G, that is to the angle B A C, the part equall to the whole, and the line of the Baſe B D equall to the line B C; the part alſo equall to the whole, which is impoſſible, and is contrary to the 9 common ſentence.

This fourth Propoſition being a *Theoreme*, is onely ſpeculation and contemplation of mind, without action, and foraſmuch as *Euclide* hath in his demonſtration uſed ſome action,(as the cōparing of the one Triangle to the other,by laying the one upon the other)for the better underſtanding of his demonſtration (by reaſon, that before this Propoſition, there was none other to prove the ſame by) he hath uſed the ſame by applying and comparing one Triangle to another, the whole to the whole; but I make no doubt, but *Euclide* could have uſed words ſufficient, if he would, to have proved the ſame without ſuch action; but whereas he hath compared the whole to the whole, we may in the ſame caſe, compare part to part, namely,line to line;the more becauſe a line Mathematicall is a length without bredth *(by the ſecond Definition)* yea ſuch a length as is onely imaginary: and a Triangle being a ſuperficies (*by the 21 Definition*) whoſe extreams, or bounds, are lines *(by the ſixt Definition,)* it cannot with reaſon be thought improper, if we apply line to line, that is length to length, oney for the proof of this moſt excellent Theoreme, that ſo by the comparing the length of the one line, to the length of the other, his correſpondent line; the truth of the Theoreme might appear to the ſenſe: foraſmuch as three lines of a Triangle,whether the ſame be all equall,or two equal, and the third unequall, or all unequall howſoever, being once limited to any one poſition, the ſame lines will not concurre in any other place, then to that whereto they were firſt limited, like in all other right line figures, the the contrary may happen: It followeth then, that in Triangles onely, it is impoſſible, that the lines which doe incloſe the one Triangle, if they be in length equall to the lines, which incloſe the other Triangle (each line to his correſpondent line)that thoſe Triangles ſhould be unequall, either in form of Poſition, or quantity of Area.

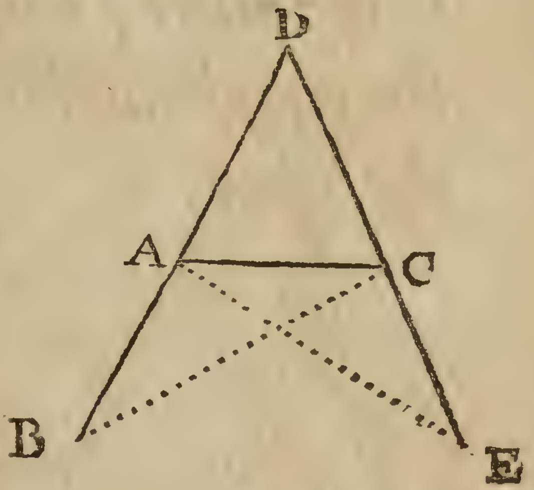
Wherefore the two ſides of the aforeſaid Triangle, namely, A B, A C, of the the one Triangle, being equall to the two ſides of the other Triangle, namely, to H I, H B, and the baſe C B, be equall in length, to the baſe G I, that then, not onely the angle A, ſhall be equall to the angle H, but alſo the other angles ſhall be equal, I to B, and G to C, which was to be demonſtrated.

Theor. 2. Prop. 5.

An Iſoſceles, *or Triangle of two equall ſides, hath his angles at the Baſe equal the one to the other, and thoſe equall ſides being produced, the angles that are under the Baſe, are alſo equall the one to the other.*

Conſtruction.

The Triangle A D C, hath the ſides A D, and D C, equall the one to the other, and therefore are the angles upon the baſe, as the angles D C A, and D A C, equall the one to the other, and the angles under the baſe, as A C E, and C A D, alſo equall the one to the other.



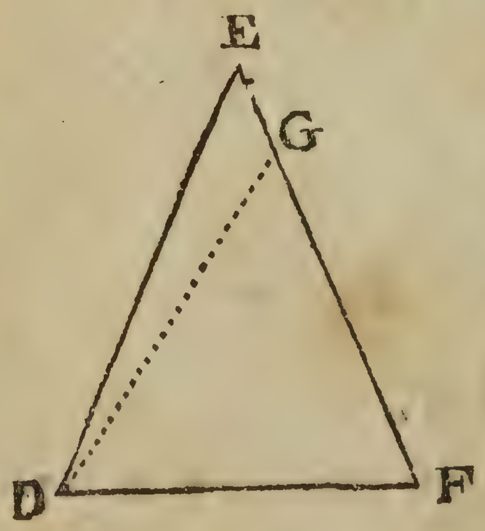
Demonſtration.

Let the two ſides DA and DC, be produced at pleaſure,

ſure, ſo that D B be equall to B E, and then draw the lines A E and C B; foraſmuch, as the two ſides B D and B C, are equall to B E and B A, and the angle D common to both of them; The Baſe C B is equall to A E, *(by the fourth Propoſition)* and the angles B and E equall, as alſo the angles D C B, and E A B, as alſo B C A, and E A C, theſe two taken from D C B, and E A D, there ſhall remain the angles upon the baſe A C D, and C A D equall one to another. Furthermore, foraſmuch as A B, and B C, are two ſides, equall to E C, E A, and that the angles B and E are equall the one to the other; the baſe equall to the baſe, and the angles equall the one to the other *(by the former Propoſition)* it followeth that the angles under the baſe, as B A C, and E C A, are alſo equall: which was to be demonſtrated.

Theor. 3. Prop. 6

If a Triangle have two angles equall, the one to the other, the ſides alſo which ſubtend the equall angles, ſhall be equall the one to the other.



Conſtruction.

In this Triangle D E F, are the angles E F D, and E D F equall, therefore are the ſides E D and E F alſo equall.

Demonſtration.

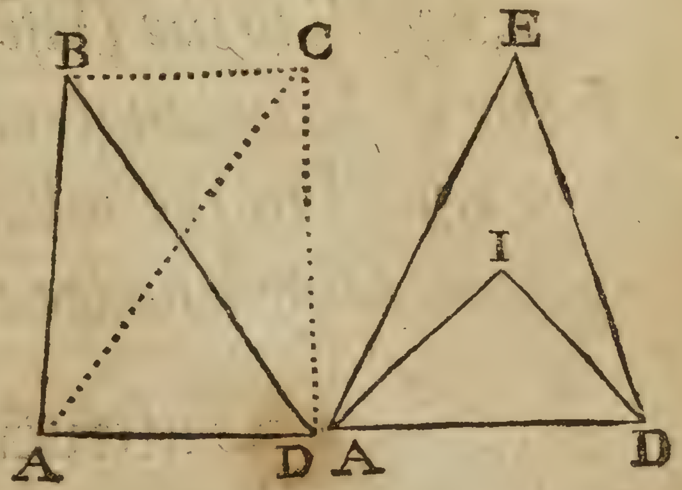
This is evident *(by the former Propo-*

Propoſition) which we otherwiſe prove in this manner, if it were poſſible, that E F were longer than E D, then let there be drawn, the line D G, cutting off F G equall to E D, then muſt the angles F D G, and F D E be equall, and the part, equall to the whole, which is falſe, and contrary to the 9 common ſentence.

Theor. 4. Proop. 7.

If from the two extream points of a line, two other lines be drawn to any point, there cannot from the ſelf-ſame points, or ends of the ſaid line (on the ſame ſide) be drawn two other lines, equall to the firſt lines, the one to the other unto any other point.

Conſtruction



From the points A and D, are drawn two lines, A B and D B, which doth concurre together in the point B, it is not poſſible, that from the point A, there ſhould be drawn a line equall to the line A B, and another from the point D, equall to B D, which ſhould come together in any other point (upon the ſame ſide of the line A D) then onely in the point B. De-

Demonſtration.

Let us take it to be poſſible, and from the point C, draw the line A C, and C D, as alſo B C, if this might happen from the point C, then muſt the angles of the two Triangles, as A B D, and A C D be equall *(by the fourth Propoſition)* being with equall ſides ſubtended, as the Triangle D A C being part of the Triangle D A B, and A D B, part of A D C, ſhould be equall to the ſame A D C, which is falſe *contrary to the 9 common ſentence.*

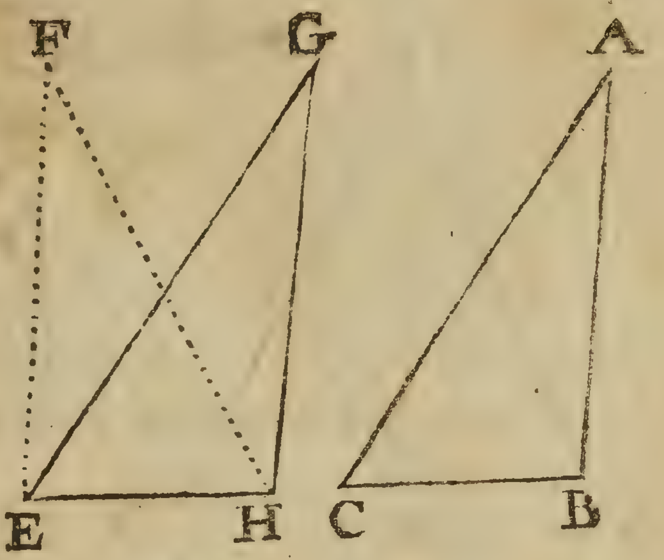
And in the ſame ſenſe, if the point fall within the Triangle, as in the other figure, in I, then muſt the angles D A I, and A D I be equall to the ſame D A E, A D E, the part equall to toe whole, which is falſe alſo.

Theor. 5. Prop. 8.

If two Triangles have two ſides of the one, equall to two ſides of the other, each to his correſpondent ſide, and alſo the Baſe of the one, equall to the Baſe of the other, then have thoſe Triangles alſo two angles, the one equall to the other, namely thoſe angles conteined under equall ſides.

Conſtruction.

The Triangle A B C, hath two ſides, namely A C, and A B, equall to the two ſid es of the Triangle G H E, namely, to the ſides G E, and G H, and the baſe C B equall to the baſe E H, wherefore, the angles conteined with equall ſides are alſo equall.



Demonſtration.

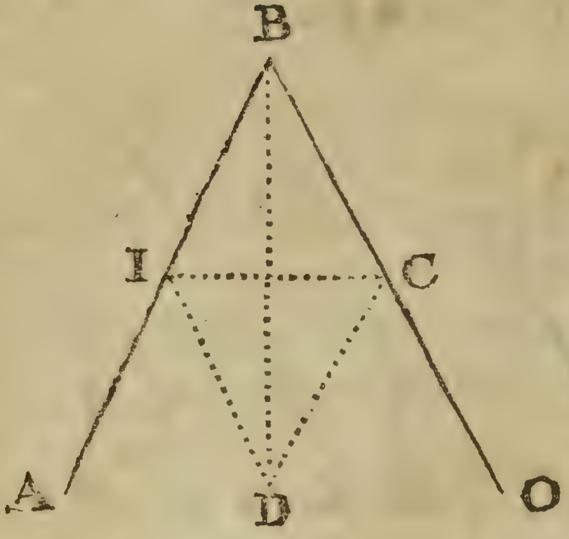
This is made manifeſt *(by the fourth Propoſition)* which may otherwiſe be thus proved. If the angle G, were leſſe than the angle A, let there be made another Triangle upon the baſe E H, as the Triangle E H F, whoſe ſides E F and F H, let be equall to C A, and A B, and the angle F, equall to A. It followeth then, that the line E F, ſhall be equall to E G, and F H, equall to G H, which *(by the former Propoſition)* is impoſſible, and the other angles ſhould alſo be equall, that is G E H, part of F E H, equall to the ſame F E H, which is falſe, and contrary to the *9* common ſentence, and this Propoſition is true.

Probl. 4. Prop. 9.

To divide a right lined angle into two equall parts.

Conſtruction.

Let the right lined angle be A B O, to divide the ſame into two equall parts, take in the two ſides, A B, B O two points equidiſtant from the point B, as the points I and C,



and draw the line from I, to C, as the right line I C, and make upon I C an equilaterall Triangle I C D, *(by the firſt Propoſition)* and then draw the line B D from B, to D, that line B D, doth divide the ſaid angle into two equall parts.

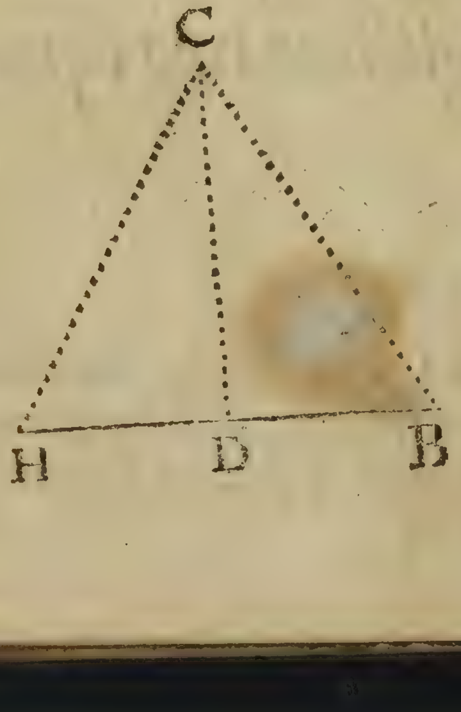
Demonſtration.

Foraſmuch as the ſides B D, B I, are equal to the two ſides B D, B C, and the baſe C D, equall to the baſe I D, it followeth *(by the 4 and 8 Propoſitions)* that the angles D B C, D B I are equall.

Probl. 5. Prop. 10.

To divide a right line being finite, into two equall parts.

Conſtruction.



The right line given is H B, make upon the ſame line H B, an equilaterall Triangle, as H C B, *(by the firſt Propoſition)* and then *(by the former Propoſition)* divide the angle C, into two equall parts, by the line C D, the ſame line C D, ſhall divide alſo the line H B, into two equall parts.

De-

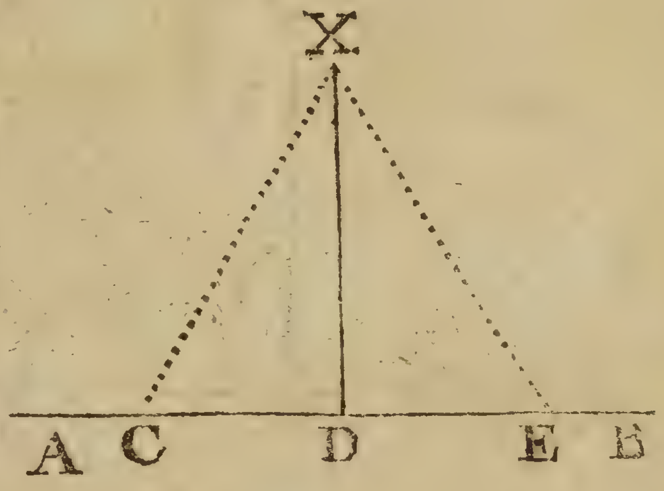
Demonſtration.

Foraſmuch, as C D and C H, are equall to C D, and C B, and the angle D C H equall to the angle D C B. It followeth *(by the 4 and 8 Propoſitions)* that H D and D B are alſo equall the one to the other.

Probl. 6. Prop. 11.

Upon a right line given, and from a point given in the ſame line, how to raiſe up a perpendicular line.

Conſtruction.



The right line given is A B, the point in the ſame line is D, from the point D, make the point C, and on the other ſide of D, make the point E, ſo that D C, be equall to D E, (each of them as long as you pleaſe) and upon C E make an equilaterall Triangle (*by the firſt*) as X C E, and draw the line X D, and that line X D ſhall ſtand perpendicular upon the given line A B.

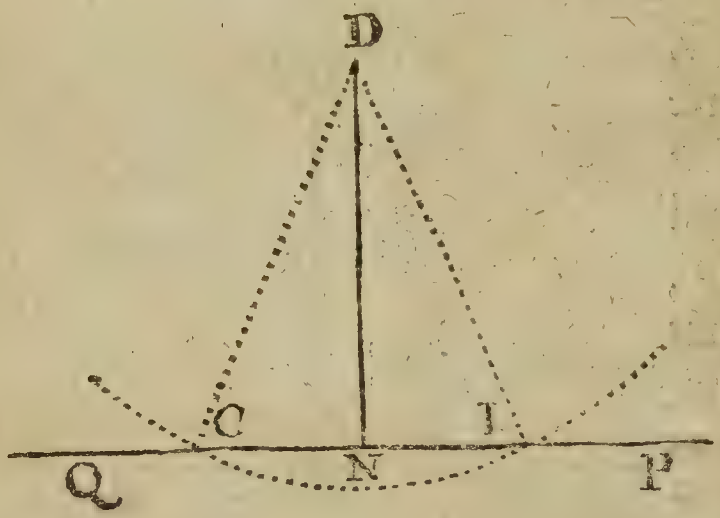
Demonſtration.

Foraſmuch as the two Triangles C X D, and E X D, and with equall lines included, they have alſo equall angles *(by the 4 and 8 Propoſitions)* and the angles X D C, and X D E, are both of them right angles, wherefore X D is perpendicular upon A B, *by the 10 Definition.*

Probl. 7 Prop. 12.

Unto a right line given, being infinite, and from a point given, without the ſame line, to draw a perpendicular line.

Conſtruction.



The right line given is Q P, and the point without the line is D, draw an arch of a Circle, which ſhall cut the line Q P, (from the given point D) in the points C, and I, from which points, draw two lines to the point D, as C D, I D, then divide the angle D into two equall parts *(by the 9 Propoſition)* with the line D N, then is the line D N, perpendicular upon the line Q P, from the point given D, in the point N.

Demonſtration.

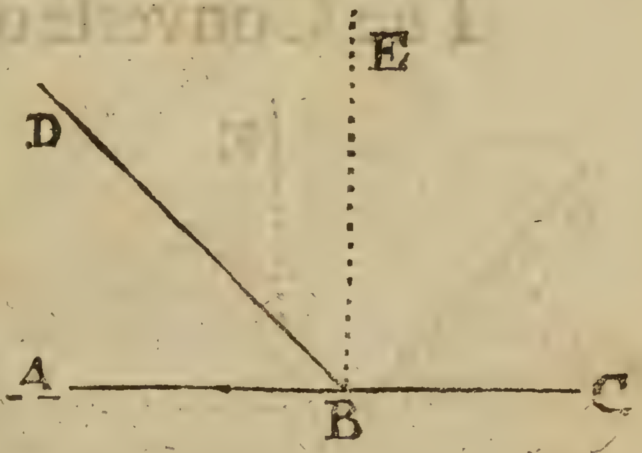
Foraſmuch as the angles N D I, and N D C are equall, then are the angles D N C, and D N I alſo equal, and right angles, and D N is perpendicular upon the ſame line Q P, *(by the former Propoſition.)*

Theor. 6. Prop. 13.

If a right line doth ſtand upon another right line, making any angles with the ſame; then are thoſe angles on both ſides of the ſtanding line, either two right angles, or equall together to two right angles.

Conſtruction.

The right line is A C, upon the line A C, doth ſtand the right line B D, the angles on both ſides of the ſtanding line B D, as the angles D B A, and D B C, are together equall to two right angles.



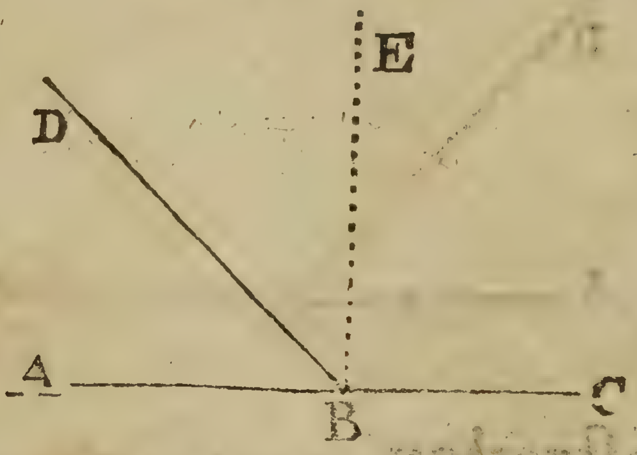
Demonſtration.

This is evident *(by the 10 Definition)* for ſo much as the one angle is greater then a right angle, the other is leſſe then right, which yet more clearly might be thus proved, let there be raiſed from the point B a Perpendicular *(by the former Propoſition)* as B E, then are the angles E B C, and E B A right angles, and the angle D B A is leſſe then a right angle, by the angle D B E; and by ſo much is the angle D B C, greater then a right angle, it is then evident that D B A and D B C together, are equall to two right angles.

Theor. 7. Prop. 14.

If from a point in a right line two right lines be drawn, if then thoſe two angles touching one another at the ſame point be together equall to two right angles, then doe thoſe two lines come ſo together, that they both together make one right line.

The Converſe of the former.



From the point B in the line B E, is drawn two other lines, as B C, B A in ſuch ſort that the two angles E B A, and E B C together, are equall to two right angles, then do thoſe two lines come ſo together, that they make one right line.

Demonſtration.

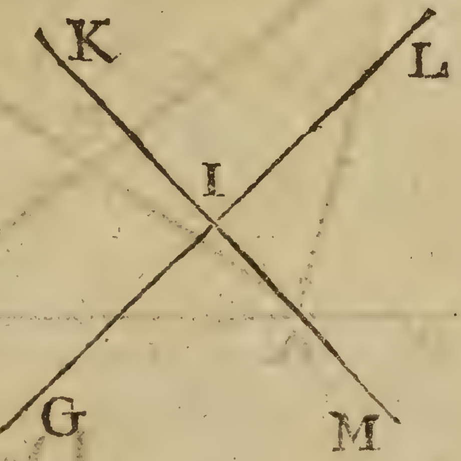
Let it be ſuppoſed, that A B is not in a right line with B C, but let B D be the ſame, then it followeth *(by the former Propoſition)* that the angles E B D, E B C together, ſhould be equall to two right angles, which is falſe, for E B A and E B C, are together equall thereto, and the angles E B A and E B C, are together leſſe, by the angle D B A, wherefore the propoſition is true.

Theor. 8. Prop. 15.

If two right lines do cut through one another, then are the two angles, which are oppoſite, equall the one to the other.

Conſtruction.

The lines K M, L G, doe cut through one another in the point I, the angles oppoſite, as K I L, M I G are equall, as alſo K I G and L I M.



Demonſtration.

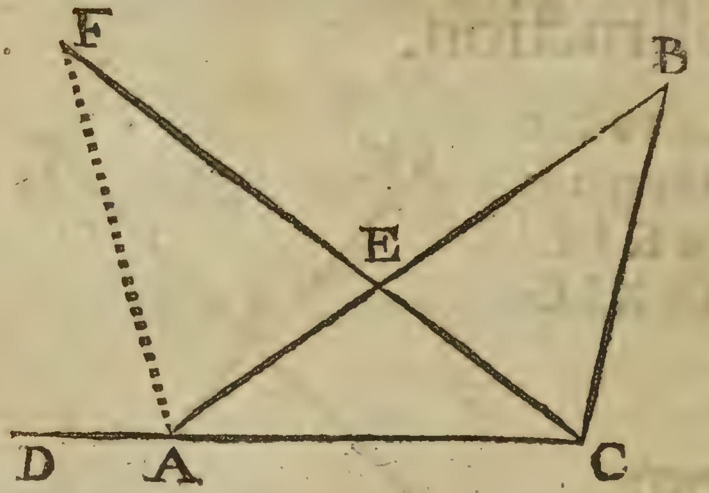
Foraſmuch, as the two angles L I K, L I M, are equall to two right angles *(by the 13 propoſition)* as alſo the ſame L I M and M I G, it followeth then, that if we take from both, the angle L I M, that then the angles remaining, K I L and M I G, are equall, the ſame is alſo to be proved, of the angles L I M and G I K, being equall.

The-

Theor. 9. Prop. 16.

In all Triangles, if one ſide be produced forth, then is the outward angle greater then one of the inward angles, that ſtand oppoſite to that angle, which is produced.

Conſtruction.



In this Triangle A B C, is the baſe A C produced to D, the outward angle B A D, is greater, then one of the inward angles A B C, or A C B.

Demonſtration.

Let the ſide A B, be divided into two equall parts in the point E, and let a right line be drawn from C, through E to F, ſo that E F be equall to E C, and draw another line from F to A, then are the two Triangles C E B, and F E A equall *(by the former Propoſition)* and the ſides E B and E C equall to the ſides, E A and E F, it followeth then *(by the 4 propoſition)* that the lines B C and A F, are equall, as alſo the angles with like lines ſubtended, as the angles E B C and E A F: and E A F is a part of B A D, and therefore leſſe *(by the 9 common ſentence:)* in like manner,

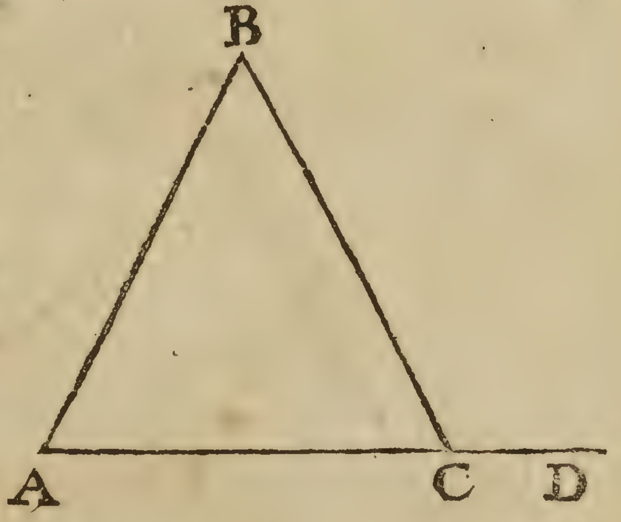
ner may be proved, that the ſame outward angle B A D is greater then A C B.

Theor. 10. Prop. 17.

In all Triangles, two angles which ſo ever be taken together, are leſſe then two right angles.

Demonſtration.

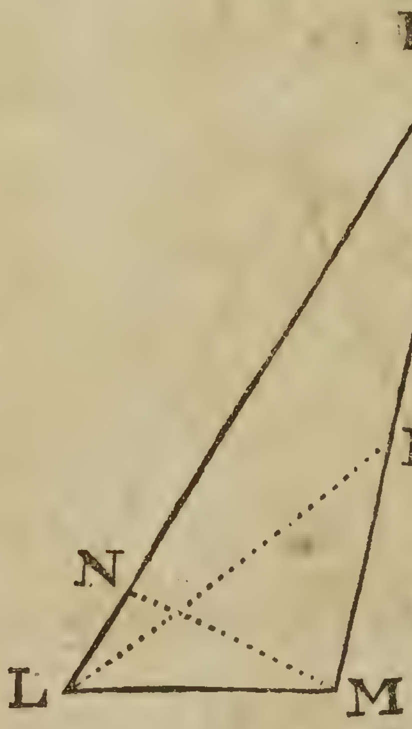
In this Triangle A B C, is the baſe produced to D, making the outward angle B C D, which with the inward angle B C A are together, equall to two right angles *(by the 13 Propoſition)* now, *(by the former Propoſition)* is one of the inward angles C B A, or C A B, leſſe than the outward angle B C D. It followeth then, that the angle A or B, with the angle A C B, together are leſſe then two right angles. In like manner can be proved, that the two angles A and B, together are leſſe then two right angles.



Theor. 11. Prop. 18.

In all Triangles, the greateſt ſide ſubtendeth the greateſt angle, and the leaſt ſide the leſſe angle.

Conſtruction.



In the Triangle, M K L is K L, the longeſt ſide, therefore is the angle M the greateſt angle

Demonſtration.

From M let there be drawn a line to the ſide N, ſo that K N and K M be equall, the angles then upon the baſe N M, as K M N and K N M are equall, *(by the 5 Propoſition:)* But K N M is greater then N L M *(by the 16 Propoſition:)* and K M L, being yet greater then K N M, is then greater, as the angle N L M in like manner alſo may be proved of the other angles. And if we will prove, that the ſhorteſt ſide L M, is under the leaſt angle, then let there drawn a line from L to I, ſo that M I, be equall to M L, the angles then M L I and M I L are equall, and M K L is leſſe then M L K. It followeth that M I is under the ſhorteſt ſide, or leſſe ſide.

The-

Theor. 12. Prop. 19.

In every Triangle under the greater angle is ſubtended the greater ſide.

In the former Triangle M K L is M the greateſt angle, therefore is K L the longe ſt ſide.

Demonſtration.

If K L were not the longeſt ſide, let it then be K M, then ſhould the angle L, be the greateſt angle , greater then M, *(by the former Propoſition :)* which is contrary to the thing it ſelfe, therefore is K L the longeſt ſide, and thus can we alſo prove, that all Triangles leaſt angle, is under the ſhorteſt ſide comprehended.

Theor. 13. Prop. 20.

In all Triangles, two ſides, (which ſo ever be taken) are greater then the third ſide.

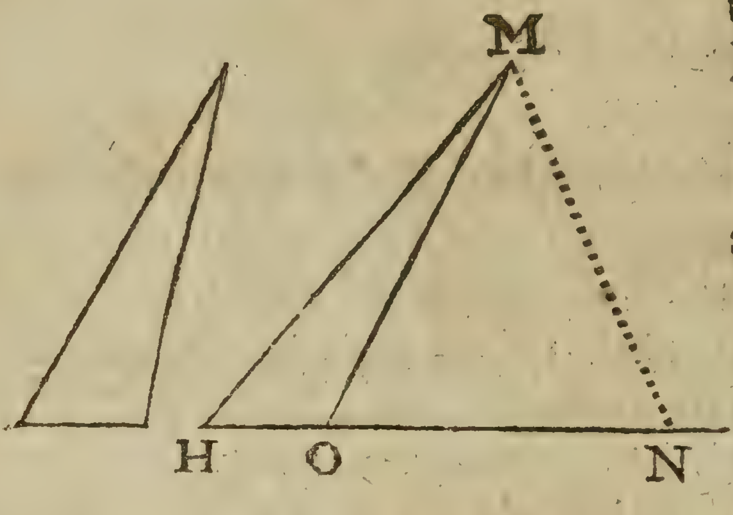
Conſtruction.

In this Triangle H O M, are the ſides O H and O M, taken together, longer then the third ſide H M.

Demonſtration.

This is evident *(by the fourth Definition,)* which otherwiſe might thus be proved: H O is prolonged to N, ſo

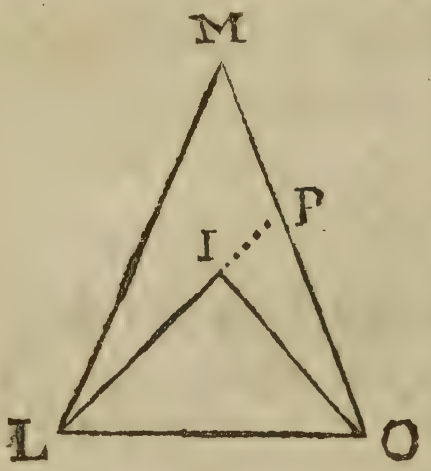
that



that O N is equall to O M, & the line M N being drawn, the angles O M N and O N M are equall, *(by the 5 Propoſition:)* but the angle N M H is greater then O N M: It followeth, that the line H N, (being equall to O H and O M together) is longer then H M, *(as may be underſtood, by the 18 Propoſition.*

Theor. 14. Prop. 21.

If from the two uttermoſt points of the ſides of a Triangle, be drawn, two lines to a point, within the Triangle, then are thoſe two lines together, ſhorter then the two other ſides of the Triangles, but do include a greater angle.



Conſtruction.

The Triangle is L M O, the point within the Triangle is I, the lines drawn are L I and O I, which together are ſhorter then the two ſides of Triangle, as L M and O M, and the angle L I O is greater, then the angle L M O.

Demon-

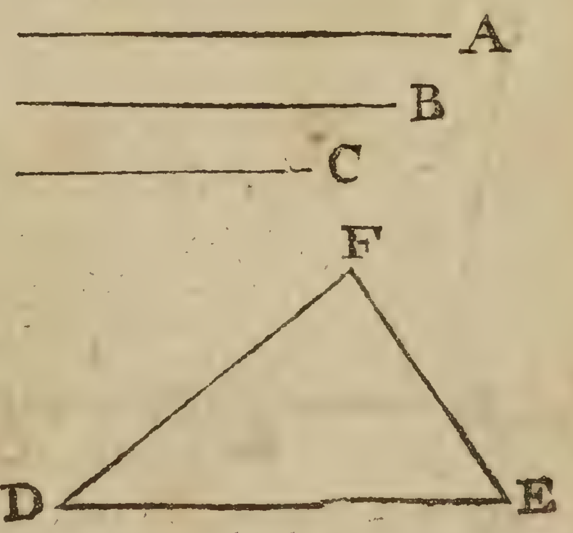
Demonſtration.

Let L I be prolonged to P, now *(by the former Propoſition:)* L M and M P, together are longer then L P and I P and O P, longer then P O, wherefore L M and M P and alſo P O and P I, that is the two ſides of the Triangle L M and M O, with I P are together longer then O L and L P, that is, the lines drawn L I and O I, with the ſame P I, and if we take from both of them P I, there ſhall then reſt the ſides, L M and M O, together longer then there two drawn lines L I and O I. Furthermore, is the angle L I O greater, then the angle I P O and I P O, yet greater then the angle L M O, *(by the 16 Propoſition:)*

Probl. 8. Prop. 22.

How to make a Triangle, of three right lines which ſhall be equall to three given lines, of which it is required, that two of them, together, ſhall be longer then the third.

Conſtruction.



The lines given are A B and C, let there be drawn a line, as D E, equall to A, and with your Compaſſes, take the length of B, ſet one foot in D, and with the other foot, make the arch of a Circle over the line D E, and then with your Compaſſes, do the like with the line C, ſetting one foot in E, and with the other foot, (with the width of the line C) draw an arch line, cutting the former arch in the point F, and from that point F, draw the two lines D F, and E F, then is the Triangle D E F, made of rhe three lines, equall to the three lines given.

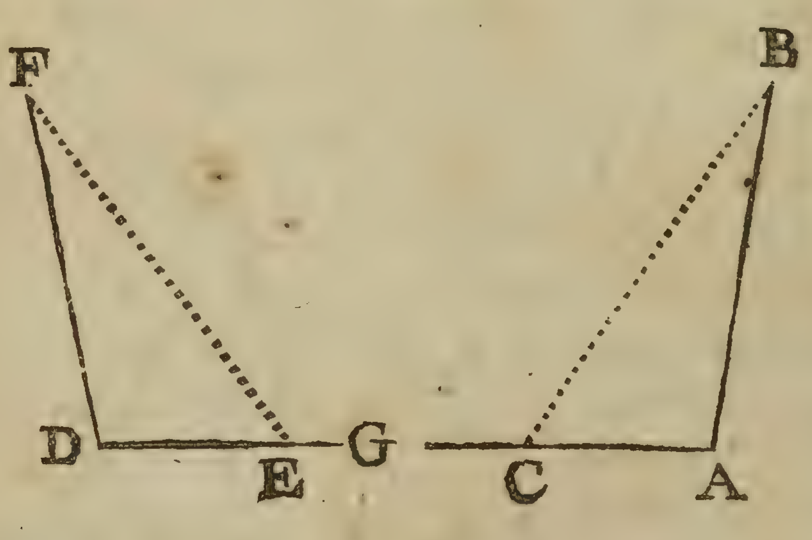
Demonſtration.

This is manifeſt by the working that the lines D E and D F, and E F, are equall to the three lines given A, B, and C.

Probl. 9 Prop. 23.

Upon a point in a line given, to make a right lined angle equall to a right lined angle given.

Conſtruction.



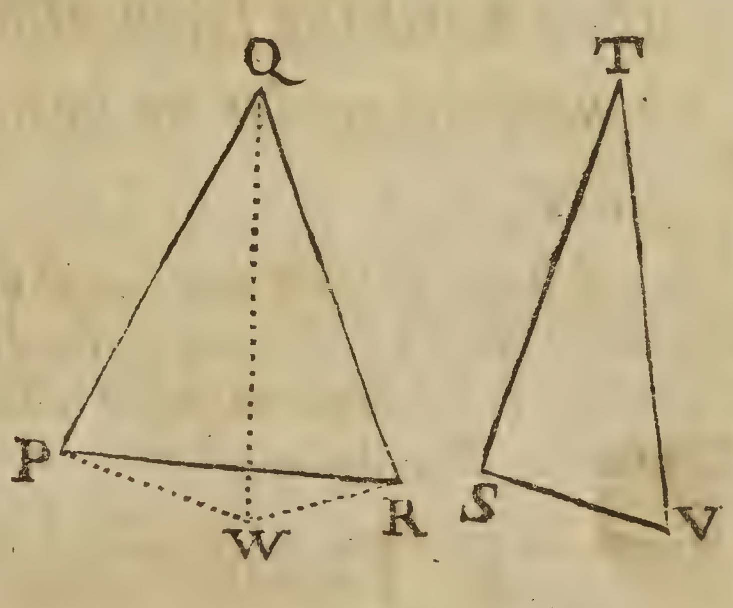
The line given is D G, and the point given is D, the angle given is B A C. Firſt upon the angle given, draw a line at pleaſure, which ſhall make with the angle given, a Triangle, as the line B C. Then with the length of thoſe three lines A B, A C, and B C, make a Triangle, *(by the former Propoſition)* provided always that the lines A B and A C, their lengths be ſet from the point D, and ſo make the Triangle D E F. It followeth *(by the 4 and the 8 Propoſitions)* that the two Triangles A B C, and D E F, are equall, and therefore the angles D and A are alſo equall.

Theor. 15. Prop. 24

If in two Triangles right lined, two ſides of the one be equall to two ſides of the other, but the one hath a greater angle included of thoſe ſides, then the other, then hath the ſame alſo a longer baſe.

Conſtruction.

Of ſuch Triangles, theſe two Triangles P Q R, and S T V, are the ſides P Q and Q R, equall to T S and T V, and the angle Q is greater than the angle T, alſo is the baſe P R longer than the baſe S V.



Demon-

Demonſtration.

This is evident, *(by the 4 Propoſition)* which otherwiſe, may thus be proved, *(by the former Propoſition :)* is the angle R Q W made equall to the angle V T S and Q W, equall to T S and drawing the lines, W R and W P, being then that Q P and Q W are equall, the angles Q P W and Q W P are alſo equall, and the angle P W R greater then Q P W, and yet more greater then the part of the ſame, namely, the part W P R: It followeth *(by the 19 Propoſition :)* that P R is longer then W R and W R is equall to S V, wherefore the baſe P R, is alſo longer then S V.

Theor. 8. Prop. 25.

If in two Triangles, the two ſides of the one, be equall to two ſides of the other, but the one hath a greater Baſe then the other, then hath the ſame alſo, a greater angle conteined under or between, thoſe equall ſides.

In the two former Triangles, P Q R and S T V, are the ſides Q P and Q R, equall to the ſides T S and T V, but the baſe P R is longer then the baſe S V, then is the angle Q greater, then the angle T.

Demonſtration.

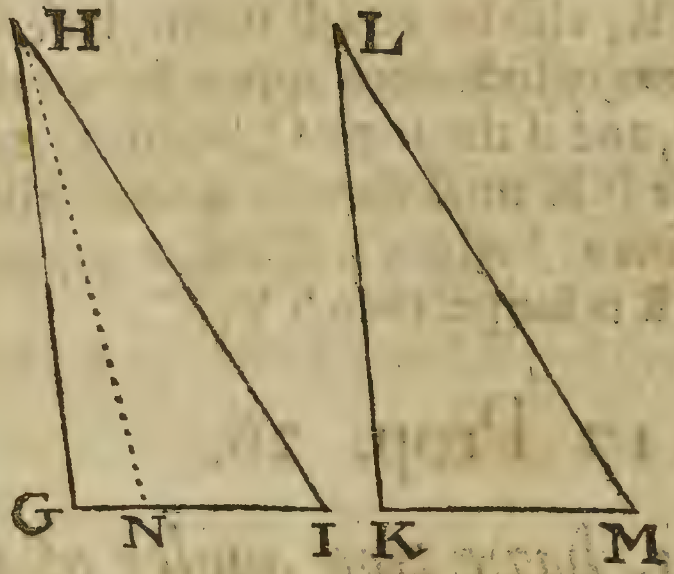
Let us take the angle Q to be equall to the angle T, then muſt the baſe P R, alſo be equall to the baſe S V, *(by the fourth Propoſition:)* which doth appear by the thing it ſelfe, to the contrary, and if the angle Q be imagined, to be leſſe then T, the baſe P R muſt then be ſhorter, then S V *(by the former propoſition :)* which is alſo contrary to the open appearance, for P R is longer then S V.

Theor. 17 Prop. 26.

If two Triangles, have two angles of the one, equall to two angles of the other, each to his correſpondent angle, and have alſo one ſide of the one, equall to one ſide, of the other, either that ſide, which lyeth between the equall angles, or that which is ſubtended under the one of the equall angles, the other ſides alſo of the one, ſhall be equall to the other ſides of the other, each to his correſpondent ſide, and the other angle of the one, ſhall be equall to the other angle of the other.

Conſtruction.

In theſe two Triangles, are the angles H and L equall, and alſo I and M, and the ſides on which the angles ſtan-



deth(or is ſubtended) as H I and L M are equall alſo, then ſhall the other ſides as HG and L K, as alſo G I, and K M be equall, and the angle G equall to the angle K.

Demonſtration.

If G I be ſuppoſed longer then K M, let there be made I N equall to K M, and draw the line H N. Foraſmuch as the ſides H I and N I, are equall to L M, and M K, and doth include equall angles I and M. It followeth *(by the fourth Propoſition)* that the line H N is equall to L K, and the angles of the one Triangle equall to the angles of the other Triangle, as I H N equall to M L K, and conſequently equall to I H G, which is falſe, *and contrary to the 9 common ſentence, that the part ſhall be equall to the whole.* Wherefore is G I equall to K M, and *by the ſame reaſon is alſo proved*, that G H and K L are equall, which alſo is proved *(by the fourth Propoſition)* as alſo that the other angles G and K are equall.

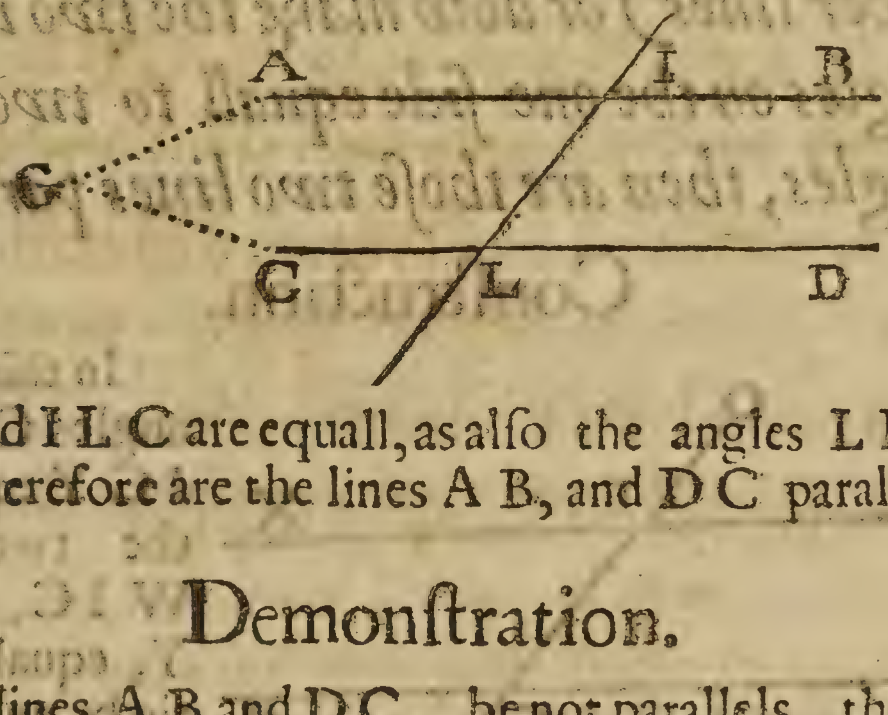
Prop.

Theor 18. Prop. 27.

If a right line be drawn through two other right lines, making the alternate angles, equall the one to the other, then are thoſe two right lines parallel.

Conſtruction.

Through theſe two lines A B and D C, there is drawn the line I L, and the angles oppoſite one to another, as



A I L, and I L C are equall, as alſo the angles L I B and I L D, therefore are the lines A B, and D C parallel.

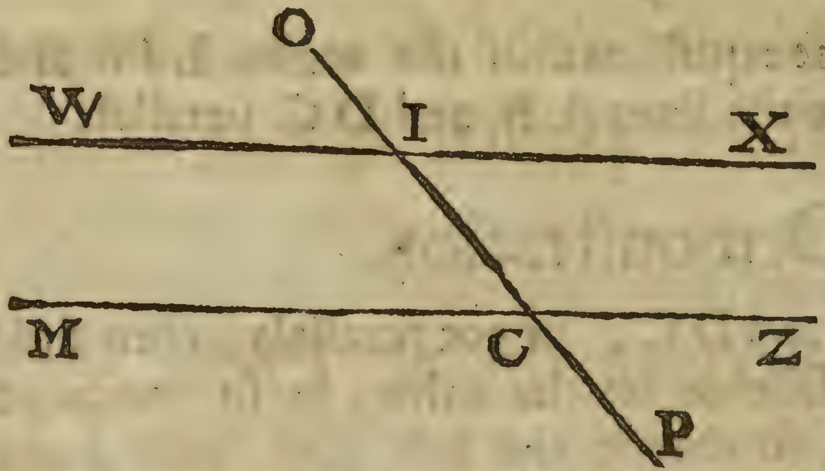
Demonſtration.

If the lines A B and D C, be not parallels, then ſhall they (on the one ſide or on the other) be ſo produced or prolonged forth, ſo that at the laſt, they ſhall come together, and make a Triangle: ſuppoſe it were poſſible, that ſuch a thing might happen in G, the angle L I B ſhall be then greater than the angle I L D, and I L C greater than L I A *(by the 16 Propoſition)* the which is contrary, the lines then being extended, ſhall not concurre, or come together, therefore *(by the 35 Definition)* the lines A B and D C are parallel.

Theor. 19. Prop. 28.

If a right line be drawn through two other right lines, making the outward angle of the one line, equall to the inward angle of the other line (which ſtandeth on the ſame ſide of that line which goeth through the other lines) or doth make the two inward angles on the one ſide equall to two right angles, then are thoſe two lines parallel.

Conſtruction.



In this are the angles C I W and I G Y equall, and the two angles W I C, and I C Y, equall to two right angles, by either of which two reaſons are the two lines W X and Y Z parallel.

Demonſtration.

By the 15 Propoſition, is X I C equall to O I W, and the ſame O I W is alſo equall to I C Y: therefore *(by the former Propoſition)* are the lines W X and Y Z parallel, becauſe the angles W I C and I C Y are equall to two right angles: underſtand alſo that the angles X I C and W I C are equall to two right angles *(by the 13 Propoſition)* if then from both theſe, were taken away the common angle W I C, the angles oppoſite as I C Y and X I C are equal, and the lin es W X and Y Z parallel.

Theor. 20. Prop. 29.

If one right line cut through two parallel right lines, then are the angles oppoſite, one againſt another equall, and alſo the outward angles are equall to the inward, that ſtandeth on the ſame ſide, over againſt the ſame, and the two inward angles that ſtandth on the one ſide of the line that cutteth through, are equall to two right angles.

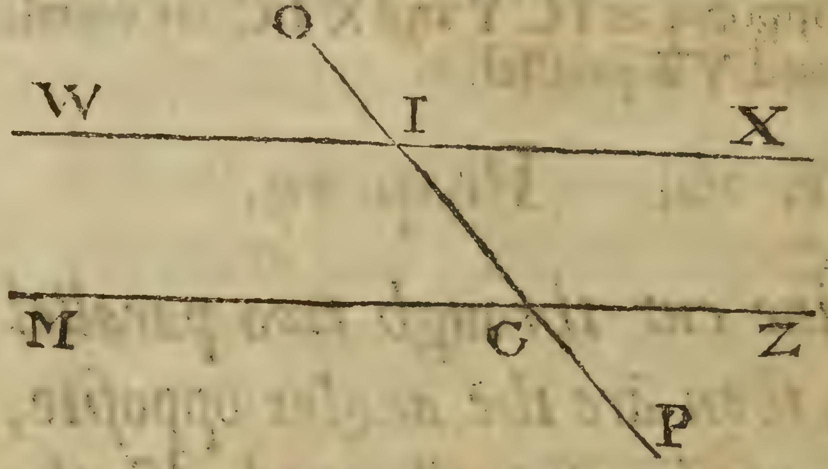
The Converſe of the 28 Prop

In the former Scheme are the two lines W X and Y Z, parallel, therefore are the angles X I C and I C Y equall, as alſo X I O, and I C Z, and the inward angles X I C, and I C Z, are together equall to two right angles.

Demon-

Demonſtration.

All this is evident *(by the two former Propoſitions)* which otherwiſe might be thus proved. The two angles X I C, and W I C are equall to two right angles, as alſo I C Y and I C Z. If then the Angle X I C be leſs then I C Y It followeth that W I C, is as much greater then I C Z, and X I C and Z C I leſſe then two rightangles, wherefore the lines W X and Y C, on the ſide X Z *(by the 11 common ſentence)* may be extended, till at the laſt they ſhall concurre together, the which is contrary *to the 46 Definition*; therefore the angles X I C and I C Y are equall. By the ſame is alſo proved, that the angle W I C is equall to I C Z, and ſeeing that the angle O I X is equall to W I C *(by the 15 Propoſition)* and the angle W I C equall to I C Z *(by the 27 Propoſition.)* It followeth *(by the firſt common ſentence)* that the outward angle O I X is equall to the inward angle I C Z: laſtly, ſeing that the angles X I C and W I C are together equall to two right angles, and that I C Z is equall to W I C, the ſame I C Z and X I C together, are equall to two right angles.



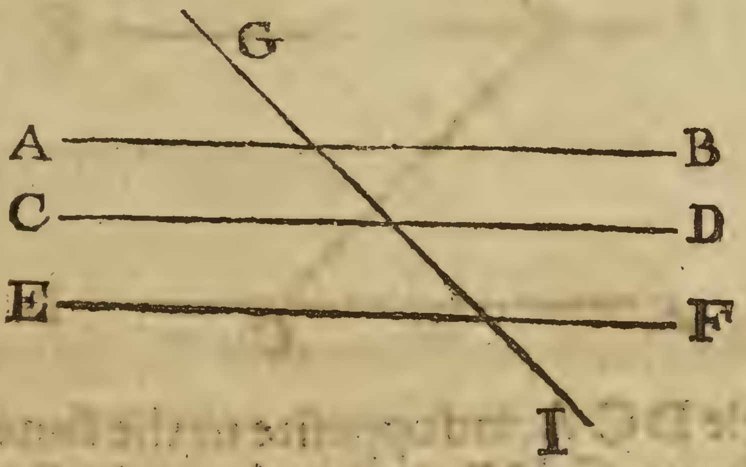
Prop.

Theor. 21. Prop. 30.

All thoſe right lines which are parallel to one right line, thoſe lines are alſo parallel one to another.

Conſtruction.

Theſe lines A B and C D are parallel to E F, the ſame AB and C D are then parallel one to another, alſo.



Demonſtration.

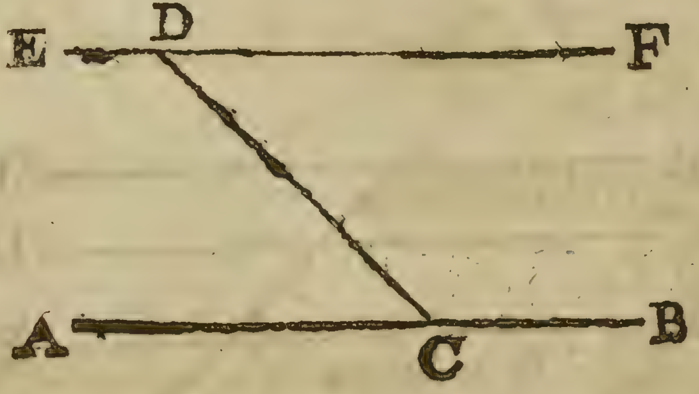
Through the three lines A B, C D, E F, there is drawn another line G I, for as much as A B and C D are parallel to E F, the angles A G I and C H I, are each of them equal to the oppoſite angle G I F, *(by the 27 Propoſition)* wherefore the angles A G I, and C H I are alſo equal *(by the firſt common ſentence)* and the line A B parallel to C D *(by the 28 Propoſition)* the ſame may be proved by many more lines alſo: As appeareth by this Demonſtration.

Prop.

Probl. 10. Prop. 31.

From a given point, to draw a right line, which ſhall be parallel to a right line given.

Conſtruction.



The given line is A B, the point given is D, from the point D draw a line at pleaſure upon A B, as the line D C, then from the point D make an angle equall to the angle D C A, and oppoſite to the ſame *(by the 23 Propoſition)* that is, C D F, and prolong the line D F, then ſhall the line D F be parallel to the line A B.

Demonſtration.

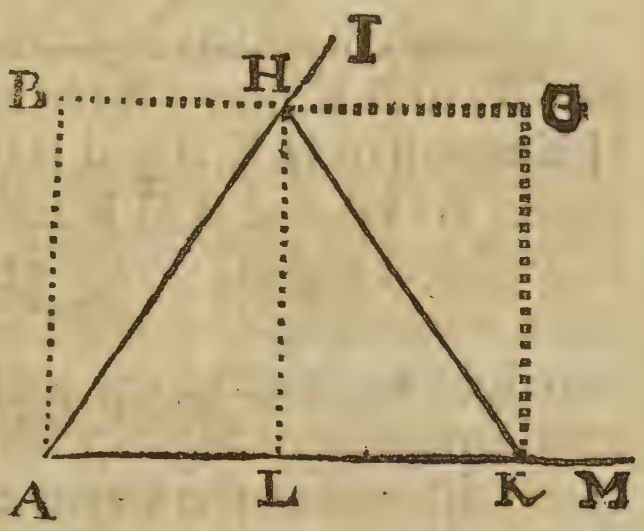
For as much as the angles oppoſite, as D C A and C D F are equall: therefore are the lines A B and D F alſo parallel *(by the 27 Propoſition.)*

Theor. 22. Prop. 32.

In all Triangles, if the one ſide be produced forth, then is that outward angle equall to both the inward angles, oppoſite againſt the ſame, alſo all the three inward angles together, are equall to two right angles.

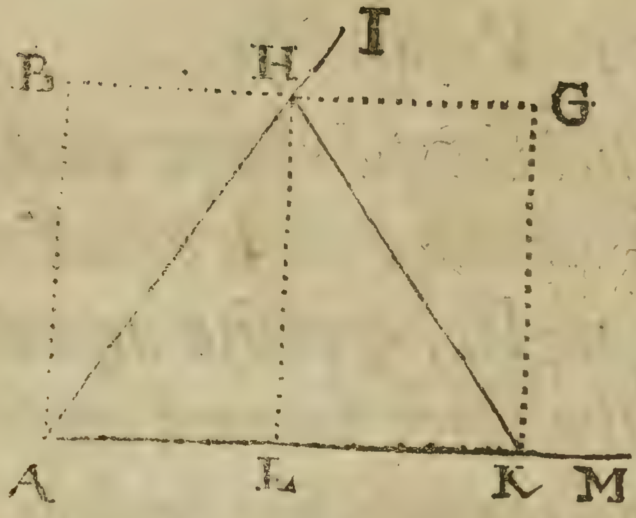
Conſtruction.

In the Triangle A H K, is the outward angle H K M, equall to the two angles oppoſite, that is, to the inward angles K H A and K A H, or the angle K H I equall to the inward angles H K A, and H A K both together.



Demonſtration.

Let there be drawn from the three angles of this Triangle three perpendicular lines upon A K, as A B, L H, and K G, theſe lines are parallel *(by the 28 Propoſition)* and therefore the angles oppoſite, as A H L and H A B are equall *(by the 29 Propoſition:)* as alſo K H L and H K G, it is evident, that the two right angles B A K and G K A, are together equall to the three angles of the Triangle



A H K, and foraſmuch, as the angles H K A and H K M, are alſo together equal to two right angles, *(by the 13 Propoſition:)* it followeth, that if from both, there be taken away the common angle H K A, that then the outward angle H K M is equall to the two inward angles K A H and K H A.

Another way.

Through the point H is drawn the line B H G, which is parallel to A K, now *(by the 29 propoſition:)* is the angle G H K equall to H K A and B H A, equall to H A K and the angles B H A and A H K and G H K, are together equall to two right angles, *(by the 13 Propoſition:)* wherefore, the three inward angles of the Triangle together, are equall to two right angles; and the angles K H A and K H I, are alſo equal to two right angles, wherefore the outward angle K H I is equall to the two inward angles HK A and H A K.

A third way proved.

H G is parallel to A K, wherefore *(by the 29 Propoſition:)* is the angle I H G equall to H A K and G H K equal to H K A: it is evident, that K H I is equall to H A K and H K A together, and the three angles of the Triangle A H K, are equall to two right angles.

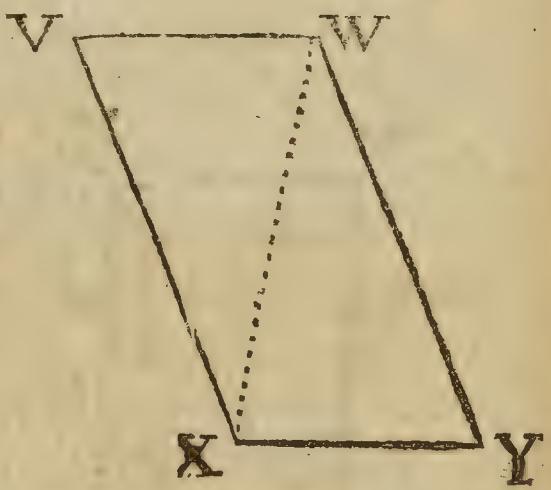
The-

Theor. 23. Prop. 33

If two equall parallel lines, be joyned together with two other right lines, then are thoſe lines alſo equall, and parallel.

Conſtruction.

The two equal parallel lines, are X V, W Y, and they are joyned together with the lines V W, and X Y, I ſay then, that theſe lines W V and X Y, are alſo equall, and parallel.



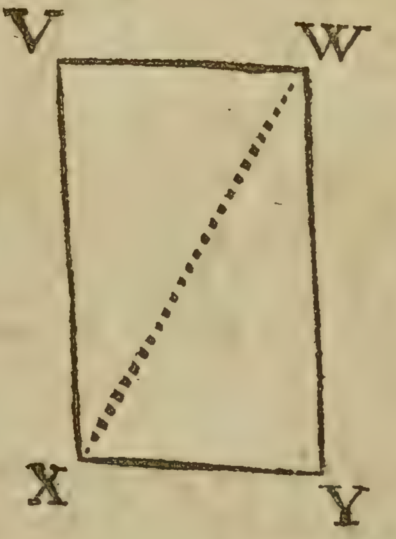
Demonſtration.

From X is a line drawn to W which doth fall upon the two parallels X V & Y W making the oppoſite angles W X V, and X W Y equal *(by the 29 Propoſition)* & the two angles V W & X W Y have two ſides, the one equal to the other, as X W & X V equal to W X & W Y. & the angles which are included with thoſe ſides, are alſo equall: it followeth *(by the fourth Propoſition)* that the third ſides as W V and X Y, are equall, and the other angles included of equall ſides, are equall the one to the other, as the angles oppoſite X W Y and W X Y equall the one to the other, wherefore *(by the 27 Propoſition)* are the lines X Y and V W alſo equall and parallel.

Theor. 24. Prop. 34.

In all Parallelograms, the angles and ſides that are oppoſite one to another, are equall, and the Diameter, or Diagonall line, doth divide the ſame, into two equall parts.

Conſtruction.



In the Parallelogram X V W Y are the angles V and Y, as alſo W X equall the one to the other, and the ſides X Y and V W alſo equal, and the ſides X V and Y W are alſo equall, and the Diameter, or Diagonall line W X, doth divide the Parallelogram into two Triangles, as X W V and W X Y, the one equall to the other.

Demonſtration.

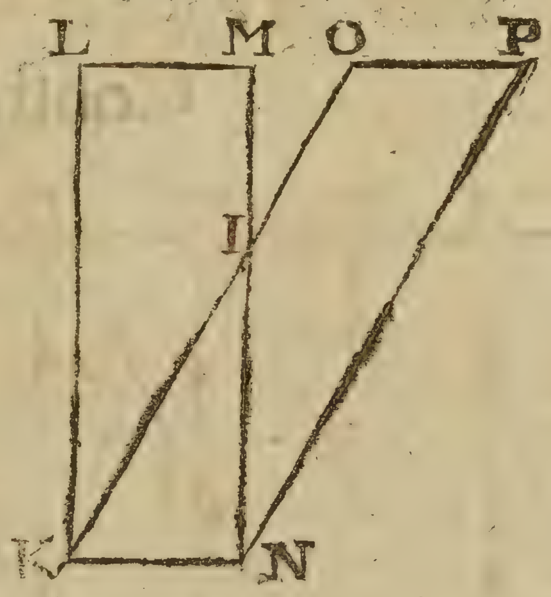
The angles oppoſite, as V W X and Y X W are equall, *(by the 29 Propoſition)* alſo Y W X and V X W, it followeth, that the whole Triangles X W Y and Y X V are alſo equall. Secondly, foraſmuch as the two Triangles, hath each of them two angles, the one of them equall to the other and the ſides X W common to them both, the ſides, and other angles ſhall be equall the one to the other, *(by the 27 Propoſition)* that is, the angle V equall to the angle Y, and

Y, and the ſide W V equall to the ſide X Y, and X V equal to W Y, the two Triangles X W V and W X Y are equal, then doth the Diagonall W X divide the Parallelogram into two equall parts.

Theor. 25 Prop. 35.

Parallelograms, which doe ſtand upon one and the ſame baſe, and between the ſame parallel lines, are equal the one to the other.

Conſtruction.



Upon the ſelfe ſame baſe K N doth ſtand two Parallelogrames N L and N O, between the two parallel lines K N and L P, theſe two Parallelograms are equall.

Demonſtration.

The two ſides K L an N M, are equall, as alſo, K O N P, *(by the 34 Propoſition:)* the Triangle K O L is equall to N P M: from theſe two Triangles, take away the Triangle M I O, common to them both, and there ſhall remain the two Trapeſias, K I M L and N I O P, equall the one to the other, to which if we adde the common Triangle K I N, it is then evident, that the two Parallelograms, N L and N O are equall

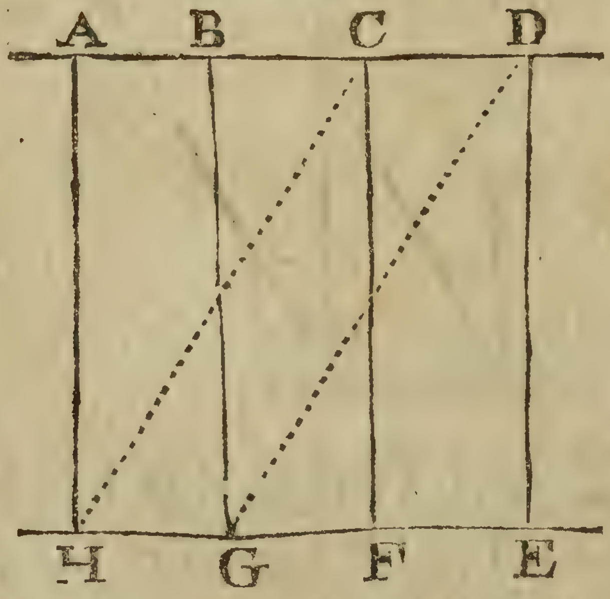
Note.

Here is to be noted, that that which is ſaid here, and ts other Propoſitions following, concerning Parallelograms, Triangles, (and ſo forth) ſtanding between two parallel lines, is therewith given to underſtand, that ſuch figures have equall height, for equall height, doth cauſe equall diſtance, and equall diſtance cauſeth parallel lines, and parallel lines doth cauſe equall height in their ſtanding, &c.

Probl. 26. Prop. 36.

All Parallelograms, which ſtandeth upon equall baſe, and between two parallel right lines are equal.

Conſtruction.



Between the two parallel lines, A D and H E doth ſtand theſe two Parallelograms A G and C E, and the baſe H G is equall to the baſe F E, theſe two Parallelograms therefore are equall.

Demonſtration.

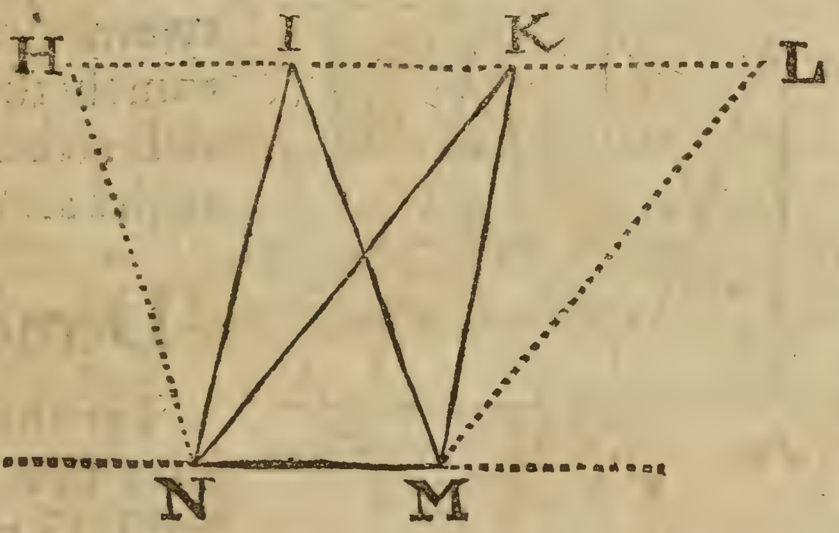
Let there be drawn the two lines H C and G D, theſe lines are parallel, *(by the 33 Propoſition)* and doth make the Parallelogram C G, which *(by the former Propoſition)* if equal to A G, and alſo equall to C E: it followeth, that the Parallelograms A G and G E are equall, *(by the 1 common ſentence.)* The-

Theor. 27. Prop. 37.

All Triangles that have one and the ſame Baſe, and ſtand between two parallel lines are equall.

Conſtruction.

The Triangles N I M and N K M hath one and the ſame baſe N M, and ſtand between the two parallel lines N M and HL: theſe Triangles therefore are equall.



Demonſtration.

Let there from the point M to the point L be drawn the line M L parallel to N K, and from the point N, draw alſo the Line N H parallel to M I: it is then evident *(by the 35 propoſition)* that the parallelograms M H and N I are equall, and that the Triangle N I M is the half of the Parallelogram, and the Triangle N K M is the half of the other Parallelogram: it followeth then that the Triangles are alſo equall *by the 7 common ſentence.*

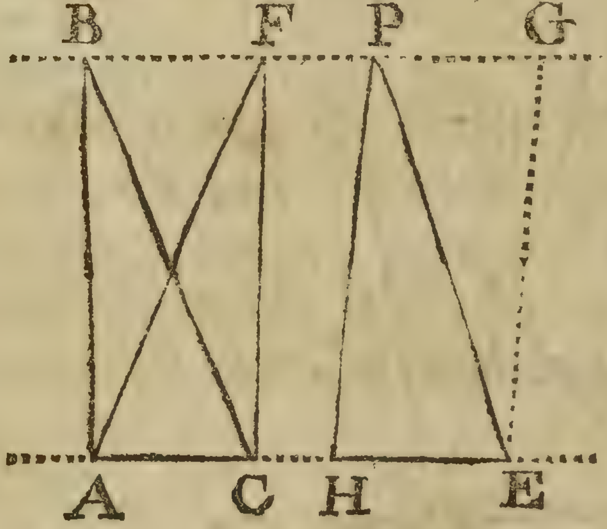
The-

Theor. 28. Prop. 38.

All Triangles, which do ſtand upon equall Baſes, and between two parallel lines, thoſe Triangles are equall.

Conſtruction.

The two Triangles A B C and H D E doth ſtand upon equall baſes, as upon A C, and H E, and they ſtand between two parallel lines, namely the lines B G, and A E: therefore theſe Triangles are equall.



Demonſtration.

Let there be drawn the lines C F, parallel to A B, and E G to H D, now foraſmuch as the Parallelogram A E, is equall to H G, *(by the 36 Propoſition,)* the Triangles then being the half thereof *(by the 34 Propoſition)* are alſo equall.

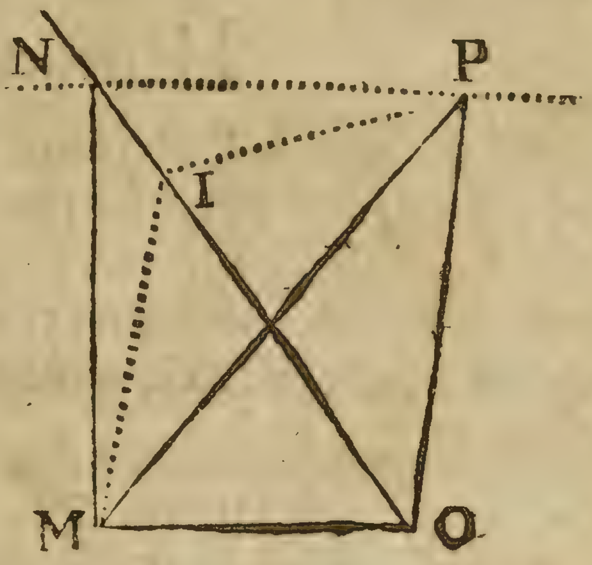
Theor. 29. Prop. 39.

All Triangles, which are equall, and ſtand upon one Baſe, thoſe Triangles ſtand alſo between two parallel lines.

Conſtruction.

Upnn the baſe M O doth ſtand two Triangles as M N O and

O and M P O, which Triangles are equall, wherefore thoſe lines are parallel which thoſe Triangles doth ſtand between, namely the lines M O and N P.



Demonſtration.

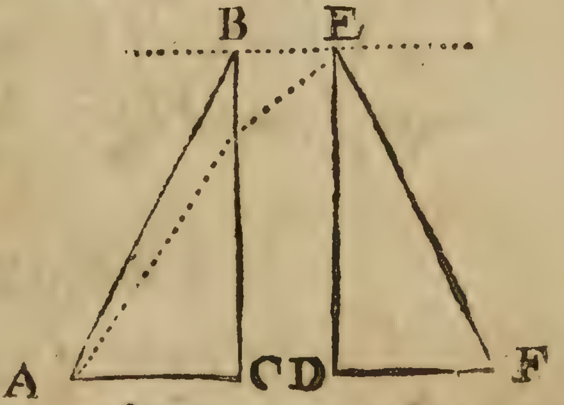
Suppoſe that N P were not parallel with M O but P I, and draw the lines I P and I M, the two Triangles M P O and M I O ſhall then be equall *(by the 37 Propoſition :)* and then ſhould they both be equall alſo to M N O ; it followeth , that the part M I O ſhould be equall, to the whole M N O which is contrary *(to the 9 common ſentence)* wherefore the line N P, (and not I P,) is parallel to M O.

Theor. 30. Prop. 40.

All Triangles that are equall, and ſtand upon equall Baſes, they ſtand alſo between two parallel lines.

Conſtruction.

The two Triangles A B C and D E F are equall , and their baſes A C and D F are equall, and the lines B E and A F , which they ſtand beween, are then parallel.



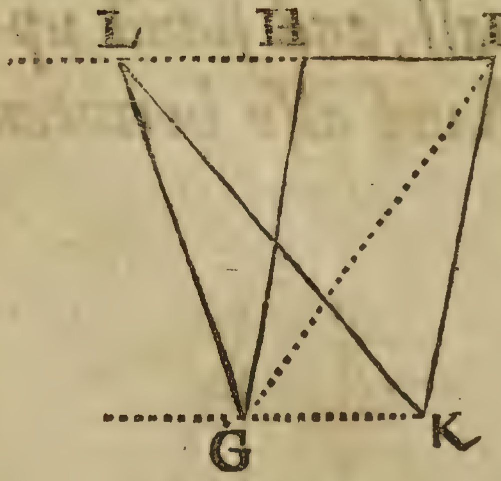
Demonſtration.

If B E and A F were not parallel. Take it then, that E I be drawn, and I A, and that E I be parallel to A F, and the Triangle D E F were equall to A I C, *(by the 38 Propoſition)* it followeth that the part A I C, ſhould be equall to the whole A B C, which is impoſſible.

Theor. 31. Prop. 41.

If a Parallelogram, and a Triangle ſtand upon one and the ſame Baſe, and between two parallel lines, then is that Parallelogram, twice as great as (or double to) the Triangle.

Conſtruction.



The Parallelogram H K, and the Triangle G L K, ſtandeth upon one and the ſame baſe G K, and between the two parallel lines G K and L I, wherefore the Parallelogram H K is double, or twice as great as the Triangle G L K.

Demonſtration.

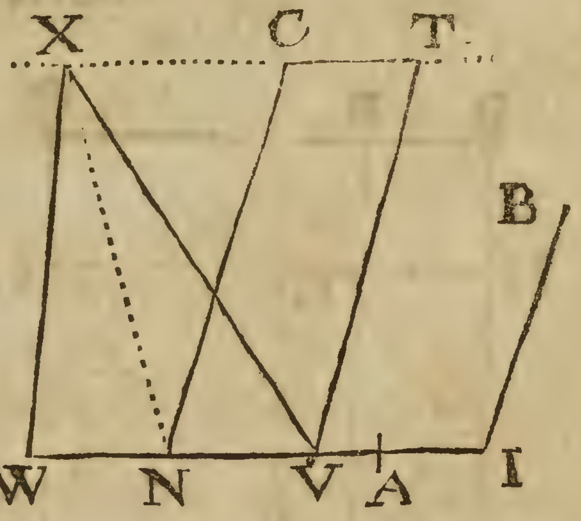
Let there be a line drawn from G to I, then is the Triangle G I K, the half of the Parallelogram H K, *(by the*

the 34 and by the 37 Propoſition :) are the Triangles G I K and G L K equall, wherefore the Triangle G L K alſo is the halfe of the Parallelogram H K, it followeth then, that the Parallelogram, is twice, as great as the Triangle.

Probl. 11. Prop. 42.

How to make a Parallelogram, equall to a given Triangle, and that the ſame ſhall have an angle, equall to a given angle

Conſtruction:



The Triangle given, is W X V and the angle given is A I B, make from the point V upon the line W V an angle, equal to the angle A I B *(by the 23 propoſition)* as the angle W V T, then divide the baſe, W V into two equall parts, the midſt is N and from thence, a line drawn to O parallel to V T, *(by the 31 propoſition:)* as N O and another from X parallel to W V, as the line X T cutting N O in the point O and V T in the point T, then is the parallelogram N O T V, equall to the Triangle given W X V.

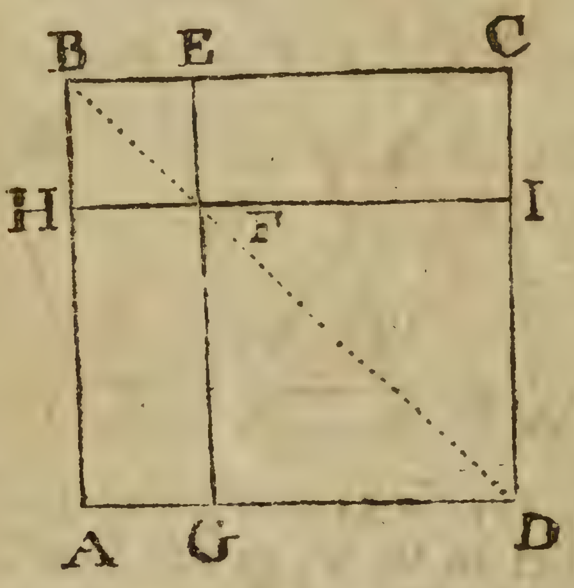
Demonſtration.

Foraſmuch, as the two triangles, W X N and N X V are equall *(by the 38 propoſition:)* and that N X V is halfe as great as the Parallelogram N T, *(by the former propoſition:)* it followeth, that the ſame Parallelogram, is equal to both the Triangles W X N and N X V, that is, equal to the given Triangle W T V.

Theor. 32 Prop. 43.

In all Parallelograms, the ſupplements, (or fillings) which doth ſtand upon the Diameter, are equall.

Conſtruction.



In the Parallelogram A B D E is the line A D the Diameter, draw from the ſide B D from the point C, a line parallel to D E or A B cutting through the Diameter, in the point G and another through the ſame point, G parallel to B D, *(by the 31 propoſition:)* as the lines C F and H I, making on each ſide of the Diameter, A D a Parallelogram, as G B and G E which are called ſupplements, and thoſe be equal the one to the other.

Demonſtration.

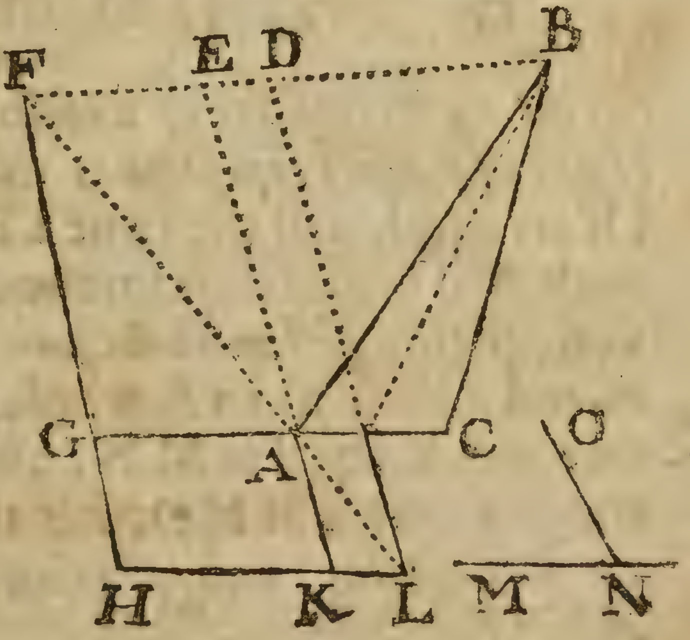
Foraſmuch, as A D doth divide the Parallelogram A

B D E into two equal parts, *(by the 34 Propoſition)* as alſo the two Parallelograms I F and C H, and foraſmuch, as *(by the before named 34 Propoſition :)* the Triangles A D B and A D E are equal, as alſo A G I and A G F and likewiſe G D C and G D H: it followeth, that if we take away equal things from equal things, that is, if we from the Triangle A D B, take away the two Triangles A G I and G D C, and likewiſe from the Triangle A D E, the two Triangles A G F and G D H, that then the ſupplements G B and G E ſhall remain equal, *(by the third common ſentence.)*

Probl. 12. Prop. 44.

Upon a right line given how to make a Parallelogram, that ſhall be equal to a Triangle given, and alſo ſhall have an angle, equal to a right line angle given.

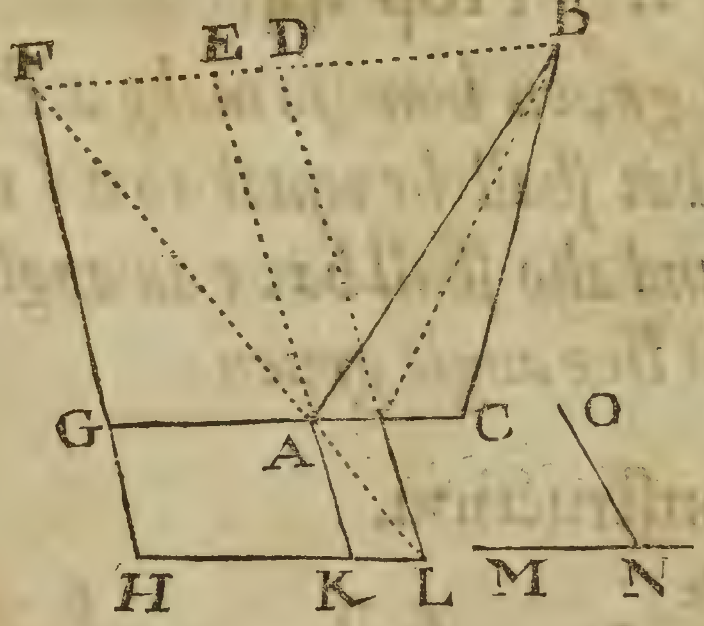
Conſtruction.



The given Triangle is A B C, the right line angle given is M N O, and the right line given let be M N, make firſt a Parallelogram, upon the line M N equal to the Triangle A B C *(by the 42 Propoſition,)* which is the Parallelogram A D, let the angle A I D

A I D be equal to the given angle M N O, then prolong D I to L ſo that I L be equal to the given line M N, and from L draw a line parallel to A I as L H, and another line through the point A as L F, cutting the extream of the extended line D E in the point F, and from thence draw a line parallel to D I as F H, ſo that the ſame doth touch L H in the point H, and extend A I to F H in G, and extend alſo E A to H L in K, then is the Parallelogram K A G H equal to the given Triangle M N O.

Demonſtration.

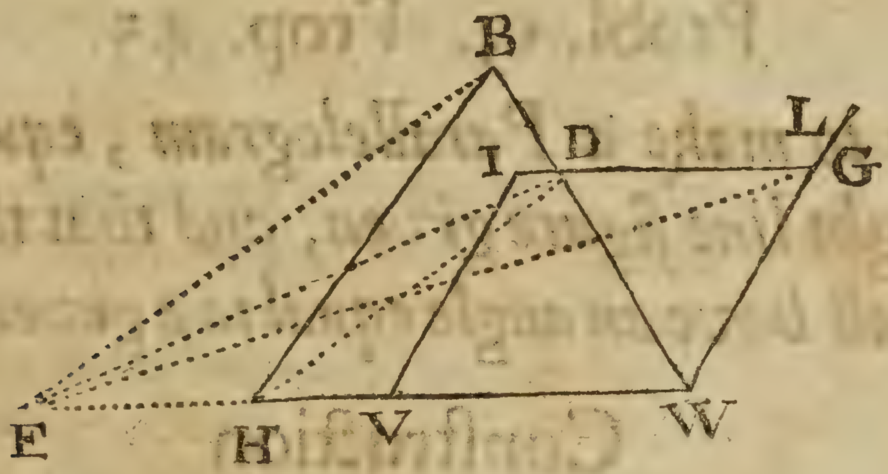


F D, G I and H L are three parallel lines, and alſo D I, E K and F H, which together doth make four Parallelograms, namely, G E, G K, K I and I E, all of them included in the Parallelogram D H which the line F L, doth divide into two equal parts, *(by the 34 Propoſition:)* as alſo G E and K I, it followeth, *(by the former Propoſition:)* that the ſupplements G K and I E are equal, and the ſame I E is equal to the given Triangle A B C, and therefore equal alſo to the Parallelogram G K, *(by the firſt common ſentence,)* and I L is equal to M N and alſo equal to A K, *(by the 34 Propoſition:)* wherefore is A K equal to M N, and the angle A I D equal to the given angle M N O, it is alſo equal to A G H, *(by the 29 Propoſition:)* and therefore equal to the given angle M N O.

The

The ſame another way.

The given Triangle is W B H, the line given is W G, and the angle given is G W H, prolong the baſe W H to E, and make (upon the point W and the baſe W H) the given angle as H W L, and upon the line W L: note the line given comes to G, from thence draw a line parallel to the baſe W H, which is G H cutting through the ſide of the Triangle W B, in the point D and from D draw a line to H as D H, and another line parallel to the ſame, from B to E, as B E, and from E to D draw the line E D, then is the Triangle W E D equal to the given Triangle W B H, then divide the baſe W E in two equall parts, the midſt is the point V, from thence a line drawn parallel to W G as V I, cutting the line G I in the point I, then is the Parallelogram W G I V, equal to the given triangle W B H having an angle equal to the given angle, and the height equal to the given line, as was required.



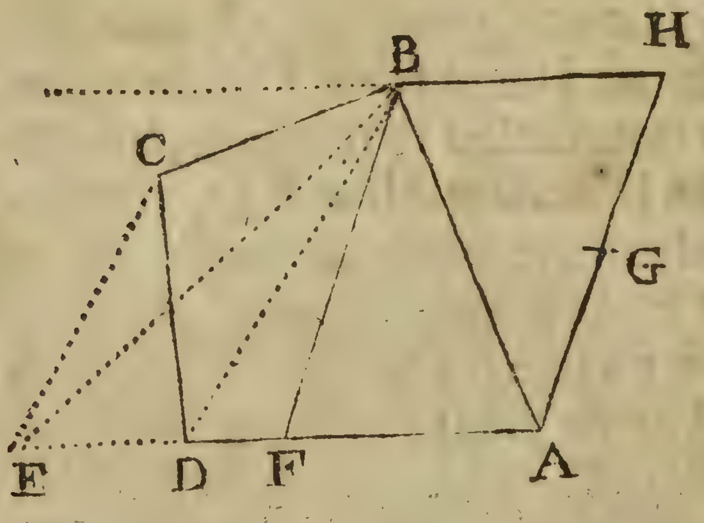
Demonſtration.

The triangles H E D and H B D ſtand upon one and the ſame baſe D H, and between two parallel lines D H and B E wherefore they are equal (*by the 37 propoſition*) to either of theſe being added the Triangle H D W, it appeareth, that the Triangle W D E is equal to W B H, (*by the ſecond common ſentence,*) and by the ſame reaſon, are the Triangles W E G and W E D equal, for they ſtand upon the baſe W E, and between two parallel lines D G and W E, (*by the 42 propoſition:*) wherefore it is alſo equal to the Triangle W B H.

Probl. 13. Prop. 45.

How to make a Parallelogram, equal to a right line figure given, and that the ſame ſhall have an angle equal to a given angle.

Conſtruction



The right line figure given is A B C D, the angle given is D A G, prolong A D to E, and draw a line from D to B, and from C draw another line parallel to B D, cutting the prolonged line D E in the point E, then is the Triangle A B E equal to the right line figure A B C D, then make upon the point A, an angle equal to the given angle D A G, (*by the 23 proposition*) that is, in this the angle D A H, and from the point B, draw the line K B parallel to A D, and prolong the ſame to H, then divide the line

A E into two equall parts, the midſt is F, and draw the line B F parallel to A H (which this doth cut the line K H in the point B.) Then is the Parallelogram A F B H, equall to the given fourth angle A B C D, having an angle juſt equall to the given angle.

Demonſtration.

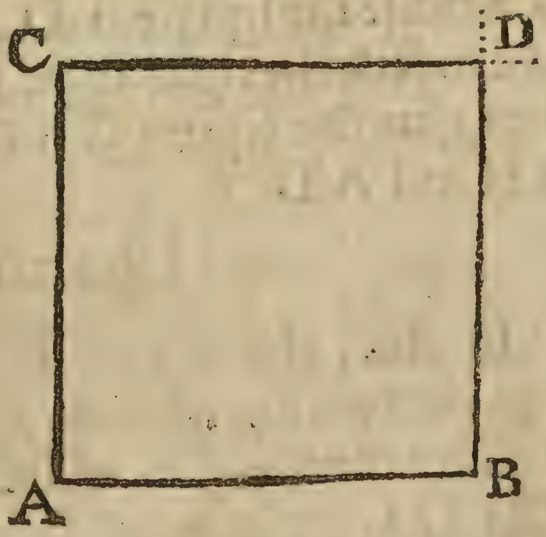
The two triangles B D C, B D E, ſtand upon one baſe D B, and between two parallel lines B D and C E. Therefore they are equall *(by the 37 Propoſition)* and the triangle BCD, being taken from the ſaid fourth angle, and in ſtead thereof, the triangle B D E added: it is evident, that the triangle A B E is then equal to the fourth angle A B C D. The reſt of the Demonſtration is ſhewed in the ſecond way of the former Propoſition.

Probl. 14. Prop. 46.

Upon a right line given, to make a Square.

Conſtruction.

The right line given is A B, from the points A and B raiſe a perpendicular, *(by the 10 Propoſition)* equall to A B, which are the lines A C and B D. Then draw a line from C to D, and ſo is the ſquare made.



Demonſtration.

The lines A C and B D are each of them equall to A B, therefore equall one to another, by the firſt common ſentence, and the line C D is equall to A B, *(by the 33 Propoſition)* and the angle C equall to B, and the angle D to A, *(by the 34 propoſition)* that is all the lines equall, and all the angles right angles.

Theor. 33. Prop. 47.

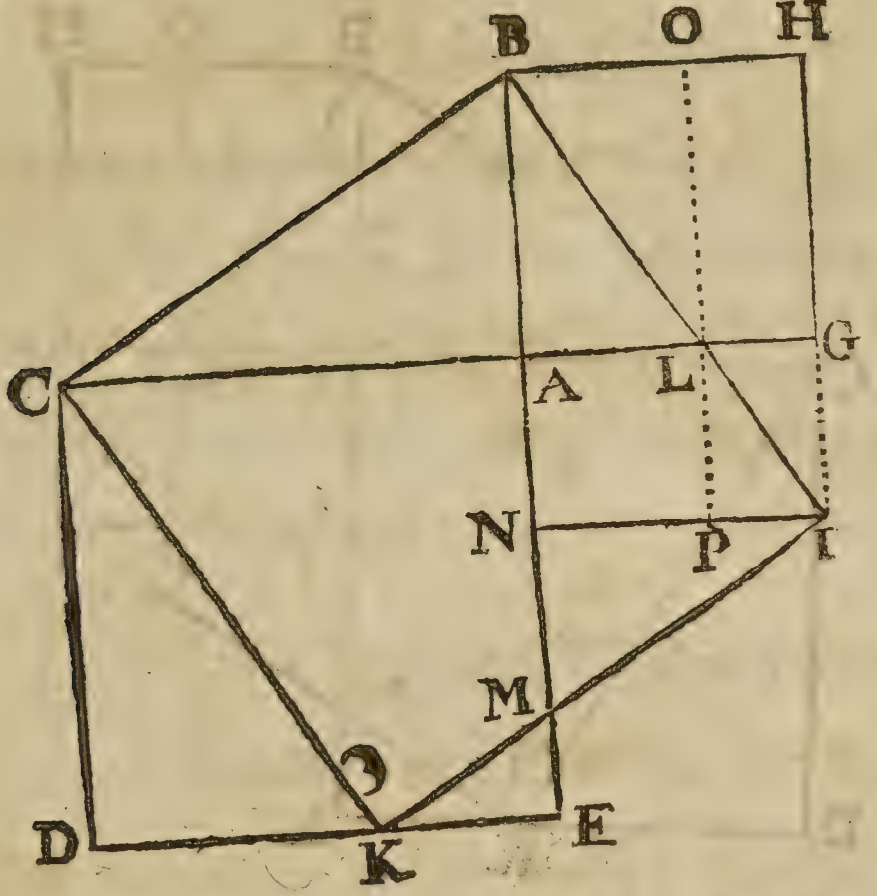
In right angled Triangles, the ſquare which is made of the ſide ſubtending the right angle, is equall to the ſquares which are made of ſides conteining the right angle.

Conſtruction.

In this figure is the triangle A B C, right angled in A, and upon the three ſides of this triangle, is made three ſquares: upon A B is the ſquare A H made. And upon A G is made the ſqnare A D, and upon the ſide B C, which ſubtendeth the right angle, is made the ſquare CBIK: I ſay then, that the ſquare C B I K is equall to the two ſquares A H and A D.

Demonſtration.

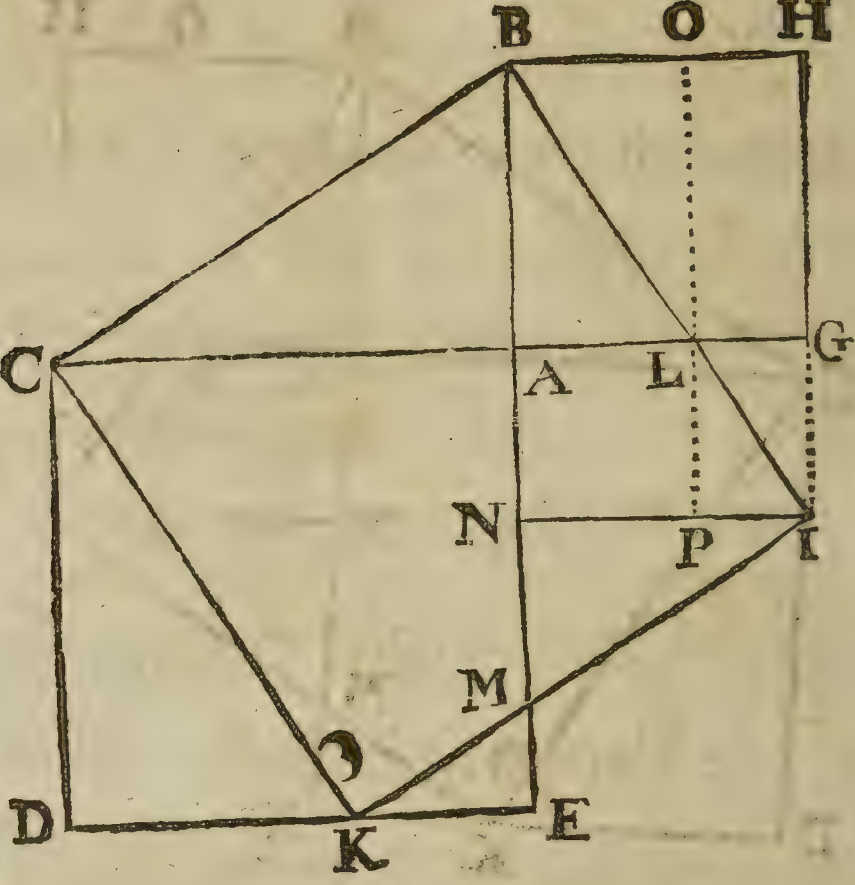
In this, the whole figure C B H I M E D is greater then the two ſquares that are made of the ſides A B, and A C by the triangle A B C together with the *Trapezia* A G I M.



And the triangle C D K is equal to the triangle C A B (*by the 4, 8, and 26 Propoſition*) and the *Trapezia* B H G L, together with the little triangle M E K (being equall to the little triangle L G I) doth make up the triangle B H I, alſo equall to A B C, the ſide H I being equal to A C, and H G equal to A B, and G I the difference, between the ſides A B and A C.

Now, we muſt prove, that the *Trapezia* A L I M is equal to the *Trapezia*, B H G L, and the little triangle M E K, that is, to the triangle B H I, which we thus prove.

Firſt, let a right line be drawn from the point I, parallel to A G, cutting the line A E in the point N, then is A N equall to G I the difference: then is N E equall A B, or to H G, or to N I. Let alſo a right line be drawne from the point L, paraliel to A B, or H G, as the line L O, and extend the ſame to P, cutting the line N I in the point P: then is the Parallogram H N, divided into his ſupplements



and triangles by the Diameter B I, ſo that each is equal to other of his kind, *(by the 43 Propoſition)* that is, the triangle A B L, equal to BOL, and L P I equal to L G I, or M E K, and the ſupplements L N equal to L H.

And laſtly, the triangle I N M, whoſe ſide N I is equal to A G or L O, and the ſide N M equall to B O (for as much as M E is equal to L G or O H) it followeth, that if we exclude the triangle C D K, and inſtead thereof, include A B C. And,

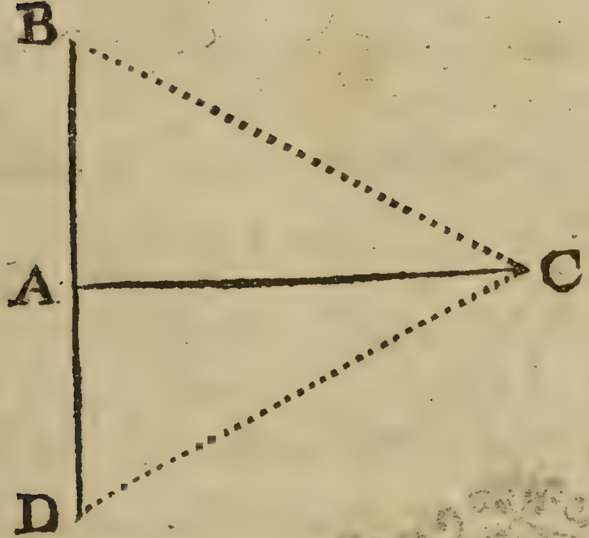
Again, exclude the *Trapezia*, B H G L) with the triangle M E K, that is (if we exclude the triangle B H I) and in his place include his *Trapezia* A L I M, equal to the ſame *(by the ſecond and third common ſentences)* there ſhal remain the ſquare of B C, equal to the two ſquares, namely, of A B and A C together, which was to be demonſtrated.

The-

Theor. 34. Prop. 48.

If the ſquare which is made of one ſide of a triangle be equall to the ſquares which are made of the two other ſides of the ſame triangle, the angle comprehended under thoſe ſides, is a right angle.

The Converſe of the 47 Propoſition.



In rhe triangle A B C, is the ſquare of B C equal to the two ſquares, made of the ſides A B and A C, therefore is the angle A right.

Demonſtration.

Let there upon A C be drawne another perpendicular line, from the point A, as A D equall to A B, and then drrw the line D C, and for as much as the ſquares A D, A C (that is equal to A B, A C) are equal to the ſquare of D C (*by the 47 Propoſition*) it followeth that the ſquare of C D ſhal be equal to the ſquare of B C, and the line C

D equal to B C, and the triangle D A C equal to
the triangle B A C, the angle
C A B equall to C A D,
that is, both right
angles.

The end of the firſt Book.

◆

EUCLIDES ELEMENTS.

The Second Booke.

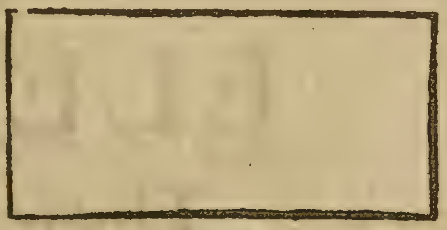
THE ARGUMENT.

IN this Book *Euclid* ſheweth what a *Gnomon* is, and a right angled *Parallelogram*, alſo in this Booke are ſet forth the power of lines divided evenly and unevenly, and of lines added one to another, the power of a line is the ſquare of a line, that is a ſquare every ſide of which is equall to the line, ſo that here are ſet forth the properties and qualities of the ſquares and right lined figures, which are made of lines and of their parts. The *Arithmetician* alſo out of this Book gathereth many compendious rules of reckoning, and many rules alſo of *Algebra*, with the *Equations* therein uſed, the grounds alſo of theſe rules are for the moſt part by this ſecond Book Demonſtrated.

This Book moreover conteineth two *wonderfull Propoſitions* one of an obtuſe angled *triangle*, and the other of an acute, which with the aid *(of the* 47 *Propoſition of the firſt Booke,)* which is of a rectangled Triangle: Of how great force and profit they are in matters of *Aſtronomy*, they know which have travailed in that Art, wherefore, if this Book had no other profit beſide onely theſe two Propoſitions, it were diligently to be embraced & ſtudied.

DEFINITIONS.

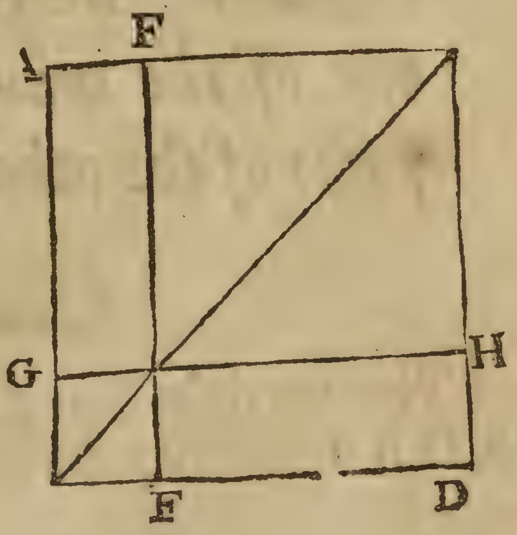
1 *Every rectangled* Parallelogram *is ſaid to be conteined under two right lines, comprehending a right angle.*



A *parallelogram* is a figure of four ſides, whoſe two oppoſite ſides are equall one to another. There are of *Parallelograms* four kinds. *A Square*, a *long Square*, a *Rombus*, and a *Romboides*. Of theſe four ſorts, the *Square*, and the *long Square* are onely *Right angled Parallelograms*, and ſuch a one is the Parallelogram here inſerted.

2 *In every* Parallelogram, *one of thoſe* Parallelograms, *which ſoever it be, which are about the* Diameter, *together with the two ſupplements, is called a* Gnomon.

In the Parallelogram A D, the Parallelogram G E with his two Complements F H, and G E, are called γνωμῶν (*i.e.*) *Gnomon.*



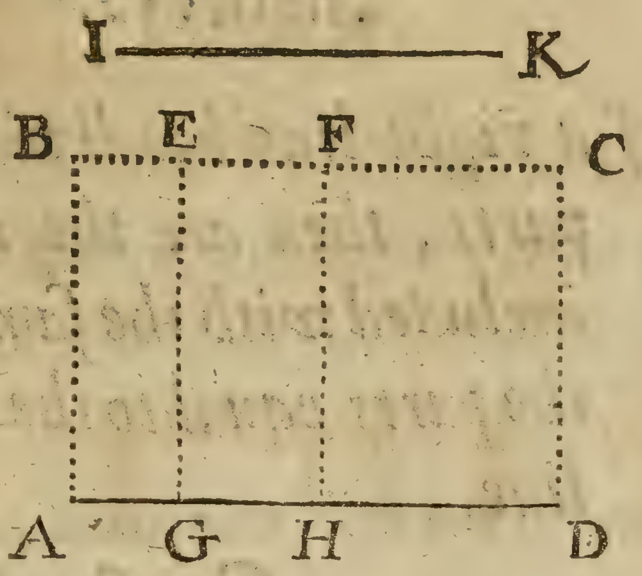
EUCLIDES ELEMENTS.

Theor. 1. Prop. 1.

Two right lines being given, whereof the one is divided by chance into as many parts as you will, then is the right angle figure comprehended of thoſe two whole lines, equall to all the right angle figures together, that are comprehended of the one of the whole lines, and each of the parts of the other line.

Conſtruction.

The two right lines given, are I K, and A D, of which A D is divided by chance in the points G and H, then is the right angled figure comprehended of I K, and A D, equall to the three right angle figures, comprehended of the ſame I K, with each of the three parts of A D, as of A G, G H, and H D.



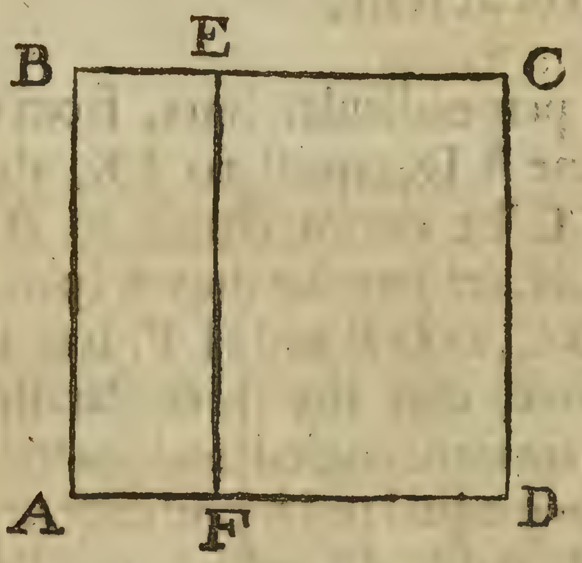
Demonſtration.

Let there be drawn two perpendicular lines, from the points A and D, upon the line A D, equall to I K, thoſe be A B and C D, and let B C be drawn equall to A D, then from the points G and H, let lines be drawn (parallel to A B or D C) to the line B C, as G E and H F. It is evident *(by the 29 of the firſt Book)* that the three Parallelograms G B, H E and D F, are right angled and included, with the whole line AB(being equal to IK)and each of the parts of A D, as A G, G H, and H D, which Parallelograms altogether, doth make the whole right angle figure B D, therefore they are equall to the whole.

Theor. 2. Prop. 2.

If a right line be divided by chance, into parts, then are the right angled figures included with the ſame line, and each of the parts equal to the Square of the whole line.

Conſtruction.



The line is A D, divided in F, upon the line A D, make a ſquare, as A B C D, and draw the line E F right angled upon A D, then is the ſquare A B C D, equall to the two Parallelograms, or right angle figures F B, and F C together.

Demonſtration.

The line F E *(by the 28 Propoſition of the firſt Book)* is parallel to A B, and D C, and doth make in the ſquare, the two right angle figures, which both being included with the whole line (equall to A D) and each with a part of the ſame, as A F and F D: it is evident, that the right angled figures F B and F C together, doth make the ſquare A B C D, and is therefore equall to the ſame.

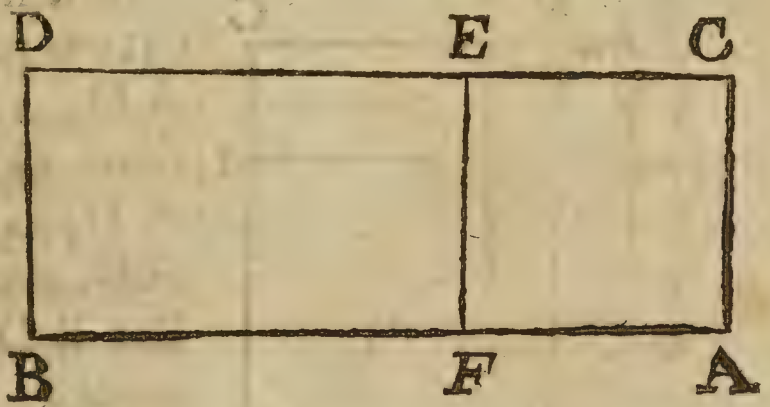
The-

Theor. 3. Prop. 3.

If a right line be divided by chance, into two parts, then is the right angle comprehended of the ſame line, and one part of the ſame, equall to the right angle figure, made of the parts, with the Square of the ſaid part.

Conſtruction.

The right line is A B, divided by chance, in the point F, as A F and F B, and of the line A B, and the part A F is included the right angle figure ACDB, which is equall to the right angle figure, made of the parts A F and F B, as F D, with the ſquare of A C, as A E.



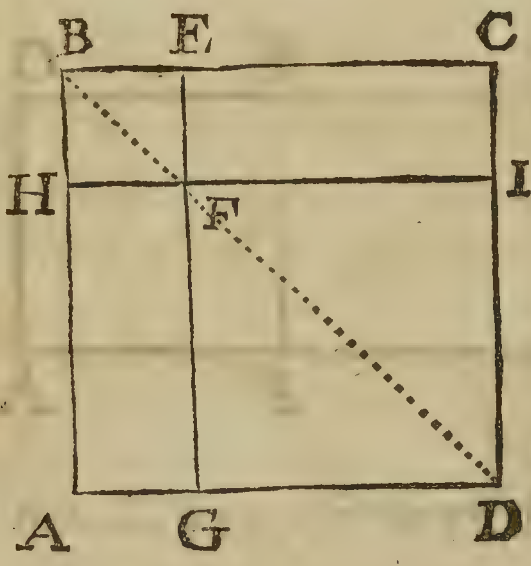
Demonſtration.

This is evident, and needeth no demonſtration, for the two figures, A E and F D (*according to this Propoſition:*) are incloſed in the figure A C D E, and are therefore equall to the ſame.

Theor. 4. Prop. 4.

If a right line be divided by chance, whether it be into equall or unequall parts, then is the ſquare of the whole line, equall to the ſquares of the parts with two right angle figures comprehended of the parts of the ſame.

Conſtruction.



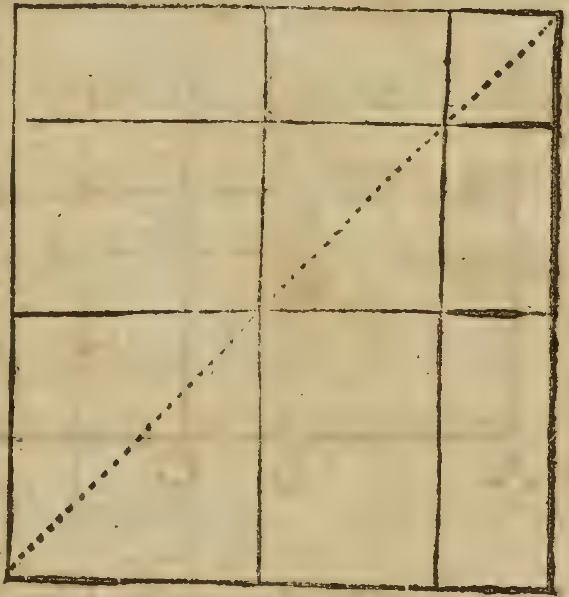
The line A D is divided into two parts, in G, and upon A D, is made a ſquare, and draw G E right angled upon A D, ſo that it be parallel to A B, draw alſo the diagonall line B D, cutting G E in the point F, through which point F, let be drawn a line parallel to A D, and equall to A D, as H I, making ſo upon the parts A G and G D, two other ſquares within the ſquare A B C D, as the ſquares G F I D, and H B E F, and alſo two right angled figures, included with the ſame parts, As F A and F C, theſe altogether are equall to the ſquare A B C D.

De-

Demonſtration

It is evident *(by the 33 and 34 Propoſitions of the firſt book)* that the parts H F, B E, E F, and I C, each of them are equall to A G, and F I, E C, F G, and H A, each of them equall to G D, inſomuch that the two ſquares made of the parts, with the two right angle figures included with the ſame parts of the line, thoſe taken altogether *(according to this Propoſition)* are equall to the ſquare of the whole line, wherein they are incloſed.

Note

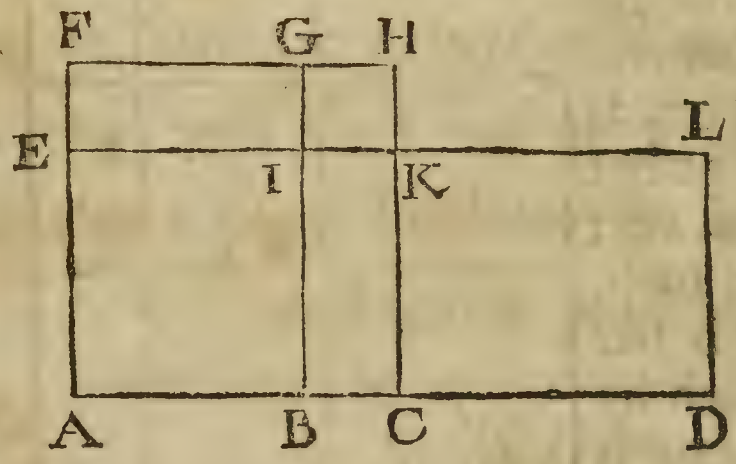


This Propoſition is not to be underſtood of the dividing of a line into two parts onely, but alſo into as many parts as one will, which parts together hath the ſame properties, that the former Propoſition hath, and the Demonſtration is like the former, as appeareth by this figure, where the line is divided into three parts, the ſquare of the whole line, is equall to the three ſquares of the parts, and alſo to twice three right angle figures made of the parts of the ſame line.

Theor. 5 Prop. 5.

If a right line be divided in two equall parts, and in two unequall, the right angle figure included with the unequall parts, with the Square of the difference of the parts, are equall to the Square, that is made of the half line.

Conſtruction.



The line A D is divided in two parts, equall in C, and unequall in B, let there be made a right angled figure, of the unequall parts, A B and B D, which figure is I D, and make a ſquare of the half line A C, as F C, and prolong the line I L to E, and I B to G, then is the right angle figure D I with the ſquare of B C, as G K, equall to the ſquare of the half line A C, that is equall to the ſquare A F H C.

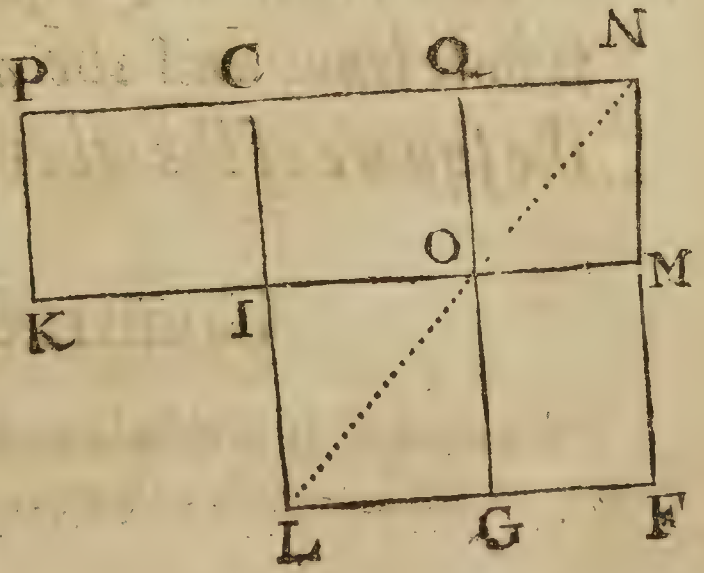
Demonſtration.

Foraſmuch as A E is equall to B I, and E F equall to B C, (that is, equall to I K, or G H) it followeth that G K is a ſquare, which with the right angle figure I D together, is equall to the ſquare F C, in aſmuch as the right angle figures F B and K D, are equall, being incloſed with equall lines.

Theor. 6. Prop. 6.

If a right line be divided into two equall parts, and then there be another line added to the ſame, directly in one right line, then is the right angle figure incloſed with the whole line and the added line (taken together for one line) with the line added with the Square of the half line, equall to the Square of the half line, together with the added line, as one line.

Conſtruction.



The line P Q, is divided in two equall parts, the midſt is C, to the ſame, is added a right line as Q N, and of the whole line P Q, and the added line Q N, is made P N, as one line, and of this line P N and the added line Q N, is incloſed the right angle figure P M, and upon the halfe line C Q and the line added Q N, is made the ſquare C F, and draw the line Q G parallel to N F and equall to the ſame, then is the right angled figure P M with the ſquare of C Q, as I G equall to the ſquare of C N, that is the ſquare C F.

Demonſtration.

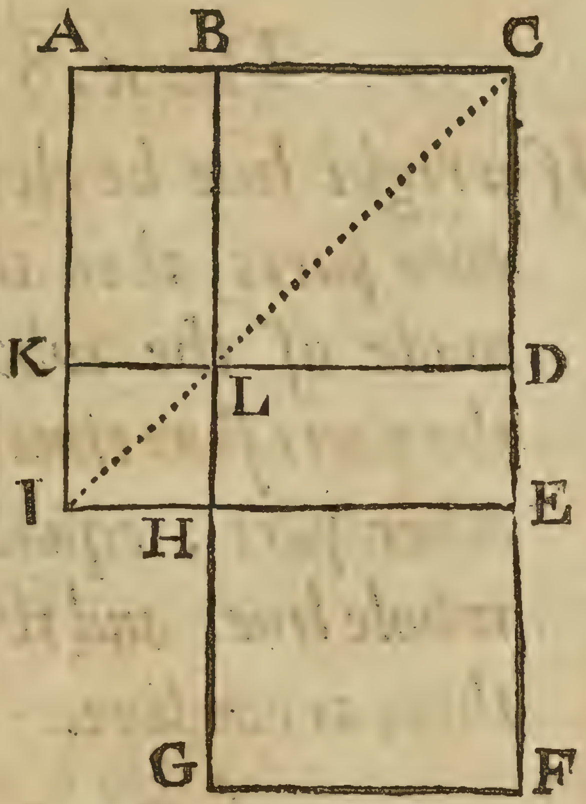
Foraſmuch, as C Q is equall to M F, which is alſo equalll to G O, I O, or L I, it followeth that I G, is a ſquare, which with the right angle figure P M is equall to the ſquare C F, becauſe the right angle figure G M, is equall to C O, which is alſo equall to C K.

Theor. 7. Prop. 7.

If a right line be divided by chance, into two parts, then is the ſquare of the whole line with the ſquare of one of the parts, equall to two right angle figures, made of the whole line, and the aforeſaid part, with the ſquare of the other part.

Conſtruction.

The line A C is divided by chance into two parts in B, and upon A C is made a ſquare, as A C E I, and C D is taken equall to B C, and K D being taken equall, and parallel to A C, and B H and CE, and alſo make a ſquare of the line B C, as H E F G, then is the ſquare of A C, as A C EI, and the ſquare of D C, as H E F G together, equall to two right line figures, included with the whole line A C, and the part B C, as A D & D G, with the ſqnare of A B, as K H together.



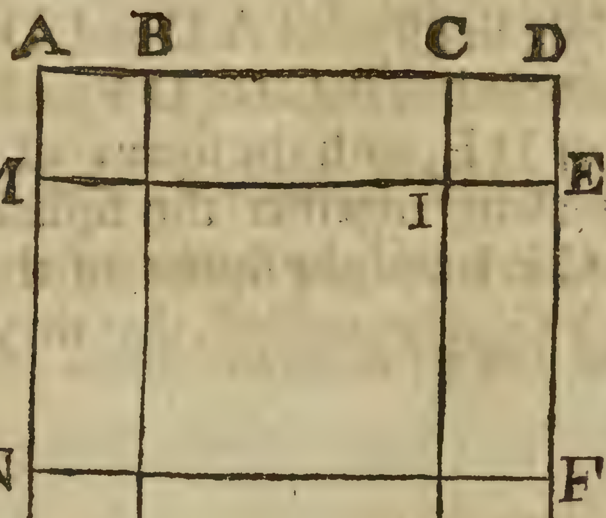
Demonſtration.

Foraſmuch as the two right line figures A D and D G, are comprehended of the whole line A C (or D F) and the part B C, equall to C D, or D L, with the ſquare of the other part A B, as K H, making together the ſquare of the whole line A C, as A C E I, and the ſquare of the aforeſaid part B C, as H E F G, it is manifeſt, that they are equall.

Theor. 8 Prop. 8.

If a right line be divided by chance, into two parts, then is the right line figure, made of the whole line, and one of the parts four times with the ſquare of the other part, equall to the Square of the whole line, and the firſt part, taken together, as one line.

Conſtruction.



The line A C is divided by chance, in B, and the line C D is added thereto equall to A B, and upon the line A D, is made a ſquare as A D G K, and then the lines B L and C H, equall and parallel being drawn to A K or D G, and alſo D E taken equall to D C, or A B, and draw E M and F N equall and parallel to A D or K G, then are the foure right angle figures comprehended of A C and A B with the ſquare of B C, equall to the ſquare of A D, that is, to the ſquare of A D G K.

De-

Demonſtration.

It is evident, that by the delineation of the figure, may be ſeen, the four right angle figures A I, C F, F L, and L M being incloſed of the whole line A C, and the part A B, with the ſquare where I ſtandeth, being made of the other part, B C; theſe I ſay altogether, doth make the ſquare of the whole line A C, with the other part added to the ſame, as one line, that is, as A C, and C D, which makes the whole line A D, as the ſquare A D G K, it is I ſay evident, that the ſame ſquare is equall to the four right line figures, with the ſquare of B C, wherein I ſtandeth.

Theor. 9. Prop. 9.

If a right line be divided into two equall parts, and into two unequall parts, then are the two ſquares of the unequall parts together, double to the ſquare of the half line, with the ſquare of the difference of the half line, and one of the unequall parts together.

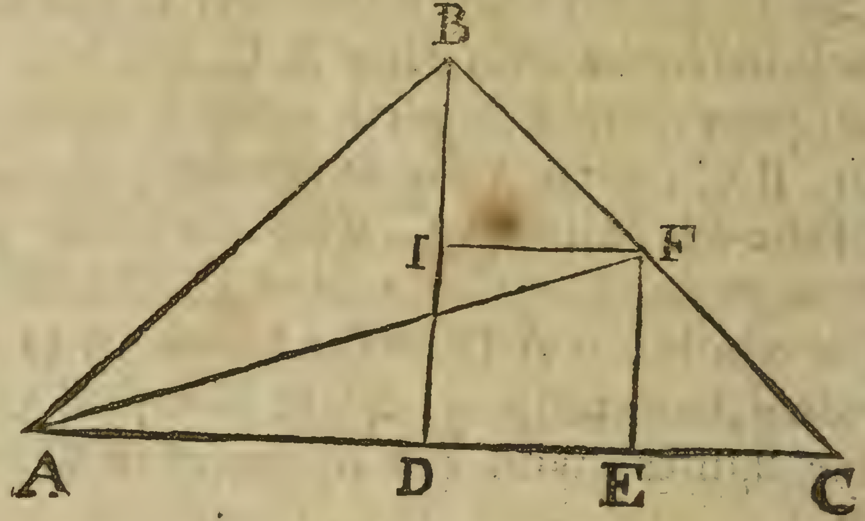
Conſtruction.

The line A C, is divided in two parts, equall in D, and in two unequall parts in E, then are the two ſquares of the unequall parts, as A E, and E C, together double to the ſquare of the half line A D or D C, with the ſquare of the difference D E.

De-

Demonſtration.

Let there from D be drawn a line perpendicular upon



A C, as D B, equall to A D, and draw the lines A B and B C. Foraſmuch as the two Triangles A D B, and C D B, have their ſides A B and B C equall, their angles on the baſe, as A D B and C D B equall, being both right angles, then are the angles D B A, and D A B, half right angles, *(by the 32 of the firſt book)* wherefore the angle A B C is a right angle, draw alſo from E a perpendicular to F, and from F a line equall and parallel to E D, it is evident *(by the 26 of the firſt book)* that E F is equall to E C, and the angles E C F and E F C, are equall, aud each half right angles, and I F and I B, are by the ſame reaſon alſo equal: and further draw the line A F. Then *(by the 47 of the firſt book)* The ſquares of A E and E F, or of A E and E C are equall to the ſquare of A F, and that ſquare of A F is alſo equall to the ſquares of F B and B A; foraſmuch then, as the ſquare of B F is double to the ſquare of B I or I F, or D E, and the ſquare of A B double to the ſquare of the half line A D, it followeth, that the two ſquares of the unequall parts A E and E C are alſo double to the ſquares of the half line A D, and the difference D E.

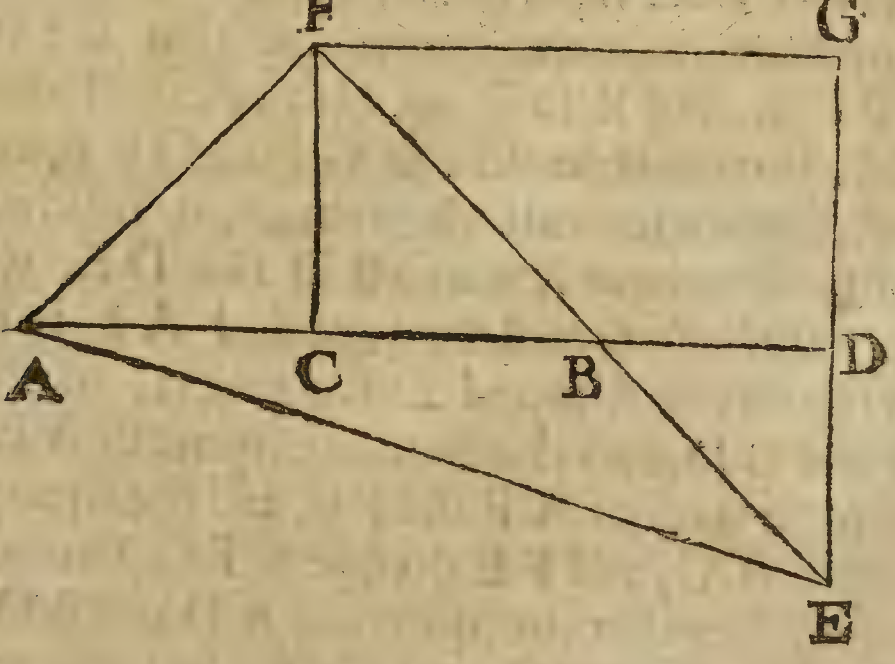
The-

Theor. 10. Prop. 10.

If a right line be divided in two equal parts, and another right line be added in a right line with the ſame, then is the ſquare of the whole line, together with the added line (as being one line) with the ſquare of the added line together double to the ſquare of the half line, with the ſquare of the half line, and added line together, as one line.

Conſtruction.

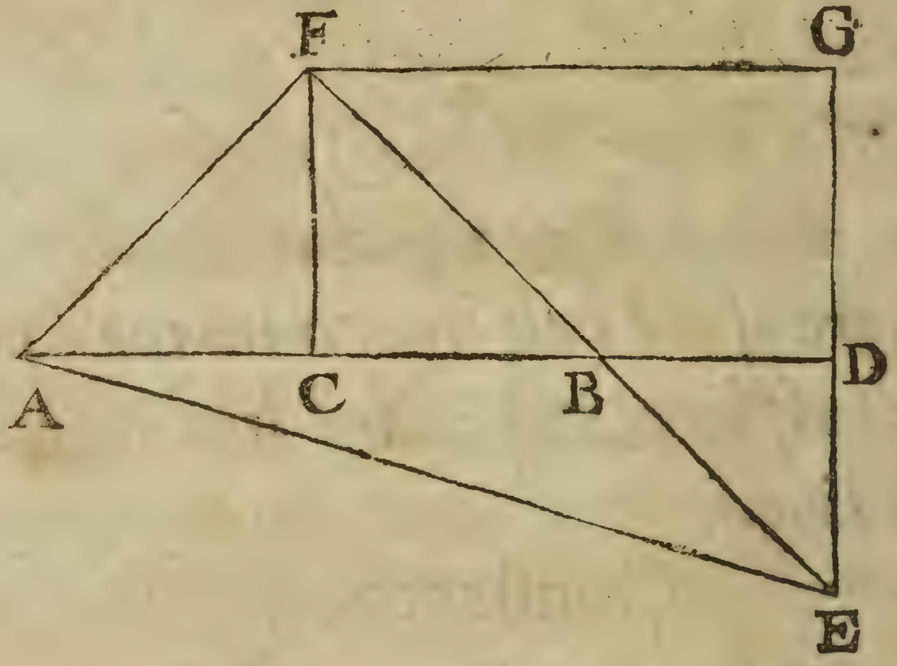
The line A B is divided into two equall parts, the midſt is C, and to the ſame is added B D, in a right line with



A B, then are the ſquares of A D and B D together, double to the ſquares of A C and C D together. De-

Demonſtration.

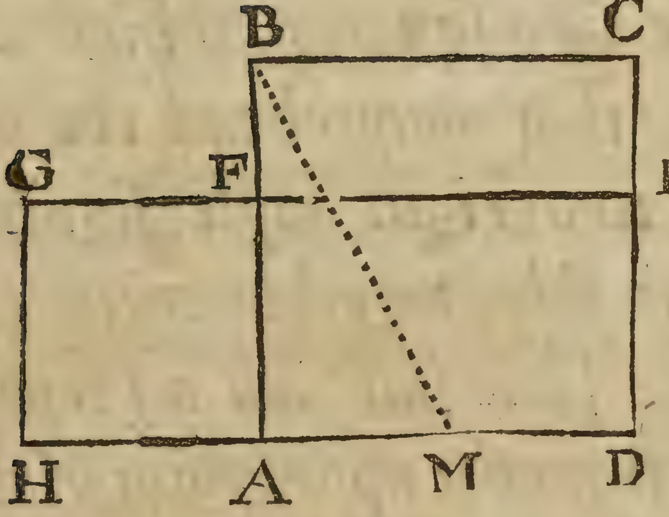
From the point C raiſe the perpendicular C F, equal to half the line A B, as C F, and the right angle figure C F G D being incloſed, and the line F A, and F B being drawn, extend F B forth till it cut the line G D in the point E, and from E, draw a line to A, foraſmuch then as the angle A F B is a right angle, being made of two half angles together, (as in the former) and that the two Triangles, F C B, and B D E are (Iſoſceles) of two equall ſides, and alſo the angles C F B, G F E, G E F, and DBE, are equall, and each a half right angle, it followeth, that F G and G E are equall, as alſo B D and D E, wherefore *(by the 47 of the firſt book)* the ſquare of A E is equall to the two ſquares of A D and D E: that is, to the ſquare of A D and B D, and alſo the ſame ſquare of A E is equal to the two ſquares of A F and F E, and the ſquare of A F, is double to A C, and F E donble to F G, that is, to C D: it followeth then that the ſquares of A D and B D are double to the ſquares of A C, and C D, which was required to be demonſtrated.



Probl. 1. Prop. 11.

To divide a right line in two parts, ſo that the right angle figure, made of the whole line, and one part, ſhall be equall to the ſquare of the other part.

Conſtruction.



The right line given, is A B, upon the ſame line AB make a ſquare, as A B C D, & divide the ſide A D in two equall parts, the midſt is M, and from M draw a line to B, and produce A D to H, ſo that M H be equal to MB, and upon A H make a ſquare, as A H G F. Then extend G F to E, and then is the right angle figure F C (being with the whole line F E, equal to A B, and the part B F) equal to the ſquare of the other part A F, as the ſquare A H G F.

Demonſtration.

Foraſmuch as *(by the 6 of this book)* the right angle figure comprehended of H D and H A, or of H D and H G, as the figure G H E D, with the ſquare of A M, are together equall to ~~the~~ ſquare of H M, being equall to B M;

it followeth, that if we take away the ſquare of A M common to both, that the ſquare of AB, as the ſquare ABCD, is equall to the right angle figure H G E D, and from them both, being taken away, the common right angle figure A E, there ſhall remain the right angle figure F C, equall to the ſquare A H G F, according to the intent of this Propoſition.

Theor. 11. Prop. 12.

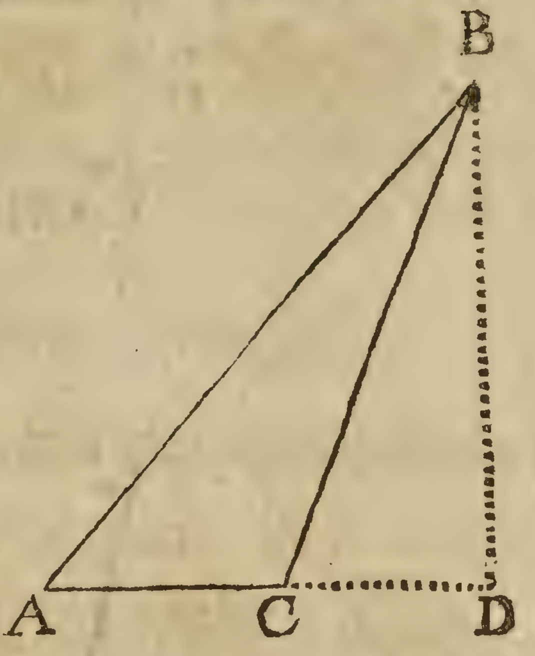
In all obtuſe angled Triangles, the ſquare of the ſide, ſubtending the obtuſe angle, is greater than the two ſquares, which are made of the ſides that comprehend the obtuſe angle, by two right angle figures, made of the one ſide, together with the extending of the ſame ſide, till a perpendiculnr falling from the upper point of the Triangle, doth cut the extended line.

Conſtruction.

In this obtuſe angled Triangle A B C, the ſide A B doth ſubtend the obtuſe C, and the ſides A C, and C B, doth comprehend the obtuſe angle C, then I ſay, that the ſquare of the ſide A B, is greater than the two ſquares, which are made of the ſides A C, and C B, by two right angle figures, comprehended under the lines AC and CD.

De-

Demonſtration.



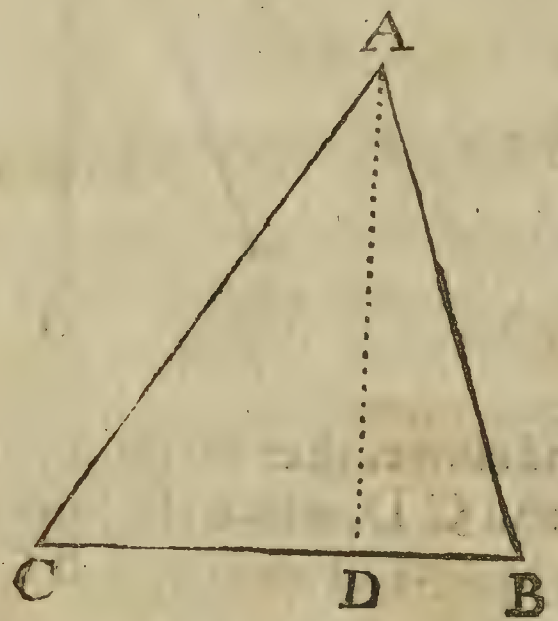
By the fourth Propoſition of this book is the ſquare of A D equall to the ſquares of A C and C D together with two right angle figures, made of the ſame A C and C D, and *(by the 47 of the firſt book)* is the ſquare of A B, equall to the ſquares of A D, and B D. If then from both were taken away, the ſquares of B D, and C D, that is the ſquare of B C, with the ſquare of A C, there ſhould yet remain over and above, the two right angle figures included with A C and C D, which the ſquare of A B, is graater then the two ſquares of A C and B C, according to the Propoſition.

Theor. 12. Prop. 13.

In all ſharp or acute angled Triang. the ſquare which is made of the ſide ſubtending the ſharp angle, is leſſer than the ſquares, made of the two ſides comprehending the ſame angle. By two right angle figures, made of the ſide (whereupon the perpendicular from the upper angle doth fall from the ut-

uttermoſt point) together with the part from the perpendicular, to the utter point of the ſharp angle of the ſaid Triangle.

Conſtruction.



In this acute angled Triangle A B C, is propounded one of the angles to be a ſharp or acute angle: namely the angle B, and the ſide ſubtending the angle is A C, and the perpendicular line is A D, and the part between the perpendicular and the ſharp angle is D B, now by this Prppoſition the ſquare of A C, is leſſer than the ſquares of A B and B C together by the two right angle figures, made of B C, and B D.

Demonſtration.

By the 47 of the firſt book is the ſquare of A C, equall to the ſquares of A D and D C, and *(by the 7 of this book)* are the ſquares of B C and B D together, equall to the two right angle figures made of B C and B D with the ſquare of D C: it followeth that if we to theſe equall things, adde the ſquare of A D, that then the two ſquares of A D and B D (being equall to the ſquare of the ſide A B) with the ſquare of B C, equall to the two ſquares of C D and A D, together with two right angle figures included with B C, and B D, now the ſquare of the ſide A C is equall to the ſquares of A D and D C, that taken from the ſquares of the other ſides A B and B C. There doth yet reſt the two right angle figures included of B C, and B D, which the ſquare of the ſide A C is leſſer than the two ſquares of the other ſides A B and B C, *according to the intent of this Propoſition.*

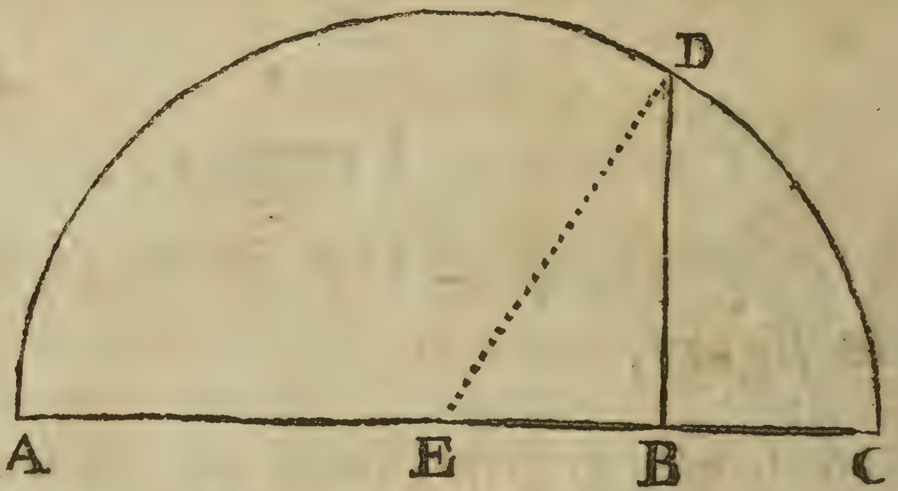
Probl. 2. Prop. 14.

To make a ſquare equall to a given right line angle figure.

It is ſhewen, *in the 45 Propoſition of the firſt book*, how to change a right line figure (of many ſides) into a Triangle. For although there be ſpoken of no more, than onely of four ſided, in that Propoſition, to bring them into Triangles, yet by that which is there ſaid, it is plain to underſtand how to do the ſame in many ſided figures alſo, which otherwiſe would be very tedious to be done.

Wherefore, if a figure of many ſides be given to be changed into a ſquare of the ſame quantity, that is equall to the ſame. Then change the ſame, firſt into a Triangle, (as before is ſhewed) and then the Triangle into a Parallelogram *(by the ſaid 45)* and laſt of all, that Parallelogram into a ſquare, as here followeth.

Let there be a Parallelogram right angled, comprehended with the two ſides or lines A B and B C, that is, as long as A B, and as broad as B C, ſet theſe two lines, the one, at the end of the other, in one right line, as here A C, then divide the line A C in two equall parts in the point E, and then from the point E, as from a center of a Circle, draw the half Circumference of a Circle, as A D C, whoſe Diameter is A C, then draw from B, a perpendicular line B D, touhing the circle in D, and upon B D, make a ſquare, and that ſquare ſhall be equall to the parallelogram incloſed with A B and B C.



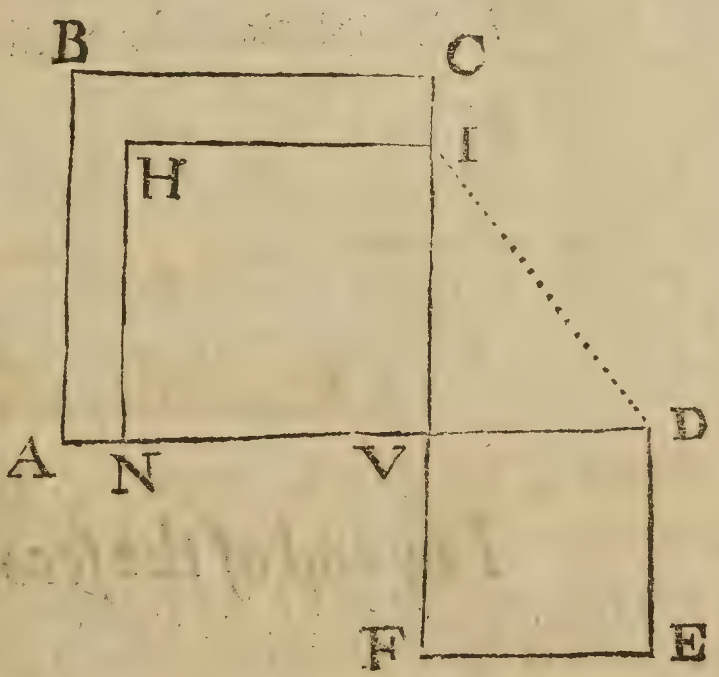
Demonſtration.

From E, let a line be drawn to D, foraſmuch as A C is divided into two equall parts in E, and unequall in B, then is *(by the 5 of this book)* the right angle figure incloſed with A B and B C, with the ſquare of E B, equall to the ſquare of E C, or E D, being *(by the Circles Definition)* equall, and the ſquare of E D, equall to the ſquares of E B and B D, it followeth, that if we take away from them both, the ſquare of E B, that the ſquare of B D, ſhall remain equall to the right angled figure, comprehended of A B and B C.

Probl. 3. Prop. 15.

To make a Gnomon about a Square that ſhal be equall to a given Square.

Conſtruction.



The ſquare given, to which the Gnomon ſhall be made equall, is the ſquare VDEF & the ſquare about which it ſhall be made, is V I H N; ſet the one ſide of each of theſe ſquares together in one right line, as N V D, and prolong VI to C, ſo that V C be equall to D I (called *Hypothenuſa*) and make a ſquare upon V C *(by the 46 of the firſt book)* which is V C B A, then is the Gnomon C H A equall to the ſquare V D E F.

Demonſtration.

Foraſmuch as the two ſquares V H and V E are *(by the 47 of the firſt book)* equall to the ſquare of D I, that is, to the ſquare of V B, it followeth that if we take from both the common ſquare VH, that then the Gnomon C H A doth remain equall to the ſquare V E.

William Holtzman, a *German*, doth teſtifie, that this laſt Propoſition is not found in the Greek Copies, nor Latine, and therefore (not without cauſe) doth doubt whether it was *Euclids* or no, as being ſet contrary to his order, ſeeing that he defineth a Gnomon in the ſecond book, and *Campen* hath ſet the ſame the laſt in the firſt book) but *Lucas Paciolus* witneſſeth, that the ſame is found in ſome old Copies, in the laſt of the ſecond book where no doubt, it ought to ſtand, if it be of *Euclides* writing.

The end of the ſecond Book.

EUCLIDES ELEMENTS.

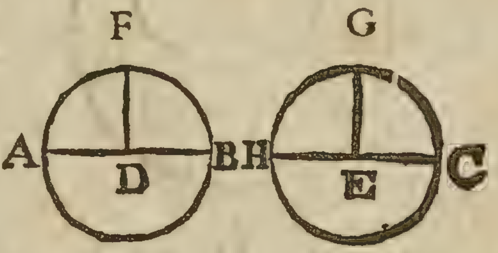
The Third Booke.

THE ARGUMENT.

This Book treateth of the moſt perfect Figure which is a Circle, wherefore it is much more to be eſteemed than the two Books going before, which did ſet forth the moſt ſimple properties of right lined Figures, for Sciences take their dignities of the worthineſſe of the matter that they intreat of, but of all Figures the Circle is of moſt abſolute perfection, whoſe properties and paſſions are here ſet forth, and moſt certainly demonſtrated. Here alſo is intreated of right Lines ſubtended to Arches in Circles; alſo of Angles ſet both at the Circumference and Center of Circles, and of the variety and differences of them, whereby the reading of this Book is very profitable, to the atteining of the knowledge of Arches and Chords. Again, it ſheweth, how in a Sphear, an Orbe, or ſuch like; may be found the two oppoſite points, which is a thing very neceſſary and commodious; chiefly for thoſe that ſhal make Inſtruments, ſerving to Aſtronomy and other Arts.

DEFINITIONS.

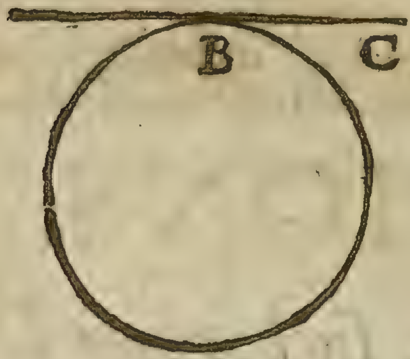
1 *EQuall Circles are ſuch whoſe Diameters are equall, or whoſe lines drawn from the Centers are equall.*



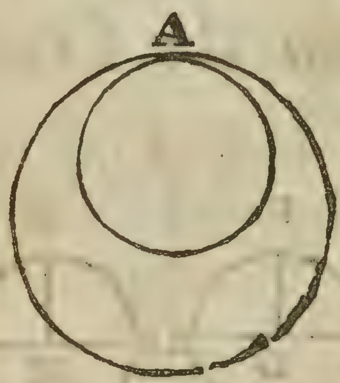
As the Circles A F B D and B G C E are equal, becauſe their Diameters A B and B C are equal, and alſo becauſe their Semidiameters DF and EG are equal. The reaſon whereof is, for that a Circle is deſcribed by one revolution, or turning about of the Semidiameter, having one of his ends fixed, as if you imagine the line D F, to have his point D fixed, and the other end, namely F, to move round, till it come to the place where it began to move, it ſhall fully deſcribe the whole Circle, wherefore if the ſemidiameters be equal, the Circles of neceſſity muſt alſo be equall, and alſo the Diameters.

2 *A right line is ſaid to touch a Circle, which touching the Circle, and being produced, cutteth it not.*

As the line B C onely toucheth the Circumference of the Circle in the point B, but cutteth it not, and ſuch lines, as the line B C, is called a Tangent line.

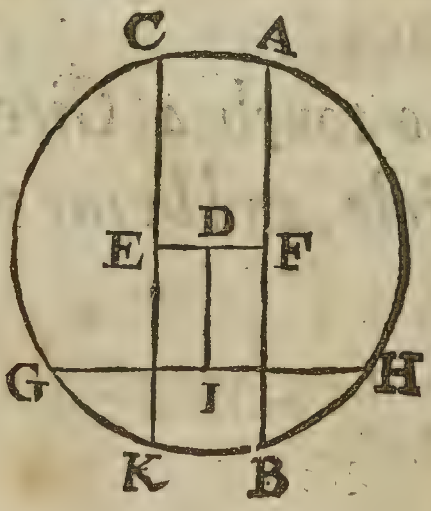


3 *Circles are ſaid to touch the one the other, which touching the one the other, cut not the one the other.*

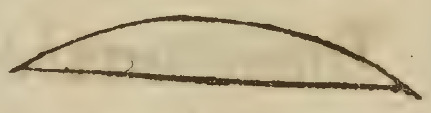


As the two Circles do not cut through, but do onely touch the one the other in the point A.

4 *Right lines in a circle, are ſaid to be equally diſtant from the Center, when Perpendicular lines drawn from the Center unto thoſe lines are equall: and that line is ſaid to be more diſtant, upon whom falleth the greater perpendicular line.*



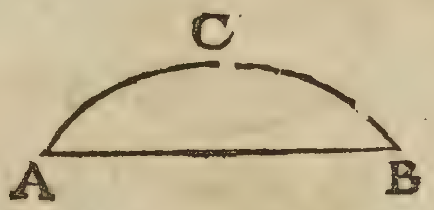
As in this Circle GCAHBK, the lines C K, and A B are equidiſtant from the Center D, becauſe that the line DE, drawn from the Center D perpendicular to the line C K, and the line D F, drawn likewiſe perpendicular from the Center D, upon the line A B, are equall the one to the other. But the line G H is farther diſtant from the Center D, then are the lines C K or A B, becauſe the line D I drawn perpendicular to the line G H from the Center D, is longer than the lines E D, or D F.



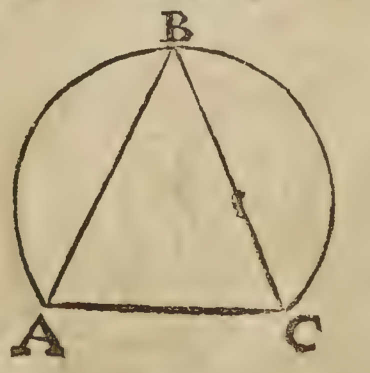
5 *A Section or ſegment of a Circle, is a figure comprehended under a right line, and a portion of the circumference of a Circle.*

6 *An angle of a Section or ſegment is that angle which is contained under a right line, and the circumference of the Circle.*

As the angle C A B in the ſection C A B, is an angle of a ſection, becauſe it is contained of the Circumference A C B, and and the right line B C.



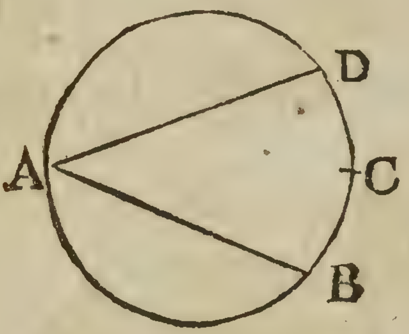
7 *An angle is ſaid to be in a ſection, when in the circumference is taken any point, and from that point are drawn right lines to the ends of the right line which is the baſe of the ſegment,*



the angle which is contained under the right lines drawn from the point, is ſaid to be an angle in a ſection.

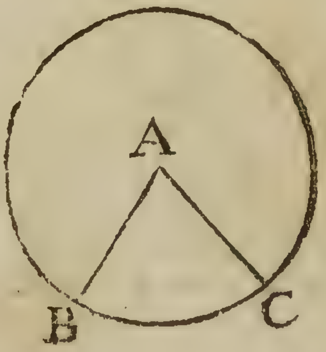
As the angle A B C is ſaid to be an angle in the ſection B A C.

8 *But when the right lines which comprehend the angle do receive any circumſerence of a Circle, then that angle is ſaid to be correſpondent, and to pertain to that Circumference.*



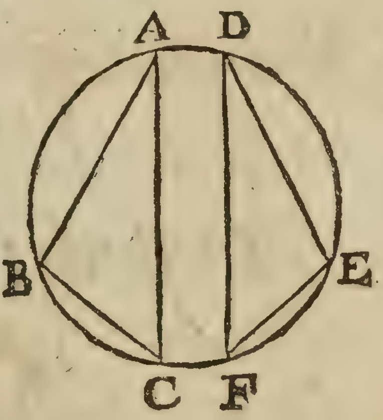
As the right lines A D, and A B, which contain the angle B A D and receive the Circumference B C D, therefore the angle B A D is ſaid to ſubtend, and to pertain to the Circumference B C D.

9 *A Sector of a Circle is (an angle being ſet at the center of a Circle) a figure contained under the right lines which make that angle, and the part of the Circumference received by them.*



As the figure A B C is a Sector of a Circle, for that it hath an angle at the Center, namely, the angle B A C, and is contained of the two right lines A B and A C, which contain that angle, and the circumference received by them.

10 *Like ſegments or ſections of a Circle, are thoſe which have equal angles, or in whom are equall angles.*



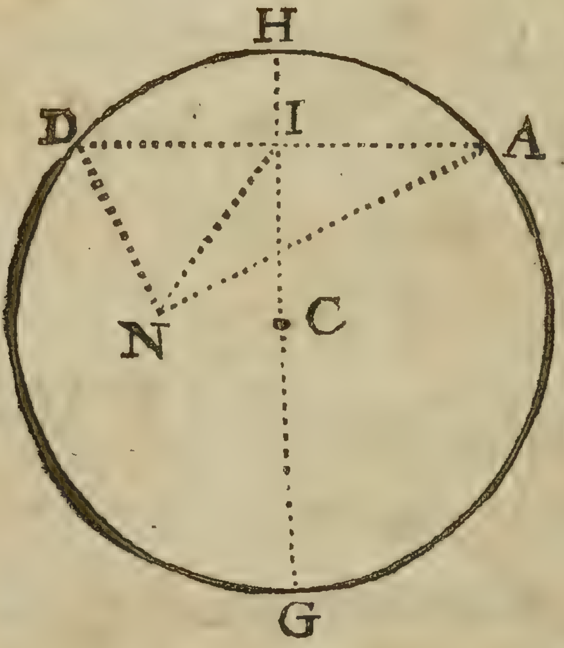
As the angles A B C, and D E F are equall, becauſe they lie in like Sections of a Circle.

EUCLIDES ELEMENTS:

Probl. 1. Prop. 1.

To finde the Center of a Circle.

Conſtruction.



To finde the Center of this Circle, draw a line by adventure, in the ſame, which with both ends doth touch the circumference, as A D, divide the ſame line in two equall parts, the midſt is I: from thence draw a line perpendicular upon A D, to the circumference, as I G, and prolong the ſame on the other ſide to H, making the right line G H, then divide the line H G, into two equall parts, the midſt is C, which is the Center of the Circle required.

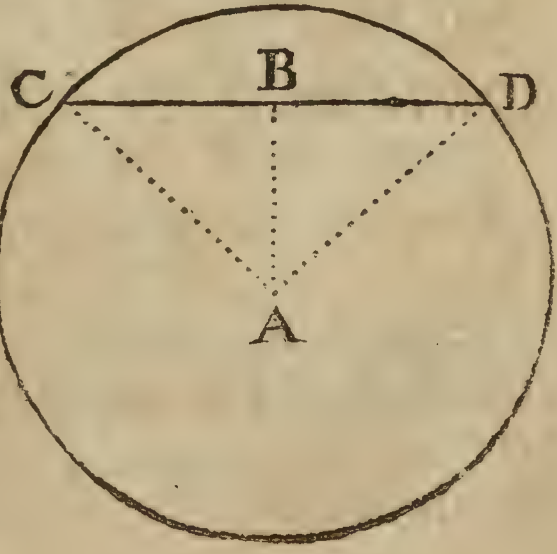
Demonſtration.

Suppoſe that C were not the Center of the Circle, but that but that N were the ſame, and draw the lines N A, N I, and N D, the lines N A and N D, *(by the Circles Definition)* muſt then be equall, aud *(by the 5 of the firſt Book)* the angles N A D and N D A equall, and the two Triangles N A I and N D I (included with equall lines) ſhould *(by the 4 and 8 of the firſt book)* be equall, and therefore muſt the angle N I A be equall to N I D, and both right angles, which cannot be; For the angles AIC and DIC, are right angles *(by the work it ſelf)* and N I A, a greater angle, then C I A ſhould be not onely equall to the ſame, but alſo a part more, which is impoſſible, and *(contrary to the* 10 *common ſentence)* wherefore not N, but C is the Center of the Circle.

Theor. 1. Prop. 2.

If two points be taken in the circumference of a Circle by chance, and from the one point to the other be drawn a right line, that line doth fall within the Circle.

Conſtruction.



The points in the Circumference are C and D, and the line drawn is C D, which line doth fall within the Circle.

Demonſtration.

Divide C D in two equall parts, the midſt is B, from thence draw a perpendicular to the center A, and from A draw two lines to the points, as to C and D: And for as much as the angle A B D is right, then is it greater then B A D, or B D A *(by the 32 of the firſt Book,)* it followeth *(by the 18 of the ſame)* that A D is longer then A B, and that the line CD is within the Circle; for if the ſame were without, then ſhould A B be longer, and upon the circumference ſhould A B and A D be equall.

Theor. 2. Prop. 3.

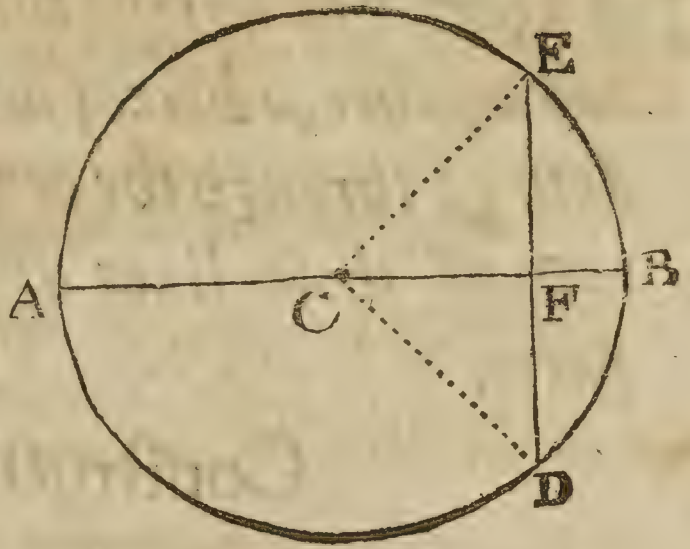
If a right line be drawn through the center of a Circle, and within the ſame Circle, it doth cut through another right line (which doth not go through the center) in two equall parts, then do thoſe right lines make (at the interſection) right angles: and if thoſe lines, in the interſection, doe make right angles, then is that line which goeth not through the center, cut in two equal parts with the other line.

Conſtruction.

The line A B doth go through the center C, and doth cut through another line E D (which doth not go through the center) in two equall parts, therefore are the angles in F all right angles.

De-

Demonſtration.



From C let be drawn the lines C E and CD, and for as much as the two Triangles F C D and F C E are equall *(by the 4 and 8 of the 1 Book)* it followeth that the angles C F E, and B F D are alſo equall, and *(by the 13 of the ſame Book)* they are alſo right angles, as alſo B F E and B F D.

And ſecondly, If then the lines A B and E D, cut one another through in right angles, then are the parts F D and F E equall.

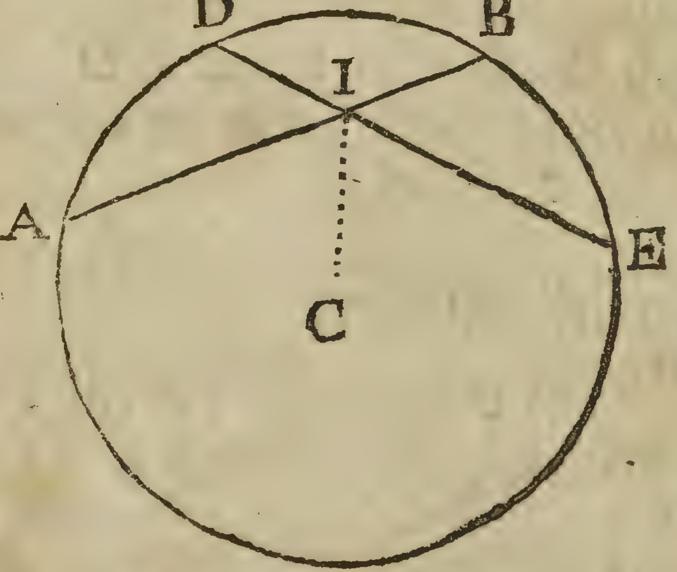
Demonſtration.

Seeing that the angles E and D are equall, as alſo C F D and C F E, and the ſides of the two Triangles, as CE and C D equall, and C F common to both: it followeth, that the other angles and ſides of thoſe Triangles are alſo equall one to another *(by the 26 of the firſt Book,)* that is, E F equall to D F, wherefore the propoſition is true.

Theor. 3. Prop. 4.

If in a Circle two right lines doe cut through one another, whereof neither of them both doth go through the center, then are neither of thoſe lines divided into equall parts.

Conſtruction.



In this Circle, the lines A B and D E, cut through one another in the point I, and neither of them both doth paſs through the center of the Circle, therefore are neither of thoſe cut through in equall parts.

Demonſtration.

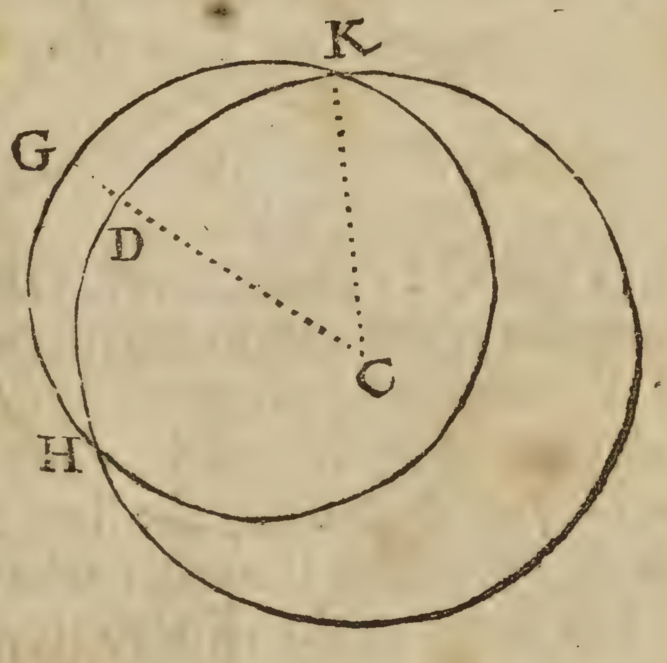
Let there be drawn from the croſſe point I, a line to the center C, as I C, if the two lines did cut through one another in equall parts, then ſhould the line C I come right angled upon both the lines A B and D E in the point I (*by the former propoſition,*) that is, the angle C I A ſhould be equall to the angle C I B, and ſeeing that all right angles are equall one to another, the ſame angle C I A, ſhould likewiſe be equall to C I D, and C I E equall to C I B, which *by the 9 common ſentence*, cannot be, For C I A is a part of C I D, and C I E a part of C I B: therefore are the lines A B and D E not both, nor any of them divided into equall parts.

Theor. 4. Prop. 5.

If two Circles cut the one the other, they have not one and the ſame Center.

Conſtruction.

Theſe two Circles cut through one another in the points K and H therefore they have not one and the ſame Center.



Demonſtration.

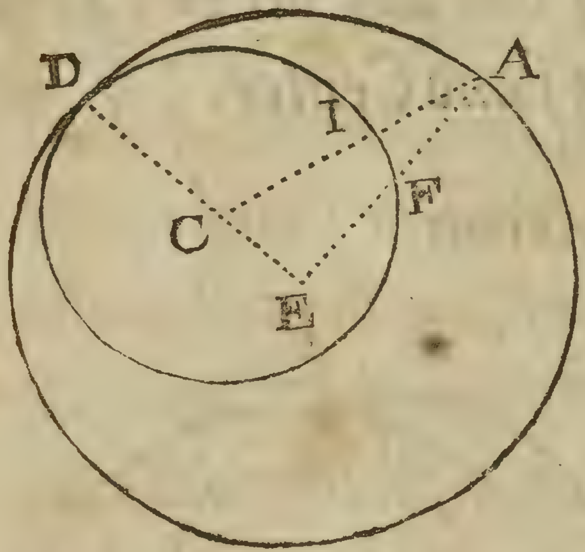
If we take C to be the center of both the Circle, and draw the lines C K, C D and C G, it followeth *(by the Circles Definition)* that theſe three lines ſhall be equall: that is, C D part of C G ſhould be equall to the ſame C G which is falſe *(by the 9 common ſentence)* wherefore, both thoſe Circles have not one and the ſame Center.

The-

Theor. 5. Prop. 6.

Two Circles which do inwardly touch one another, have not one and the ſame Center.

Conſtruction.



Theſe two Circles touch one another in the point D inwardly : therefore they have not one and the ſame center.

Demonſtration.

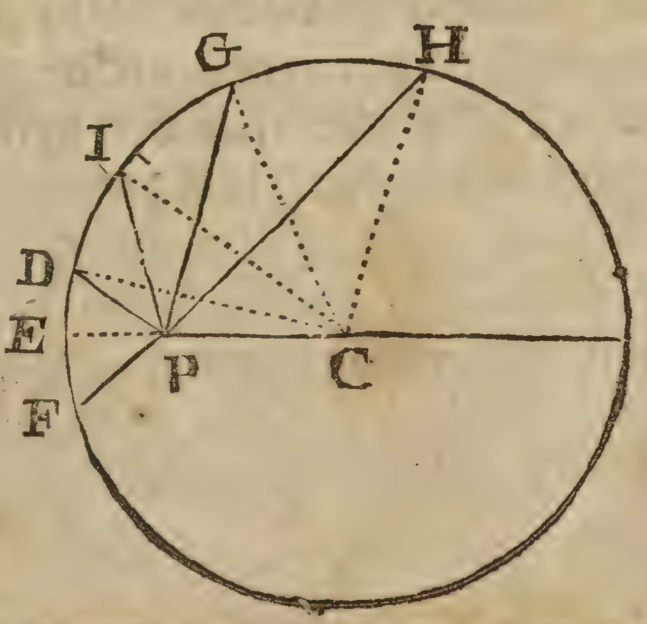
Let us take C to be the center of both the Circles and draw the lines C A, C D and C I: theſe lines *(by the Definition of the Circle)* ſhould be equall the one to the other, that is C I equall to C A, the part equall to the whole, or let us take E to be the center of both, and draw the lines E A and E D; then *(by the ſaid Definition)* the lines E A, E D and E F be alſo equall, the part E F equall to the whole E A and C D part of E D equall to the whole, which is falſ, and therefore the Propoſition is true.

Theor. 6. Prop. 7.

If in the Diameter of a Circle, be taken any point, which is not the center of the Circle, and from that point, be drawn unto the Circumference certain right lines, the greateſt of thoſe lines ſhall be that line wherein the center is, and the leaſt ſhall be the reſidue of the ſame line, and of all the other lines, that which is neareſt to the line which paſſeth through the Center is greater then that which is further diſtant, and from that point, can fall within the Circle on each ſide of the leaſt line, onely two equall right lines.

Conſtruction.

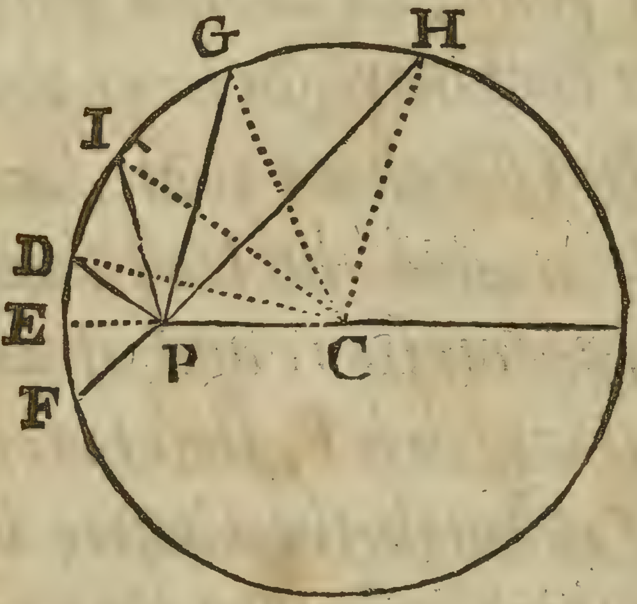
In this Circle, E K is the Diameter, and P is the taken point in the ſame, and C the center, now the line P K doth paſs through the center, & the line P E is the reſidue of the ſame, the other lines drawn frõ the



the ſame point is P H, P G and P I, we have in this four ſundry caſes to ſhew in the demonſtration.

Demonſtration.

And to the ſame end, we draw the lines from the center C to the circumference, as to H G and I D.



And firſt concerning the line P K, that that is greater than P H, or any of the other is thus proved, the lines P C and C H together, are (*by the 20 of the firſt book*) greater than P H, and C H is equall to C K: therefore is P K greater alſo then P H, and by the ſame reaſon is P K, alſo greater than P G, or any of the other lines that can be drawn from P to the Circumference of the Circle.

Secondly, that P E is leſſe than P F or P D is thus underſtood P D and P C are together greater than D C (*by the aforeſaid 20 of the firſt*) and D C equall to E C: therefore is P D and P C alſo greater than E, and if the part P C common to both, be taken away, then ſhall remain D P greater then E P *by the 5 common ſentence.*

Thirdly, that P H is greater than P G is evident (*by the 24 of the firſt book*) for the angle P C H is greater than P C G, and the lines C H and C G are equall, and P C is common, in like manner is proved, that P G is greater than P I, or any of the other lines, which can be drawn from P between G and I.

Laſtly, can there not be more then two lines equall one to another on each ſide of the line E P, which is as much as if we ſaid from one point (as from P) may on the one ſide of the line E P, no line be drawn*, which ſhall be equall to one already drawn, as to D P, and that is now proved, but on the other ſide ſuch can be done, as is to be ſeen on the line F P.

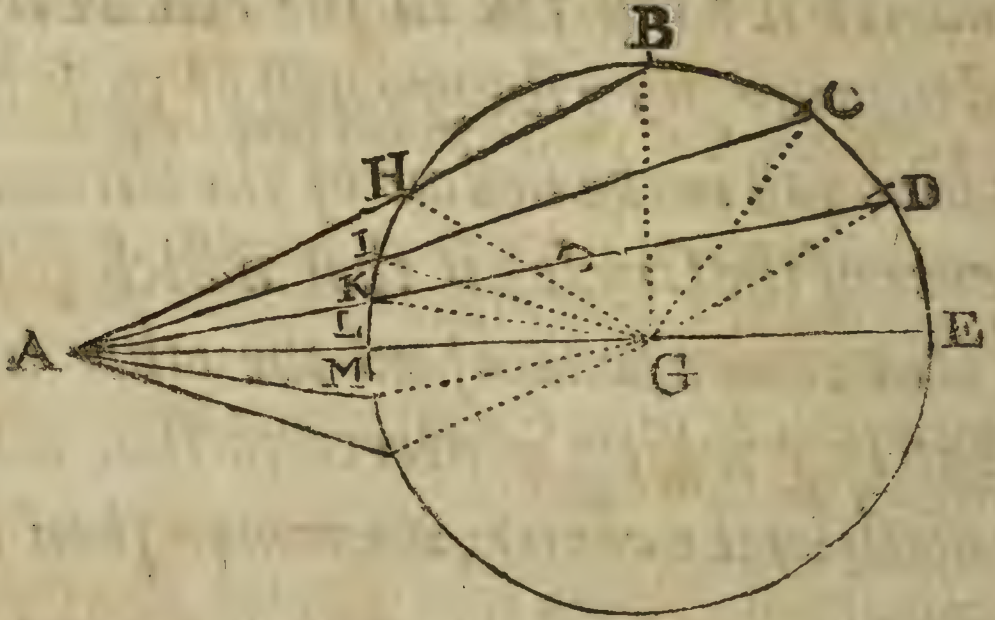
Theor. 7. Prop. 8.

If without a Circle be taken any point, and from that poin, drawn into the Circle unto the Circumference certain right lines, of which, let one be drawn by the center, and let the reſt fall in the concavity by adventure, then the greateſt of thoſe lines which fall in the hollow of the Circumference of the Circle, is that which paſſeth by the center, and of all the other lines, that line which is neerer to that which paſſeth by the center, is greater then that which is more diſtant, but of thoſe right lines, which end at the convexe ſide, or part of the Circumference, that is the leaſt which is drawn from the point to the diameter,

ameter, and of the other lines, that which is neerer to the leſſer, is leſſe then that which is more diſtant, and from that point can be drawn unto the Circumference, on each ſide of the leaſt line, onely two equall right lines.

Conſtruction.

Without this Circle, is the point A, the lines drawn through the Circle, and doth fall on the concave ſide, as A E, A D, A C, A B: & by A E goeth the circles center G, alſo the lines which fall upon the convexe ſide of the Circumference, as A L, A K, A I and A H, and A L doth fall between the point A, aud the center G, there are alſo drawn *(for demonſtration)* from G, the half diameters to the Circumference, like as doth appear in the Figure.



We have here five ſundry caſes to ſhew, and firſt that the line which doth paſs through the center is the greateſt.

De-

Demonſtration.

Foraſmuch as A G and G D together are greater then A D)*by the 20 of the firſt book)* and that the ſame is equall to the line A G E, it followeth that A G E is alſo greater then A D, and alſo greater then A C or A B or any other that can be drawn between E and B within the Circle.

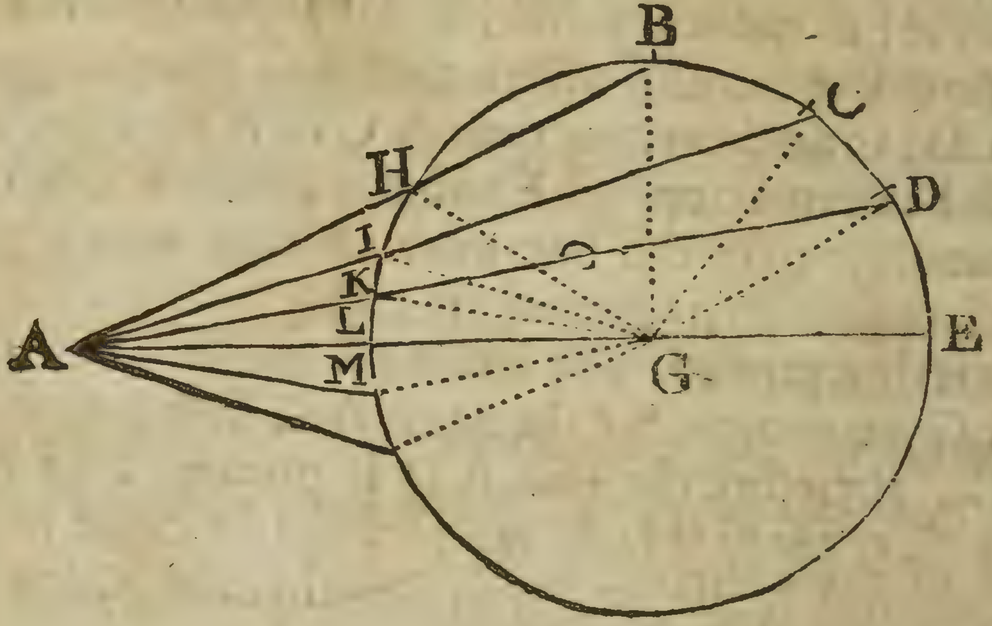
Secondly, that the line next to that which goeth through the center, is greater then that which is further diſtant, is thus proved. The lines D G and G C are equall, and A G is common, but the angle A G D is greater then A G C, therefore the line A D is greater then A C, *(by the 24 of the firſt book)* and for the like cauſe is A C greater then A B alſo, or any of the other lines that can be drawn between the ſame.

Thirdly, that the line A L, which falleth between the point A, and the Circumference (in the line of the Center as A G) is leſſer then thoſe that fall upon the Circumference without is thus proved.

Foraſmuch as A K and K G are together greater then A G, and that G K and G L are equall. It followeth, that A L is leſſer then A K, or any other line that can be drawn from the point A, to the Circumference, like as clearly appeareth by the ſame figure.

Fourthly, that the next line to that which falleth between the point and the Center upon the Circumference, is leſſer then that which falleth further from the ſame is thus proved.

Seeing that the lines A G and G K are equall to A G and G I, and that the angle A G K is leſſer then A G I: it followeth *(by the former 24 of the firſt book)* that the line A K is ſhorter then A I, and the ſame A K is alſo ſhorter then A H, or any other lines which can be drawn from the point A to the Circumference (which alſo appeareth by the Figure between the lines A I and A B.



And laſtly, there can on that ſide from A K, out of the point A to the Circumference, no line be drawn, which ſhall be equall to the ſame A K, and that is now proved, for towards L they will be ſhorter, and towards I they will be longer, but on the other ſide it can be done, if in G be made an angle equall to A G K, as the angle A G M and draw the line A M, that ſhall be equall to A K *(by the fourth Propoſition of the firſt book.)*

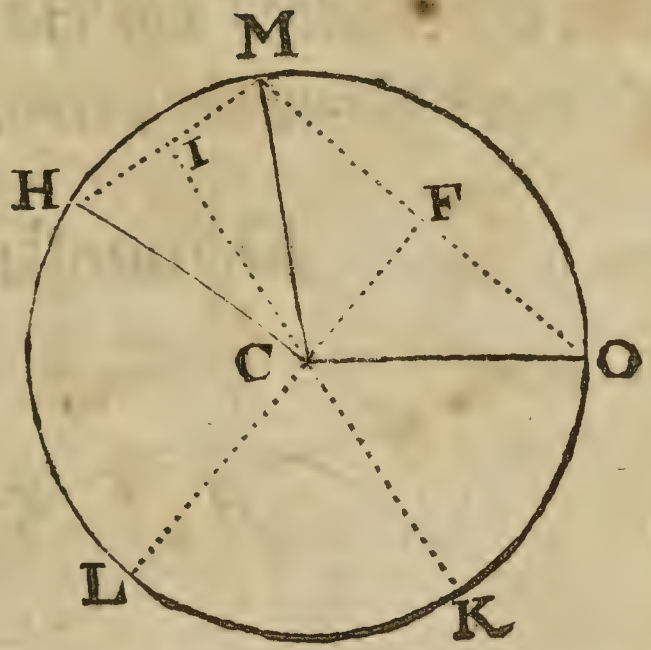
Theor. 8. Prop. 9.

If within a Circle be taken a point, and from that point be drawn unto the Circumference more then two equall right lines, then is that point the center of the Circle.

Con-

Conſtruction.

From the point C in this Circle is drawn three equall lines as C H, C M and C O: therefore is C the center of the Circle.



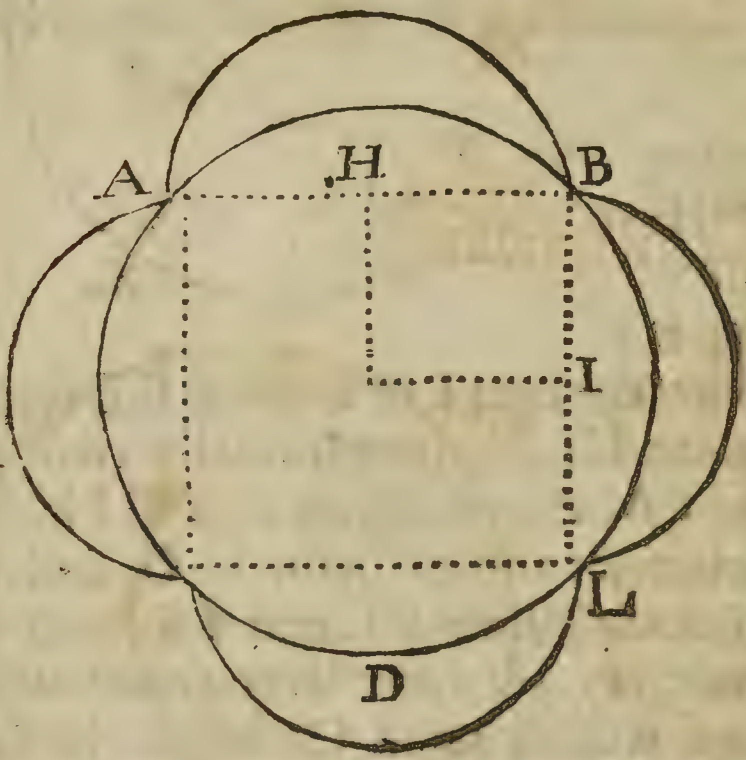
Demonſtration.

Let there be drawn the lines O M and MH, and each of them divided into two equal parts as in F and I, and draw the lines I C K and F C L cutting one another in the point C: foraſmuch as the two Triangles C F O and C F M are equall, as alſo C I M and C I H, the line L F is right angled upon M O, and K I upon H M: in each of theſe lines F L and I K muſt the center of the Circle be *(by the third Propoſition)* which muſt then happen in the croſſe point C, for if the ſame were in the line F L, in any other point from C towards L, or towards F: it were then certain, that the ſame ſhould not come upon the line I K and ſo taken in any other point upon the line I K, from C towards I or K, it would not come upon the line L F, wherefore the center of the Circle, is in the croſſe point C, which might alſo many other ways be proved.

Theor. 9. Prop. 10.

One Circle doth not cut through another in more places then two.

Demonſtration.



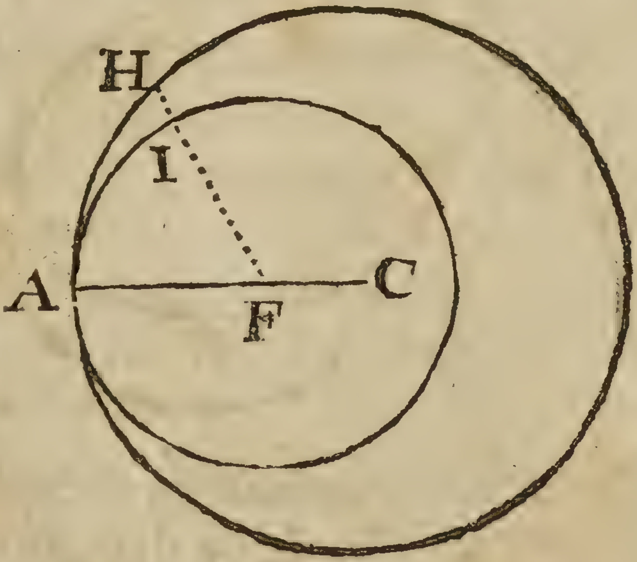
If theſe Circles might cut through one another in three ſundry places, as in ABL, let there be drawn the lines A B and B L, and thoſe lines cut into two equall parts in I and H, and from thence draw two more perpendicular lines, thoſe cut through one another in the point C, which *(by the former Propoſition)* ſhould be the center of the Circle, it is evident *(by the 5 Propoſition)* unpoſſible to be ſo, for Circles which doth cut through one another, have not one and the ſame center.

Theor. 10. Prop. 11.

If two Circles doth touch one another inwardly, the centers being given, and from the one center to the other be drawn a right line, and produced forth, the ſame ſhall cut the Circumferences in the point of touching.

Conſtruction.

Of theſe Circles, the touching is inward in A, and the centers are C and F, the right line that goeth through the centers, and produced forth, doth cut the Circumference, in the point of touching A.



Demonſtration.

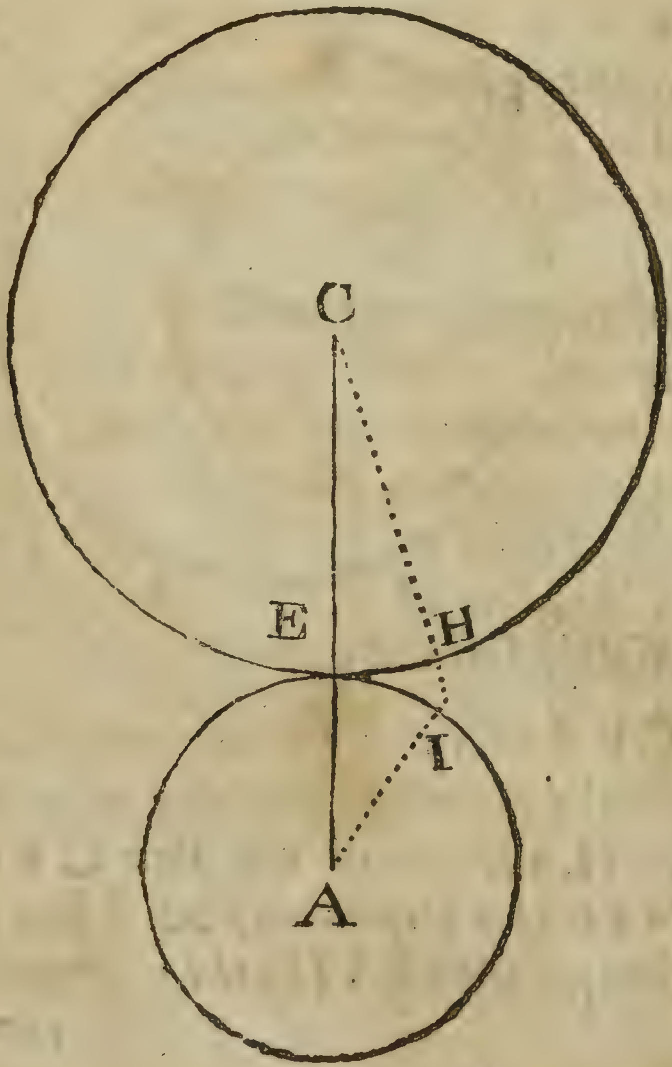
If the produced line C F doth not fall to cut the Circumference of the Circle in the point of touching, as in A, let the touch-point be in H, then muſt the line C F H be equall to C F A *(by the Cir cles Definition)* but F I is equall to F A, and F H is longer then F I *(by the 7 Propoſition)*

tion) wherefore *(by the fourth Definition)* C F H is alſo longer then C F A, being both out of one and the ſame center, which cannot be, as *contrary to the Circles Definition*, wherefore is C F H no right line, but the produced line C F doth fall in the point of touching A.

Theor. 11. Prop. 12.

If two Circles touch one another outwardly, then a right line drawn from one center to the other, doth paſſe through the touch point.

Conſtruction.



Theſe Circles touch one another outwardly, then a right line drawn from the one center to the other, doth paſſe through the touch point

Demonſtration.

If C A doth not paſſe through the point of touching, then let it be C I A, foraſmuch as *(by the Circles Definition)* C E and C H are equal, and alſo AE and AI, and that C I is longer than C E *(by the 8 Propoſition)* It followeth that C I A

I A alſo is longer then C A, wherefore C I A doth not goe right from the one center to the other, *(by the 4 Definition of the 1 Book)* but the line A C being a right line, doth paſſe through the point of touching.

Theor. 12. Prop. 13.

Two Circles cannot touch one another in more then one place, whether it be inwardly or outwardly.

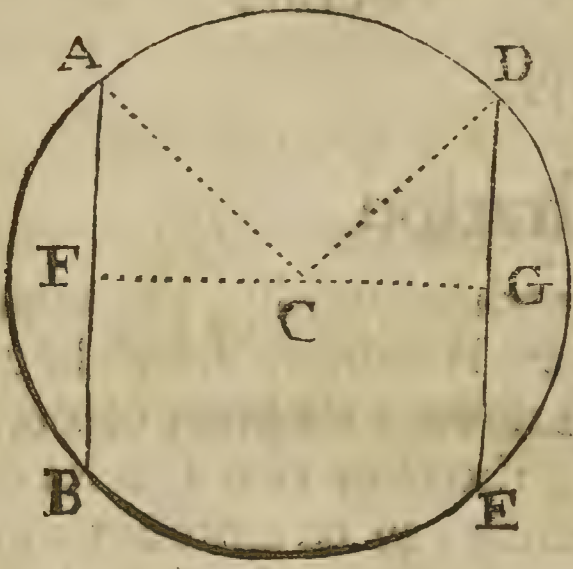
Demonſtration.

If the Circles touch one another inwardly, that cannot be but in one place, and a right line drawn from the one center to the other, doth paſſe through the point of touching, *(by the former propoſition)* therefore two Circles doe not touch one another, outwardly, in no more but in one place.

Secondly, If two Circles touch one another, inwardly, that can be but in one place *(by the 11 propoſition,)* If a right line be drawn from the one center to the other, and the ſame line prolonged forth, it ſhall paſſe through the point of touching, which cannot be in more then one place alſo, *(like as in the demonſtration of the 11 Propoſition is ſhewed,)* wherefore two Circles doe touch one another inwardly not in more then one point.

Theor. 13. Prop. 14.

In a Circle equall right lines are equally diſtant from the center, and right lines equally diſtant from the center are equall.



Conſtruction.

In this Circle A B D E, are drawn equall right lines A B and D E, theſe lines ſtand equally diſtant from the center C, and right lines which ſtand equally diſtant from the center, are equall.

Demonſtration.

Divide the line A B into two equall parts, the midſt is F, doe the like with D E in G, and from the points F and G draw lines to the center C, as the lines F C and G C: theſe lines fall perpendicular upon the lines A B and D E *(by the third of this Book,)* and furthermore, draw from the center C the lines C A and C D, theſe are equall *(by the 15 definition,)* and for aſmuch as the angle A F C is equal to the angle D G C, being both right angles, and the ſides F A and C A equall to the ſides D G and C D, it followeth, that the third ſides F C & G C are alſo equall, *(by the conſequence of the 47 of the 1 Book,)* the ſquare of

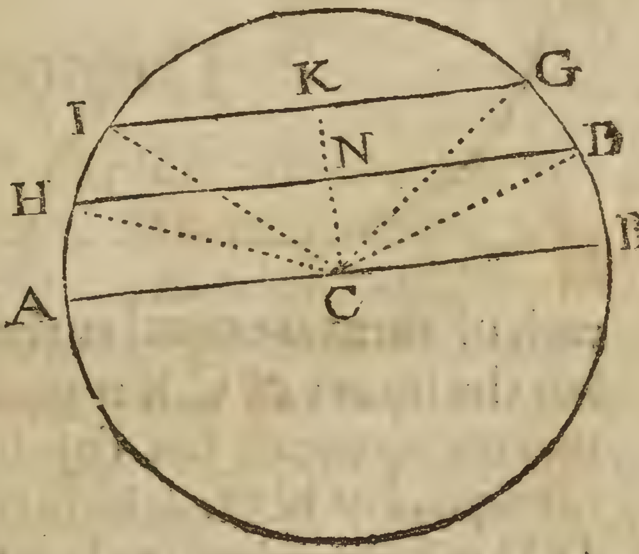
A F and F C together, being equall to the ſquare of A C, and likewiſe the ſame is to be conſidered of D G and G C, wherefore the lines A B and D E are equally diſtant from the center, *(by the 4 definition of the 3 Book.)*

And furthermore, the perpendicular lines C F and C G are proved equall, and the part A F equall to the part D G: and alſo it is proved that A B is double to A F, and D E to D G, wherefore *(by the 6 common ſentence)* the lines A B and D E are alſo equall.

Theor. 14. Prop. 15.

In a Circle, the greateſt line is the Diameter, and of all other lines, that next the Diameter or Center, is greater then that farther diſtant.

Conſtruction.



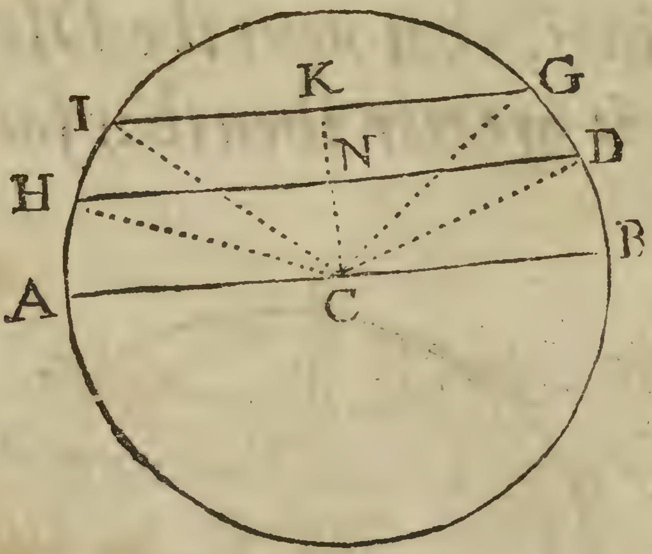
In this Circle is the line A B the Diameter, and therefore the greateſt, and the lines D H & G I leſſe; alſo D H is neerer to the center C then G I, and therefore greater then G I.

Demonſtration.

Let there be drawn the perpendicular lines C N and C K, and the lines C D, C G, C I, C H; foraſmuch as the line lines C D and C H together are equall to the diameter, and that the ſame *(by the 20 of the firſt book)* is greateſt or longeſt, and longer than D H: it followeth, that the diameter A B is alſo greater then D H, and for the ſame reaſon alſo greater then G I, or any other lines, that beſides the diameter, can be drawn.

Secondly, ſeing that the angle D C H, is greater then G C I, it followeth that *(by the 24 of the firſt book)* D H is longer than G I, or any other line that can be drawn from D H towards G I.

Another way.



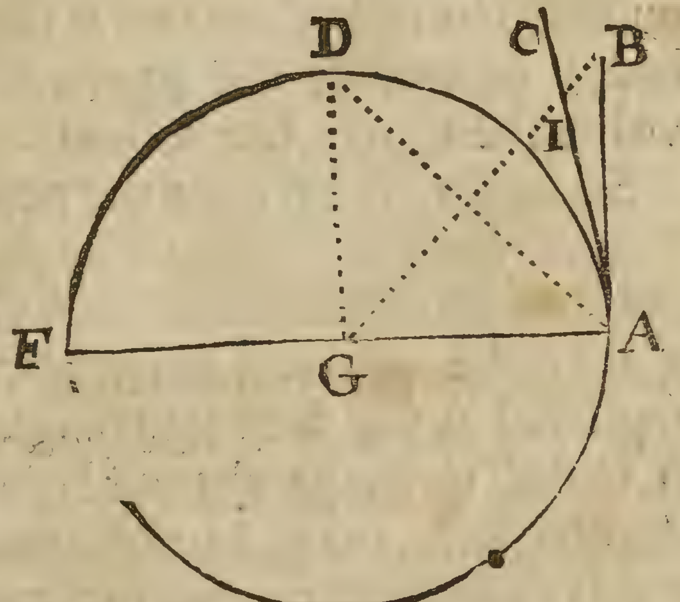
The ſquare of C H is greater then the ſquare of H N by the ſquare of N C *(by the 47 of the firſt book)* therefore is C H alſo longer then N H, that is, the diameter A B is longer then the line D H.

And the two ſquares K I and K C are together equall to the two ſquares N H and N C, ſeeing that each of them are equal to the ſquare of the half diameter, but the ſquare of C K is greater then C N: it followeth that the ſquare of K I (by ſo much) muſt be leſſer then the ſqnare of N H: it is therefore manifeſt, that the line D H is longer then the line G I.

Theor. 15. Prop. 16.

If from the uttermoſt point of the diameter of a Circle, be drawn a line perpendicular to the ſame, that right line doth fall without the Circle, and between the ſame right line, and the Circles circumference cannot be drawn another right line, and the angle of the half Circle and diameter is greater then any acute angle made of right lines, and the angle without the circumference is leſſer then any right line angle,

Conſtruction.



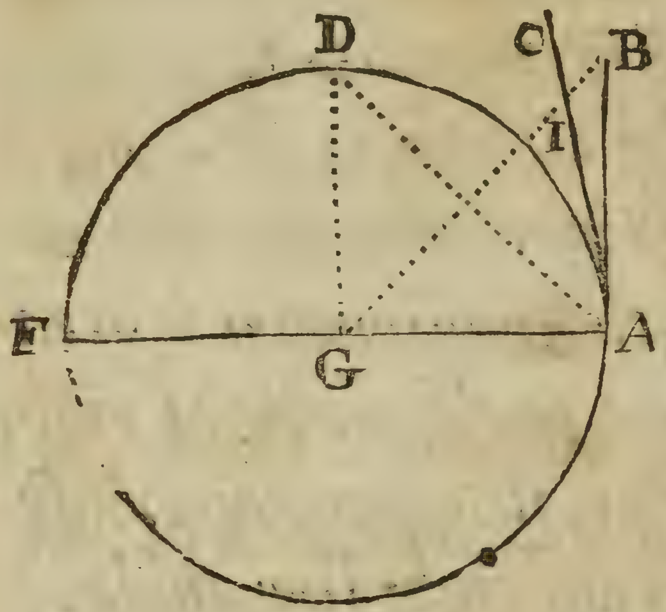
The diameter of this Circle is A E, from the uttermoſt point A, let there be drawn a perpendicular line AB that doth fall without the Circle, and from that point A cannot any other line be drawn between the Circle, and the ſaid perpendicular AB.

And the angle contained with the Diameter, and the arch A I D is greater then any right line angle, that is leſs then a right angle, and the angle conteined with the perpendicular A B, and the arch A I D is leſſer then any right

De-

Demonſtration.

In the figure are drawn theſe lines A C, G I, and G D, thereby to demonſtrate theſe ſundry parts following.



Firſt, If it were poſſible that the perpendicular from A might fall within the Circle, as A D, then ſhall A D G be an Iſoſcheles Triangle, *(by the 25 Definition of the 1 Book)* upon A D, as G D A and G A D, and the angles upon the baſe be equall, as the angles A and D *(by the 5 of the firſt Book)* but the angle G A D is taken to be right: it followeth then that G D A muſt alſo be right *(contrary to the 17 propoſition of the firſt Book,)* wherefore the perpendicular A B doth not fall within the Circle, but without.

Secondly, If between the perpendicular A B, and the circumference, there could be drawn another line from A, and then take the ſame to be A C, and from the center G, if there be drawn a perpendicular upon A C, as G, the angle G I A will be a right angle, and therefore the line A G ſhall be longer then G I *(by the 29 of the firſt Book,)* which is falſe, for G I paſſeth through the circumference, and G A doth but touch the ſame, wherefore G I is longer then G A. It followeth then that between the circumference and the perpendicular A B, no right line from A can be drawn.

Thirdly, And by the ſame reaſon, it is manifeſt, that the angle contained of the diameter & the circumference, is greater then any right line angle which is leſſe then a right angle, and the angle that is between the ſame circumference and the perpendicular is leſſe then any right line angle.

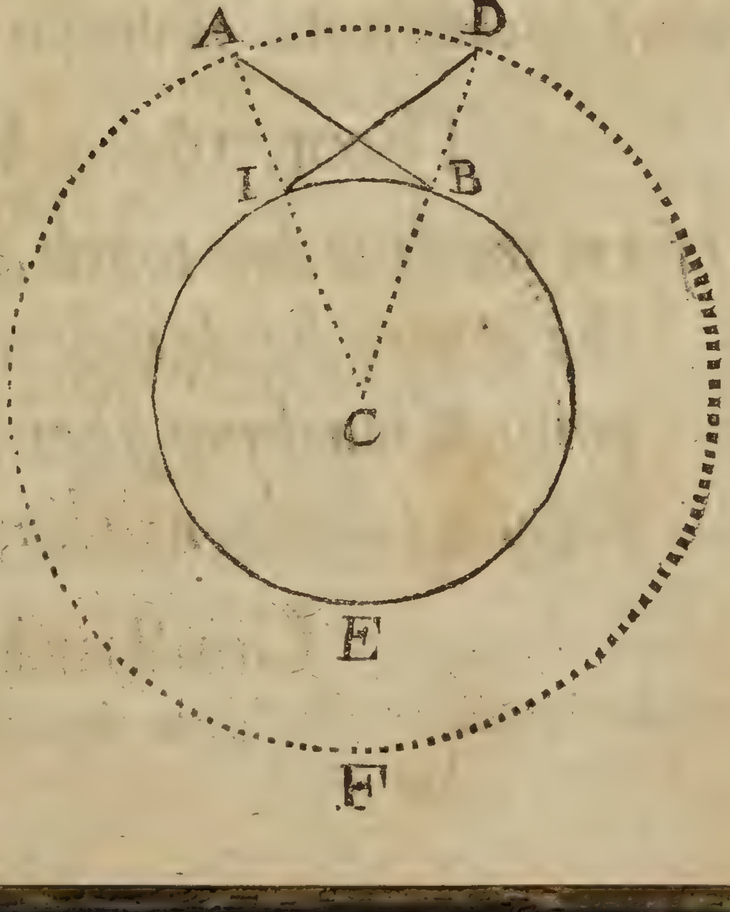
And upon this Propoſition hath *Euclide* hanged this Appendix or Corollary following.

Hereby it is manifeſt that the perpendicular line which is drawn from the uttermoſt point of the diameter of a Circle, doth touch the Circle but in one point only.

Probl. 2. Prop. 17.

From a point given without a Circle, to draw a line that ſhall touch a Circle given.

Conſtruction.



The given point is D, the Circle is IEB, the center is C, from the center C draw a line to the point D, cutting the circumference in the point B, and from C deſcribe a Circle whoſe half diameter is C D, which is the circle A F D, & from B draw a line perpendicular upon the circumference, as

A

A, and from thence a line to the center C, cutting the given Circles circumference in I, and from thence, from the point I, draw a line to the point D, the ſame line I D doth touch the Circle in I.

Demonſtration.

Foraſmuch as the two ſides C D and C I, are equall to C A and C B, and the angle C common to both, the baſe D I is equall to A B *(by the 4 of the 1 Book)* and the other angles included with equall ſides are equal one to another, that is, the angle C B A equall to C I D, and C B A is a right angle, and the line A B doth touch the circle by the former propoſition: it followeth then, that the angle CID is alſo right, and that the line D I doth touch the circle.

Another plain and eaſie way.

Lay a ruler to the point D, and to the uttermoſt of the circumference of the given circle, that it doe touch the circle, and not goe through the ſame, and draw the line to touch the circle, and no farther, as I D.

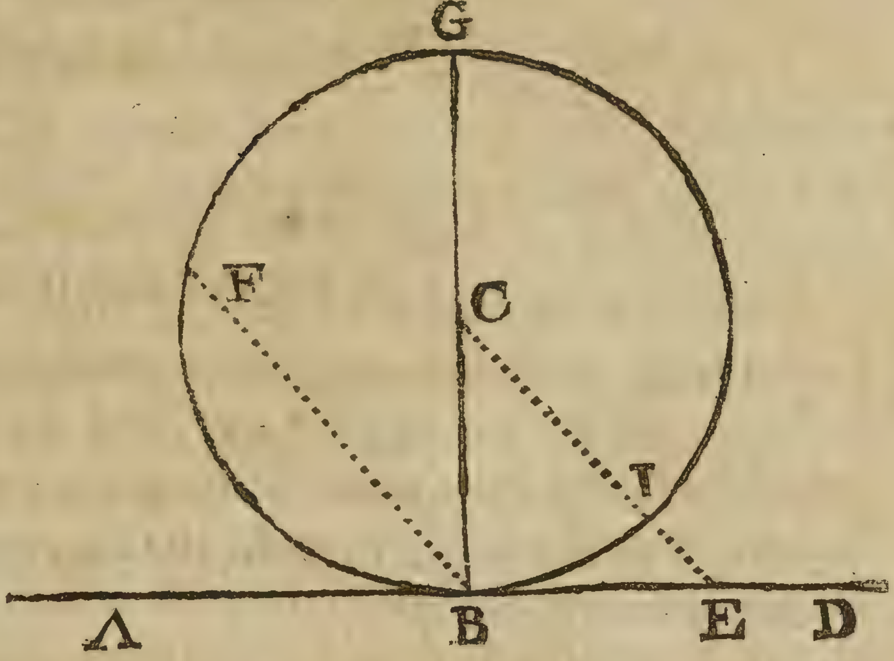
Theor. 16. Prop. 18.

If a right line doe touch a Circle, and from the center be drawn a right line to the point of touching, then is that line perpendicular to the touch line.

Conſtruction.

In the following figure, the line A D doth touch the

Circle, in the pointB,and from thence is a line drawn to the cēterC,asBC,that line B C is perpendicular to the touch line A D.



Demonſtration.

This is manifeſt *(by the 16 Propoſition)* which otherwiſe might be thus proved.

If C B D be not a right angle, let there be drawn another line perpendicular to A D, as C E, then muſt the angle C E B, and the line C B ſhall be longer then E C *(by the 9 of the firſt book)* but C I part of C E is equall to C B, wherefore C E is longer then C B; it followeth that from the center C cannot be drawn a perpendicular upon A D to any other point, then onely to the point of touching B.

Theor. 17. Prop. 19.

If a right line do touch a Circle, and from the point of touching be raiſed upon the ſame line a perpendicular though the Circle, in the ſame perpendicular line is the center of the Circle.

Conſtruction.

In the former figure, there is drawn from the point of touching in B a line perpendicular to AD, as BG, that line B G doth paſſe through the center of the Circle.

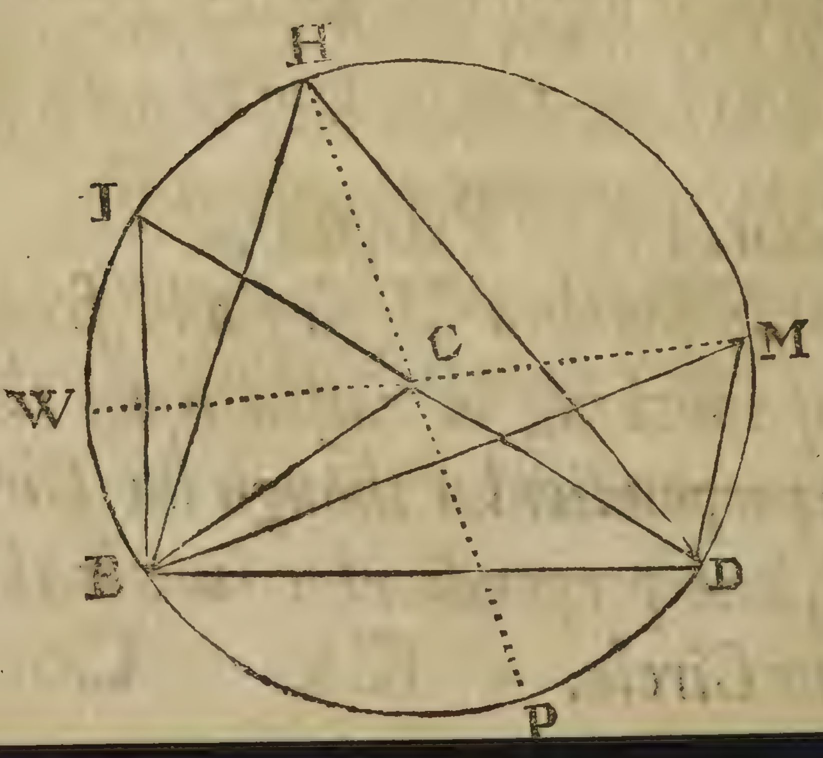
Demonſtration.

This is evident *(by the former Propoſition)* for if the centre be not in the perpendicular line B G, let the ſame be in the line B F, in the point F, then the angle F B A ſhould be equall to the angle G B A, the part equall to the whole which is impoſſible.

Theor. 18. Prop. 20.

In a Circle, an angle at the center is double to an angle at the Circumference, ſo that both the angles have to their baſe one and the ſame part of the Circumference.

Conſtructi.



In this Circle, upon one and the ſame part of the Circumference B P D doth ſtand the angle B C D in the center C, & upon the ſame part B D doth ſtand in the Circumference of the

Circle, theſe angles B I D, B H D, and B M D, that angle in the Center, as B C D is double to any of the angles in the Circumference, as B I D, B H D, or B M D.

Demonſtration.

We have here three angles in the Circumference drawn to ſhew three ſundry things, the firſt angle is comprehended with the diameter I D, and the line I B. The ſecond angle is included with the lines H B and H D, and the third angle, with M B and M D, and drawing the lines out of each angle, thorow the center C, as P H and M W.

Firſt, In regard that (by the definition of the circle) all half diameters are equall one to another, there is the in figure, certain triangles, called Iſoſceles, of two equal ſides, as D C M, D C H, B C I, B C H, B C M and B C D: which (*by the 5 propoſition of the 1 Book*) have their angles at the baſe equall one to another, (which Angles are here in the Circumference) and each angle oppoſite, are double to one of theſe) *by the 32 of the firſt Book*) wherfore, for the firſt, is the angle B C D, in the center, double to the angle B I C.

Secondly, the Angle B C P is double to B H C, and P C D, double to C H D. It followeth, that if we add P C D, and P C B together, as alſo B H C, and C H D, that the angle B C D is double to the angle B H D.

Thirdly, the angle W C D (*by the aforeſaid reaſon*) is double to the angle C M D and W C D double to C M B: it followeth, if we take away W C B from W C D, and C M B from C M D, that the angle B C D is double to B M D, according to the intent of this Propoſition.

Theor. 19. Prop. 21.

In a Circle, the angles which ſtand upon one and the ſame part of the circumference are equall one to another.

Conſtruction.

In the former figure, the angles B I D, B H D, and B M D, doe ſtand upon one and the ſame part of the circumference B P D, therefore are thoſe angles at the circumference equall one to another.

Demonſtration.

(*By the foregoing propoſioion,*) The angles at the circumference are each halfe as great as the angles in the center, therefore are they equall one to another, (*by the 7 common ſentence.*)

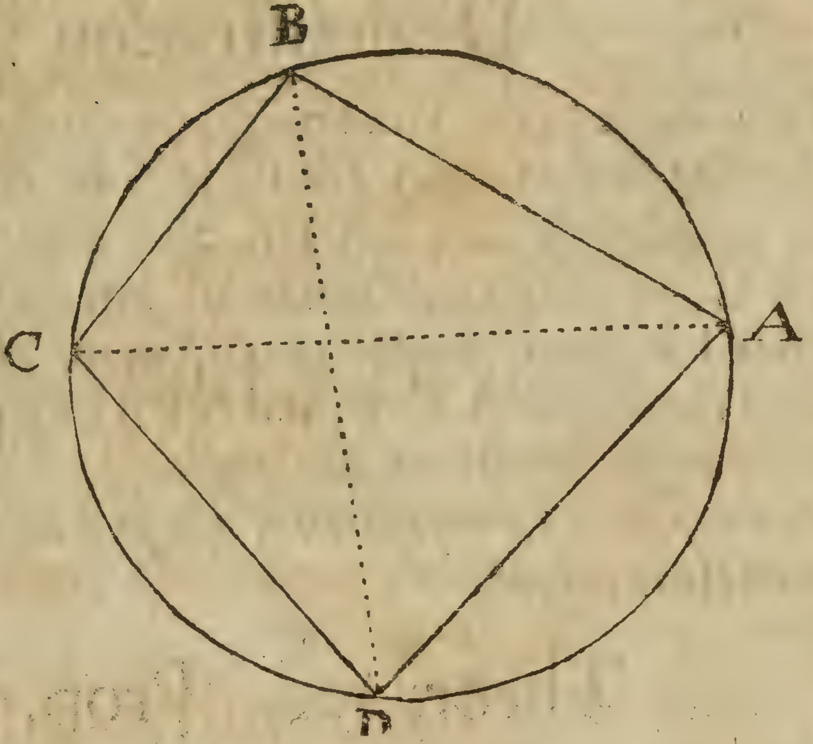
Theor. 20. Prop. 22.

If within a Circle be deſcribed a figure of four ſides, the angles thereof which are oppoſite one to another are together equall to two right angles.

Conſtruction.

This four ſided figure A B C D is inſcribed in a Circle, therefore are the two angles of the ſame which are oppoſite, as B A D and B C D together, equall to two right angles, as alſo A B C and A D C.

Demonſtration.

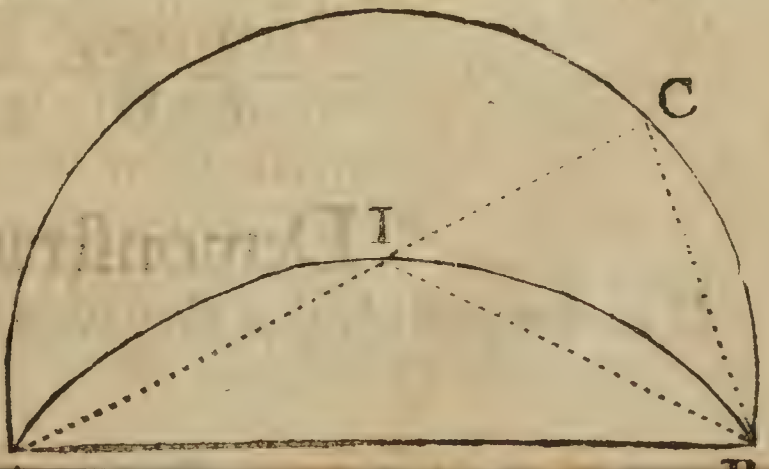


Let there be drawn the lines A C and B D, then (*by the former propoſition*) the angles B A C and B D C are equall, as alſo C A D and C B D, it followeth, that the angle B A D, which is equall to the two angles C B D and B D C, which with the angle B C D together, are equall to two right angles *(by the 32 of the 1 Book,)* therefore the angles B A D and B C D are together equall to two right angles: the ſame is alſo to be proved of the other two angles A B C and A D C.

Theor. 21. Prop. 23.

Upon one and the ſame right line cannot be deſcribed two ſegments of a Circle, which ſhal be like in form & unequal in quantity.

Conſtruction.



Upon the line A B is deſcrib'd the ſegmēt A C B, on which line no other ſegmēt can be deſcribed, like in form and unequal in quantity to the ſegment A C B De-

Demonſtration.

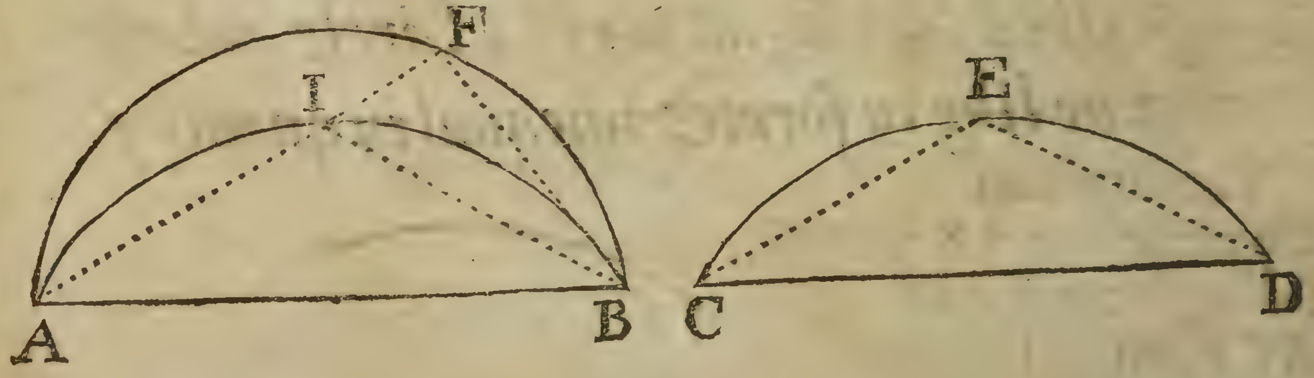
Foraſmuch as the ſame might be thought to be poſſible let there be drawn upon the ſame line A B another ſegment of another Circle A I B, and draw the lines A C and B C, and from the point of interſection in I, draw the line I B, then *(by the 10 Definition of this third book)* ſhould the angles A C B, and A I B be equall, that is, the inward angle equall to the outward, which is falſe and *contrary to the 16 Propoſition of the firſt book*, the Propoſition therefore is true.

Theor. 22. Prop. 24.

Like ſegments of Circles, deſcribed upon equall right lines are equall one to another.

Conſtruction.

Theſe Segments A I B and C E D ſtandeth upon equall right lines A B and C D, therefore are thoſe ſegments equall one to another.



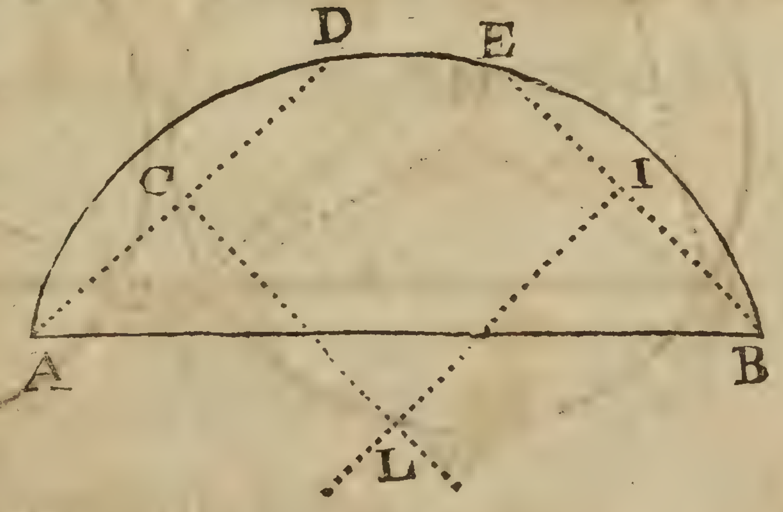
Demonſtration.

If it were poſſible that they were unequall, let there be upon the line A B, deſcribed another ſegment equall to the ſegment C E D in form and quantity,and let the ſame be the ſegment A F B, and draw the lines A F, F B and I B, alſo E C and E D; foraſmuch then as *(by the 21 of this book)* in equall ſegments are equall angles, then muſt the angles A I B and A F B each of them be equall to the angle C E D, and therefore the ſame angles A I B and A F B, alſo be equall one to another, which is falſe, for the angle A I B is greater then A F B *(by the 16 Propoſition of this book)* wherefore equall ſegments that ſtandeth upon equall right lines are equall.

Probl. 3. Prop. 25.

A ſegment of a Circle being given, to deſcribe the whole Circle, whereof the ſegment is a part.

Conſtruction.



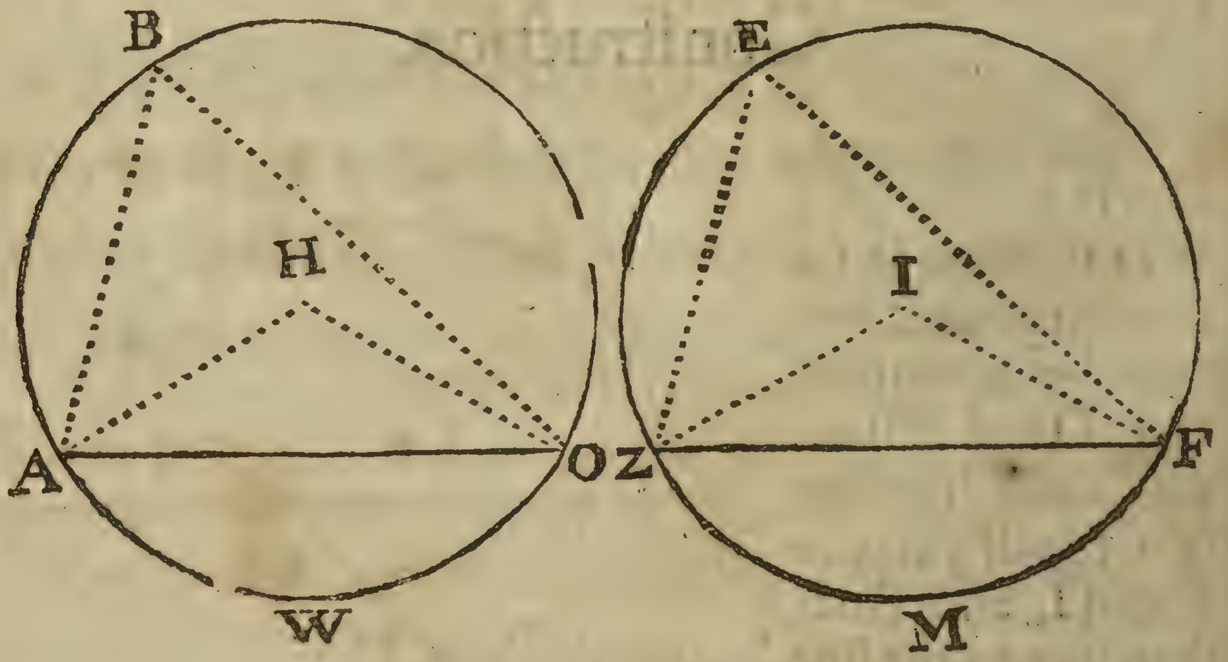
The ſegment of a Circle given is A B & D E, draw in the ſame ſegment, two lines by adventure, as A D and E B, divide each line into two equall parts in C and I, and from thence upon the line A D and E B, draw two perpendiculars, as C L and I L, which all cut through one another in the point L, therefore is the point L the center of the circle, whereof the ſegment A D B is a part.

Demonſtration.

It is evident *(by the third propoſition of this Book)* that the center of the circle muſt come in both thoſe perpendiculars C L and I L, that is, in the common interſection point L, which is alſo ſufficiently proved *(by the 9 propoſition of this Book.)*

Theor. 23. Prop. 26.

Equall angles in equall Circles doe ſtand upon equall parts of the circumference, whether the angles be in the center or in the circumference of the Circle.



Conſtruction.

Theſe two Circles are equall, and the angles A H O and Z I F inthe Center, or the angles A B O and Z E F inthe Circumference are equall, therefore by either of theſe two reaſons, are the parts of the Circumference, as A W O and Z M F, whereupon theſe angles doe ſtand equall.

Demonſtration.

Foraſmuch as the lines H A and H O, are equall to I Z and I F, and that they do include equall angles H and I, the baſe A O is equall to the baſe Z F *(by the fourth of the firſt book)* ſeeing the angles B and E *(by the 20 of this book)* are each the half of H and I, then are the ſegments equall *(by the 10 Definition of this)* as A B O and Z E F, and alike in forme, and *(by the 24 Propoſition of this book)* equall in quantity, and the reſt or remaining ſegments, or parts of the Circumference A W O and Z M F alſo equall.

Theor. 24. Prop. 27.

In all Circles the angles which ſtand upon equall parts of the circumference are equall the one to the other, whether thoſe angles come at the center or in the circumference.

Conſtruction.

In the former Circles the angles A B O and Z E F doth ſtand upon equall parts of the circumference, as upon A W O and Z M F, therefore are thoſe angles equall.

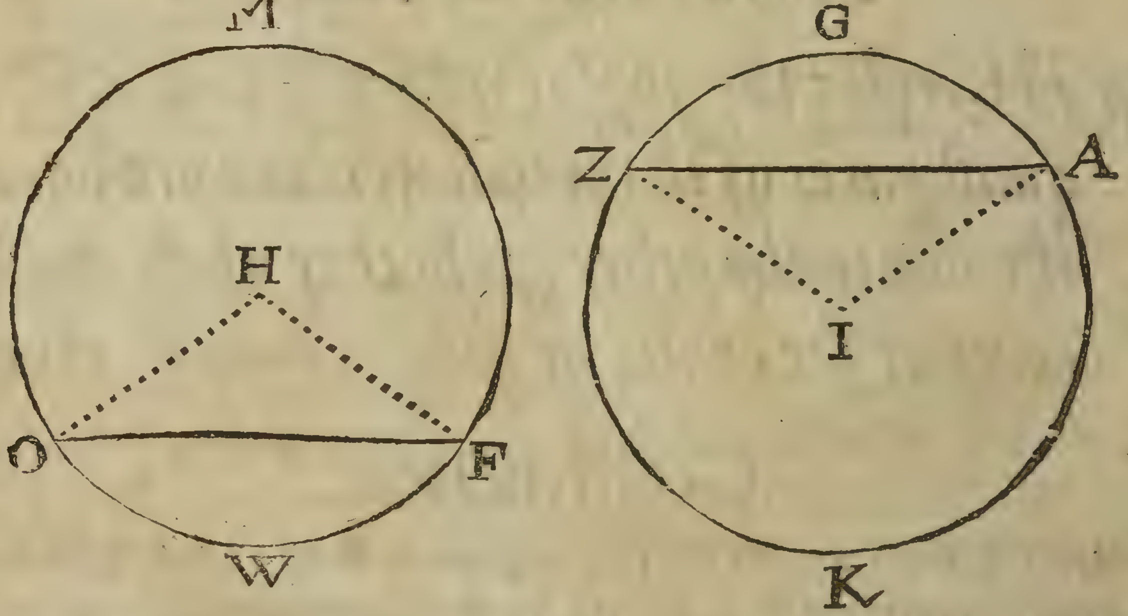
Demonſtration.

This is evident by the former propoſition: for ſeeing that the parts of the circumference cannot be unequall, if the angles be equall; it followeth, that the angles alſo cannot be equall if the parts of the circumference were unequall whereupon they ſtand, for otherwiſe might the part be equall to the whole, which is contrary to the *9* common ſentence.

Theor. 25. Prop. 28.

In equall Circles equall right lines doe cut off equall parts of the circumference, the greateſt part to the greateſt, and the leaſt to the leaſt.

Conſtruction.



Theſe two Circles are equall, as alſo the two lines Z A and O F, and therefore are the parts cut off equall, the part of the circumference A K Z, to the part O M F, and the part A G Z equall to the part O W F.

De-

Demonſtration.

From the centers H and I are drawn the lines A I, I Z, and F H, H O, making the two triangles A Z I and OFH, theſe two triangles are equall *(by the 8 of the 1 Book)* and the angle I is equall to the angle H, therefore, *(by the 26 propoſition of this Book,)* the ſegment or part of the circumference A G Z is equall to the ſegment O W F, as alſo the parts remaining A K Z, and O M F, *(by the third Common-ſentence.)*

Theor. 26. Prop. 29.

In equall Circles, are equall parts of the circumference ſubtended with equall right lines.

Conſtruction.

The circles in the former propoſition are equall, as alſo the parts of the Circumference cut off, with the lines A Z and O F, therefore are the lines A Z and O F, likewiſe equall.

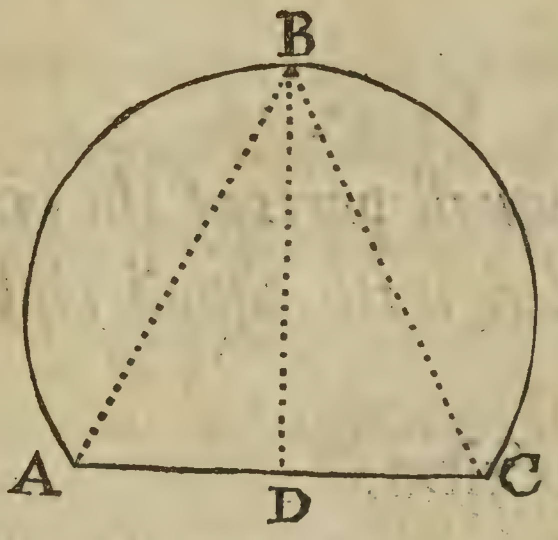
Demonſtration.

If the parts of the circumference A K Z and O M F, be taken to be equall, the parts remayning A G Z and O W F ſhall alſo be equall. *And by the 27 of this Book,* the angle I equall to the angle H, *and by the fourth of the firſt Book,* are the lines A Z and O F alſo equall.

Probl. 4. Prop. 30.

To divide the part of a Circumference of a Circle into equall parts.

Conſtruction.



The part of the circumference is A B C, from C to A draw a right line, as A C, divide A C into two equal parts in D, and from D draw a line perpendicular upon A C, as D B, this line D B doth divide the part of the circumference A B C into two equall parts.

Demonſtration.

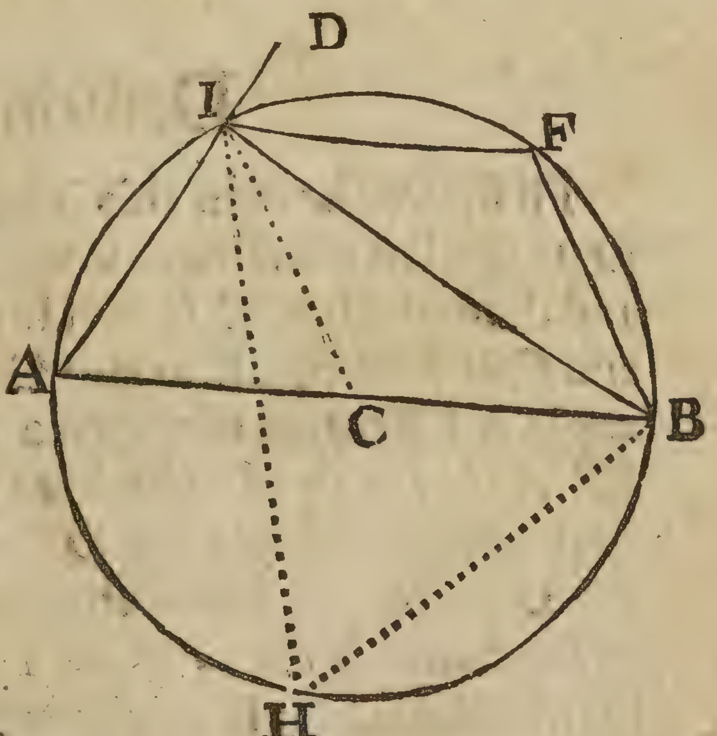
Let there be drawn the lines B A and B C, and for as much as the lines D A and D B are equall to the lines D C and D B, it followeth *(by the 4 of the 1 Book)* becauſe they incloſe equall angles, that A B and B C are equall, and *(by the 28 of this Book)* the ſegments AB & BC are alſo equal.

The-

Theor. 27. Prop. 31.

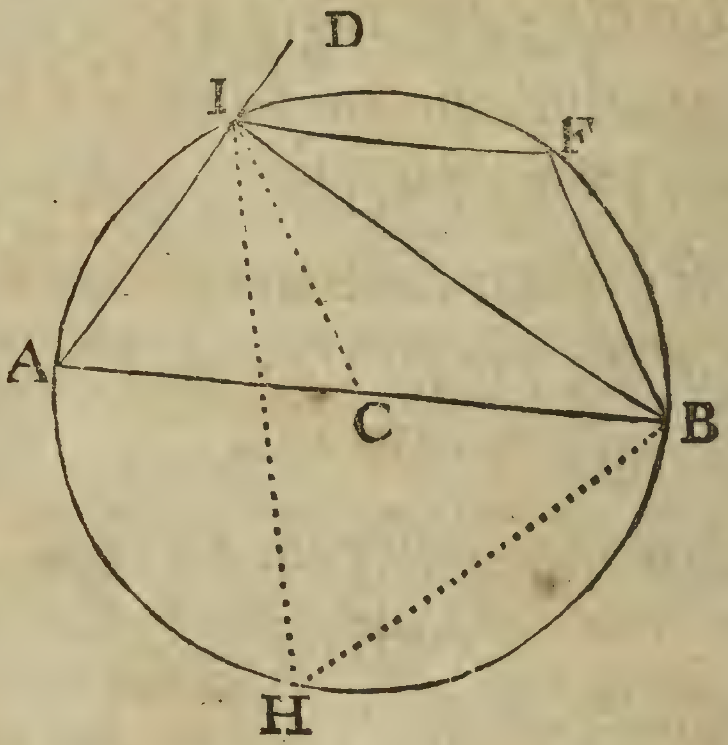
In a Circle the angle which ſtandeth upon the diameter is a right angle, but an angle that ſtandeth in a ſegment that is greater then a halfe Circle, that angle is leſſe then a right angle: And an angle that ſtandeth in a ſegment that is leſs then a half Circle, that angle is greater then a right angle: and the angle of the greater ſegment is alſo greater then a right angle, & the angle of the leſſer ſegment, is leſſer then a right angle.

Conſtruction.



In this Circle I A H B F, the center is C, and the angle in the half Circle is A I B, that angle I is a right angle, and the angle that ſtandeth in a ſegment that is greater then a halfe Circle (as in the ſegment I A H B the angles A and H) is leſſer then a right angle, and the angle F (which ſtandeth in the ſegment B I F being leſſe then a halfe

Circle) is greater then a right angle, alſo the angle of the greater ſegment, namely, the angle that is incloſed with the right line I B, and the part of the Circle I A, is greates then a right rngle comprehended of right lines; but the angle B I F comprehended with the right line I B and the part of the Circle I F, is leſſe then a right angle.



Demonſtration.

Firſt, We have in this Theorem five parts to demonſtrate, and to the ſame end we prolong the line A I to D, and draw the line C I, and foraſmuch as the angles C I B and C B I are equall, as alſo C I A and C A I, then is the angle A I B equall to the two angles together I A B and I B A *(by the 32 of the 1 Book)* and thoſe both are equall to the outward angle B I D, it followeth, that B I A and B I D are equall, and both right angles *(by the 10 Definition)* and alſo *(by the 13 of the 1 Book)* wherefore the angle in the halfe Circle is a right angle.

Secondly, Foraſmuch as the two angles of the Triangle A I B, namely, the angles I A B and I B A, are leſſe then two right angles (*by the 17 of the 1 Book*) and that the angle A I B is a right angle, and ſtandeth on the halfe Circle, therefore the angle I A B or I H B is leſſe then a right angle, and it ſtandeth in the ſegment I A H B, which is greater then a halfe Circle.

Thirdly, Foraſmuch as in the Circle is a figure of four ſides, namely, I A B F, and that the angles that are oppoſite, are equall together, to two right angles, (*by the 22 of this Book*) therefore, the angles I A B and I F B together, are equall to two right angles, but the angle I A B is leſs then a right angle, wherefore the angle remayning, as FIB is greater then a right angle, and ſtandeth in the ſegment I B F, which is leſſe then a half circle.

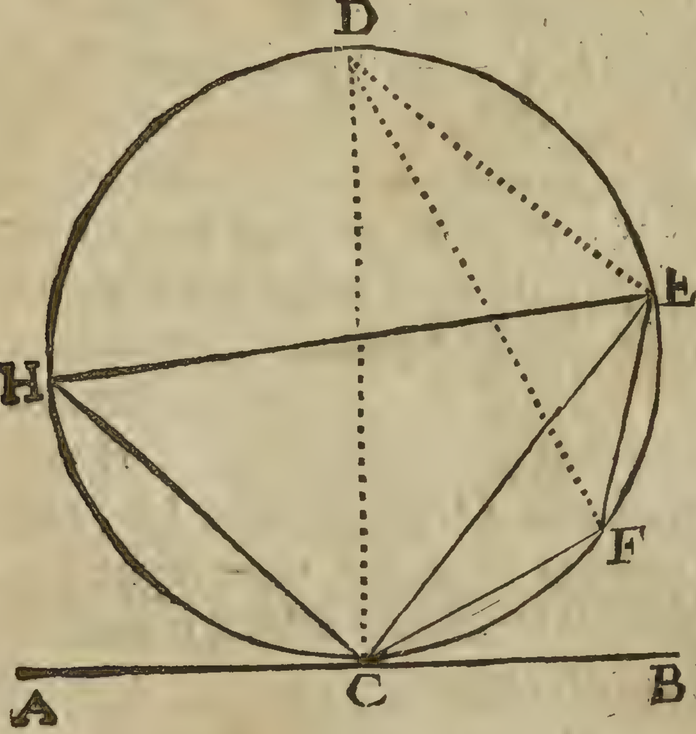
Fourthly, Foraſmuch as the angle comprehended under the right lines, I B and I D, is a right angle, and that the angle comprehended under the right line B I, and the arch I F, is leſs then the ſame. Therefore the angle comprehended under a right line, and the arch of the circle that is leſs then a halfCircle, is leſſe then a right angle. For the part is leſs then the whole, *by the 9 common ſentence.*

Fiftly and laſtly, Foraſmuch as the angle comprehended under the right lines I A and I B, is a right angle, therefore the angle that is included under the right line I B, and the circumference I A is greater then the ſame right line angle B I A, which is a right angle, for the whole is greater then his part (*by the 9 common ſentence.*)

Theor. 28. Prop. 32.

If a right line touch a Circle, and from the touch point be drawn a right line cutting the Circle through, the angles which that line and the touch line doe make in the touch point, are equall to the angles which can be made in the ſegments of the Circle, each in his alternate ſegment.

Conſtruction.



The right line A B doth touch the Circle in the point C, and from the point C is drawn a right line C E, which doth cut the Circle into two Segments E F C & E D H C, making at the touch point C, the angles B C E and A C E, then is the angle B C E equall to the angle that can be made in the alternate ſegment, that is, in the ſegment E D H C, and the angle E C A is equall to the angle that can be ſet in the ſegment E F C, being made to ſtand upon the ſaid C E and the arch of the ſegment.

Demonſtration.

Let there be drawn the Diameter C D, and alſo the lines D E and D F, foraſmuch then as the angle C E D is a right angle, and equall to E D C and E C D together, and alſo that B C D is a right angle, therefore is B C D equall to both the angles E C D and C D E together, if then we take away the angle E C D common to both, the angle remaining E C B is equall to C D E the angle remaining, which angle C D E is alſo equall to the angle that can ſtand in the ſegment C H D E (that is) to the angle C H E (or any other angle, that can ſtand upon the line C E, and in the ſegment C H D E.)

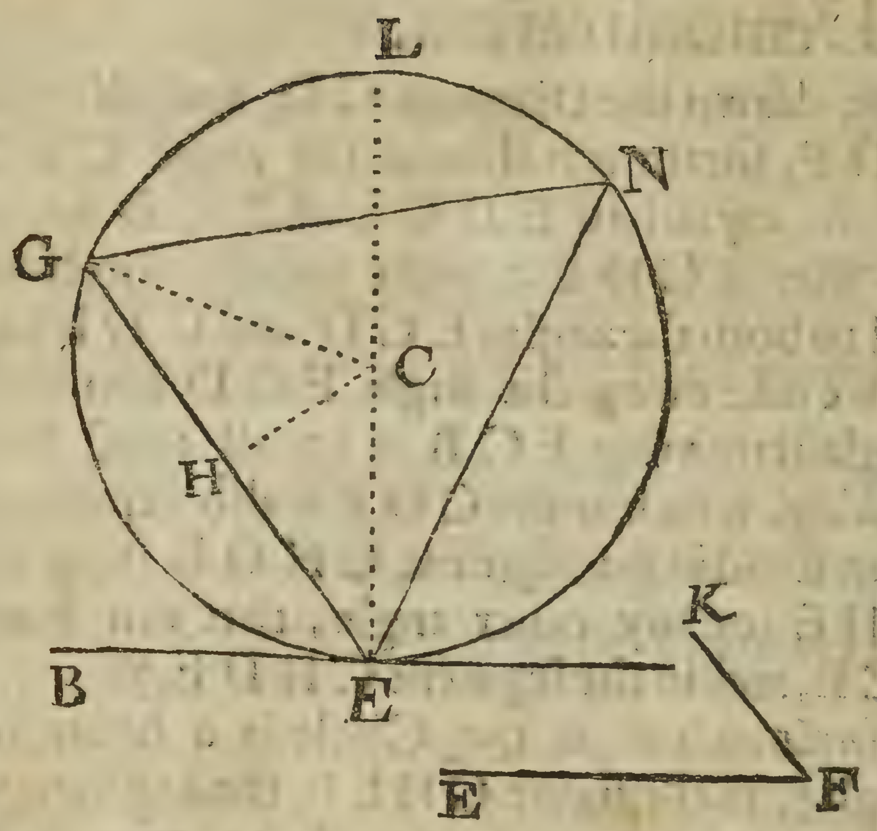
Secondly, Foraſmuch as in the Circle is a figure of four ſides or angles, as the figure C H E F, the angles oppoſite are equall to two right angles *(by the 22 Propoſition of this book)* as the angles C H E and C F E, and the angles E C B and E C A are alſo equall to two right angles *(by the 13 of the firſt book)* therefore the angles E C B and E C A are equall to the angles C D E and C F E, whereby it is proved, that the angle C D E or C H E is equall to the angle E C B, and the angle remaining E C A, is equall to the angle remaining E F C, which ſtandeth in the alternate ſegment.

Probl. 5. Prop. 33.

Upon a right line given, to deſcribe a ſegment of a Circle, which ſhall contein an angle, equall to a given right line angle.

Conſtruction.

The given right line is the line G E, and the angle given



is the angle E F K, upon the line G E, and from the point E deſcribe an angle equal to the given angle E F K *(by the 23 of the firſt book)* that is the angle G E B, and upon the line B E from the point E draw a line perpendicular, as E L, and divide G E in two equal parts, the midſt is H, and from the point H, draw a line perpendicular to E G, as H C, which doth cut the line E L in the point C, from thence, as from a Center, deſcribe a Circle, whoſe half diameter is C E, then is the ſegment E G L N, included with the line E G, therein can ſtand the angle E F K.

Demonſtration.

Let there be drawn the line C G, foraſmuch as the equall right angles C H G and C H E, have the lines H G and H E equall, and H C common to both, then is C G equall to C E *(by the fourth of the firſt book)* therefore is the line C G, in the Circle: ſeeing then that *(by the 16 of this book)* the line E B doth touch the Circle in E, and that the angle G E B is equall to the given angle E F K, it is then manifeſt *(by the former Propoſition)* that the angle which can ſtand in the ſegment E G L N (ſubtended with the line E G) is equall to the given angle E F K.

Probl. 6. Prop. 34.

From a given Circle to cut off a ſegment which ſhall contein an angle, equall to a given angle.

Let the given Circle be the former figure E G L N, and the right line angle given is the angle E F K, firſt draw to the Circle a touch line *(by the 17 of this book)* as B E F, and from the touch point E draw a line through the Circle, making an angle equall to the given angle, ſet from the point E towards B *(by the 23 of the firſt book)* which angle is B E G, then doth the line E G cut off a ſegment, as E N L G, in which may ſtand an angle equal to the given angle E F K, as the angle E N G, or E G N.

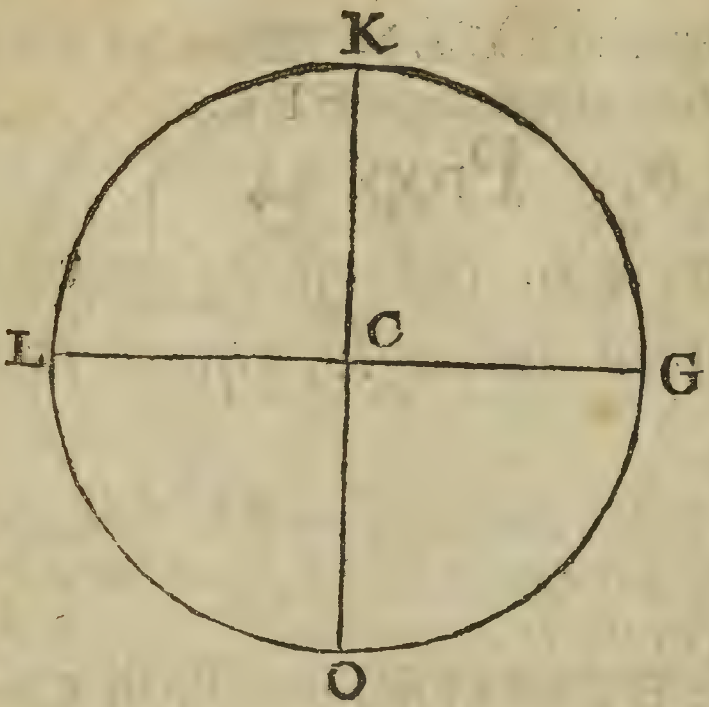
The Demonſtration hereof is evident by the former Propoſition.

Theor. 29. Prop. 35.

If in a Circle two right lines do cut through the one the other, the right angled Parallelogram comprehended under the parts of the one line, is equall to the right angled Parallelogram comprehended under the parts of the other line.

Conſtruction.

I have here ſet ſundry figures, thereby to ſhew four ſundry



dry caſes in this Propoſition. The firſt,

If both the lines be diameters of the Circle then do they cut one another through in two equall parts, as L G and K O, which doth happen in the Center C.

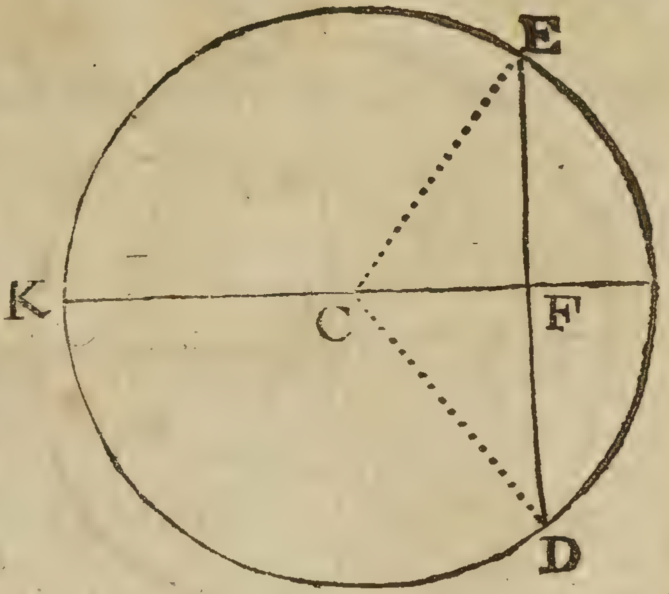
It is then evident, that the right angle figures made of ſuch parts being equall, are equall ſquares.

Secondly, if one line being drawn through the Center, and is a diameter (as in the following figure K O doth divide the line E D in two equall parts) then is the right angle figure comprehended under the parts of the one, as K F and F O equall to that of D F and F E, that is, to the ſquare of E F.

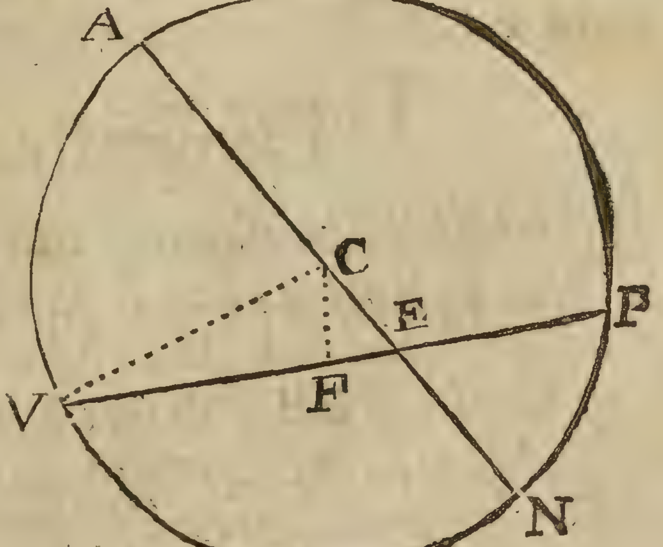
Demonſtration.

By the third Propoſition of this book, the diameter K O doth cut through the line D E right angled in F, and *(by the 5 of the ſecond book)* the right angle Parallelogram incloſed with K F and F O, with the ſquare of C F together, is equall to the ſquare of C O or C E, which is alſo equall to the two ſquares of C F and F E, if we then take from both the ſquare of C F, which is common to both, there ſhall remain the right angle figure comprehended of K F and F O, equal to the ſquare of F E, which is equal to the right angle figure, or rather the ſquare of the equal parts D F and F E.

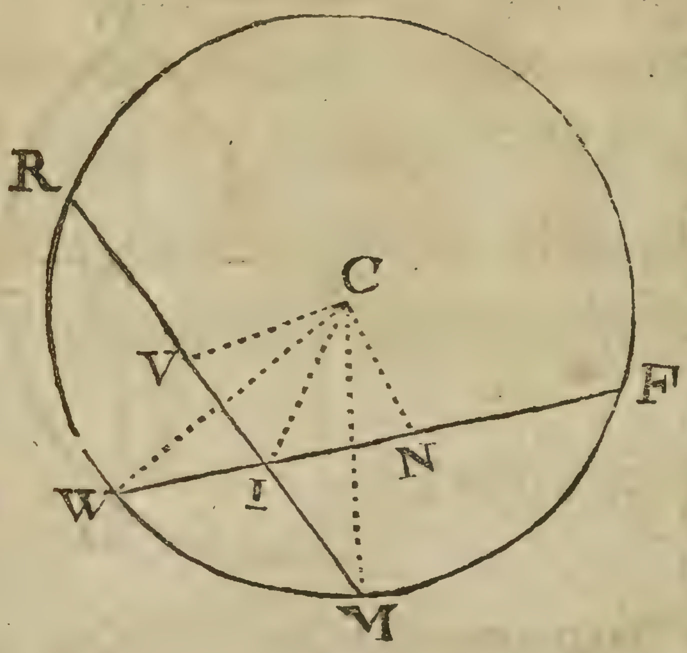
Thirdly,



Thirdly, if the one being a diameter (as in the following figure) A N doth cut through another line as V B in unequall parts, as in this third figure in E, then let there be drawn the line C V, and another line perpendicular to V B as C E *(by the third of this book)* which doth divide V B into two equall parts in F, then is *(by the aforeſaid 5 Propoſition)* the right angle figure included of A E and E N with the ſquare of E C, equall to the ſquare of C N or C V, and the right angle figure included with V E and E B with the ſquare of E F are together equall to the ſquare of V F, which together with the ſquare of C F is equall to the aforeſaid ſquare of C V, and the two ſquares of E F and C F, that is, the ſquare of C E being taken from both, there ſhal remain the two right angle figures comprehended of A E and E N, and of V E and E B equall the one to the other.



And fourthly, in this fourth figure, the lines R M and W F do cut one another through, in the point I and neither of both is a diameter of the Circle, therefore let there be drawn from the center C, theſe lines C W, C M and C I, and two perpendiculars C V and C N, they di-



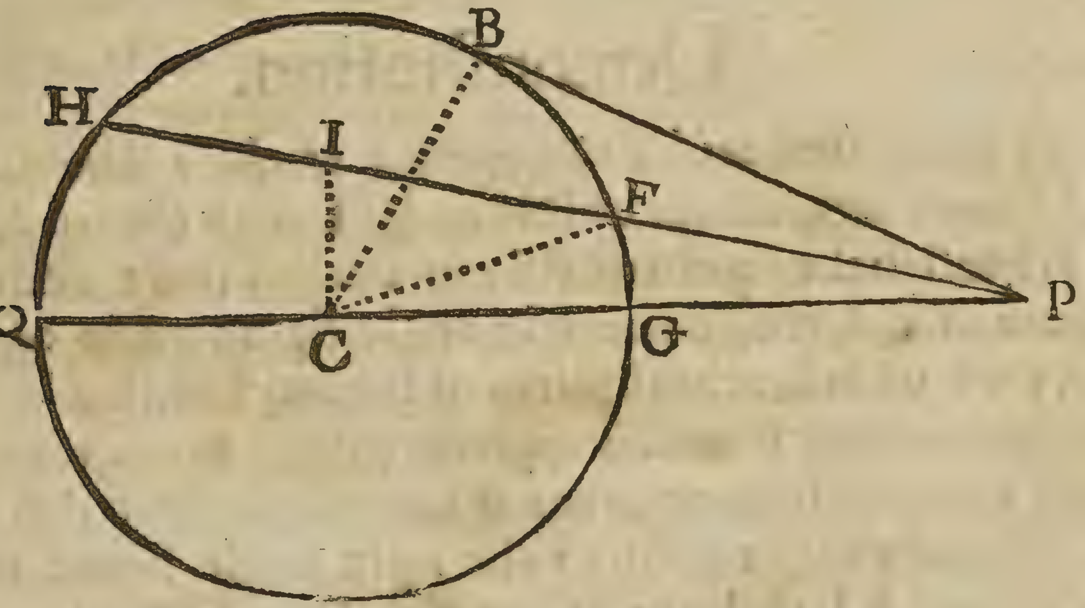
divide the two lines R M and W F in two equall parts, in V and N: now in the third difference is ſhewed that the right angle figure that is comprehended of R I and M I with the two ſquares of V I and V C, that is, with the ſquare of C I, are equall to the ſquare of C M or C W, and that the right angle figure comprehended of W I and I F, with the two ſquares of I N and N C, that is with the ſquare of the aforeſaid C I, are alſo together equall to the ſquare of C W, from each of them being taken away the ſquare of C I, which is common to them both, there ſhall remain the right angle figures comprehended under R I and I M, and of W I and I F equall the one to the other.

Theor. 30. Prop. 36.

If without the circumference of a Circle, be taken a point, by adventure, and from that point be drawn two right lines to the Circle, ſo that the one of them do cut the Circle, and the other doth touch the ſame Circle: the right angled Parallelogram which is made of the whole right line

line which doth cut the Circle, and of that part of the ſame line which lieth between the point and the utter circumference of the Circle, is equall to the ſquare that is made of the line that toucheth the Circle.

Conſtruction.



Without this Circle, the point taken is P, the line cutting through the Circle is P H, and the touch line is P B: but we have drawn another line through the Circles center, as P Q, thereby to ſhew two ſundry differences.

Now by this propoſition is the right angled Parallelogram which is made of P Q and P G, equall to the ſquare of P B.

Demonſtration.

Foraſmuch as (*by the 6 of the 2 Book*) the right angled Parallelogram comprehended of P Q and P G, with the ſquare of C G is equall to the ſquare of P C, and that *(by the 18 of this book)* the angle P B C is a right angle, and therefore *(by the 47 of the firſt book)* the two ſquares of B P and B C are together equall to the ſquare of P C. It followeth that if from both be taken away the common ſquare C G or B C, that the right angle figure, made of P R and P G is equall to the ſquare of P B.

Secondly, if the line that cutteth through the Circle, do not paſſe through the center, as the line P H, yet is the right angle figure which is made of P H and P F equal to the ſquare of the touch line P B.

Demonſtration.

By the aforenamed 6 Propoſition of the 2 book is the right angle figure made of P H and P F with the ſquare of F I equall to the ſquare of P I: to both of theſe, adde the ſquare of C I, then is the right angle figure made of P F and P H, with the two ſquares of F I and I C, that is with the ſquare of C F or C B together, equall to to the ſquare of P C: it followeth, that the two ſquares of P B and P C together are equall to the right angle figure made of P H and P F with the ſquare of C F or C B, therefore the ſquare of B C being common to both taken away, there ſhall then remain the right angle figure of P H and and P F equall to the ſquare of the touch line P B.

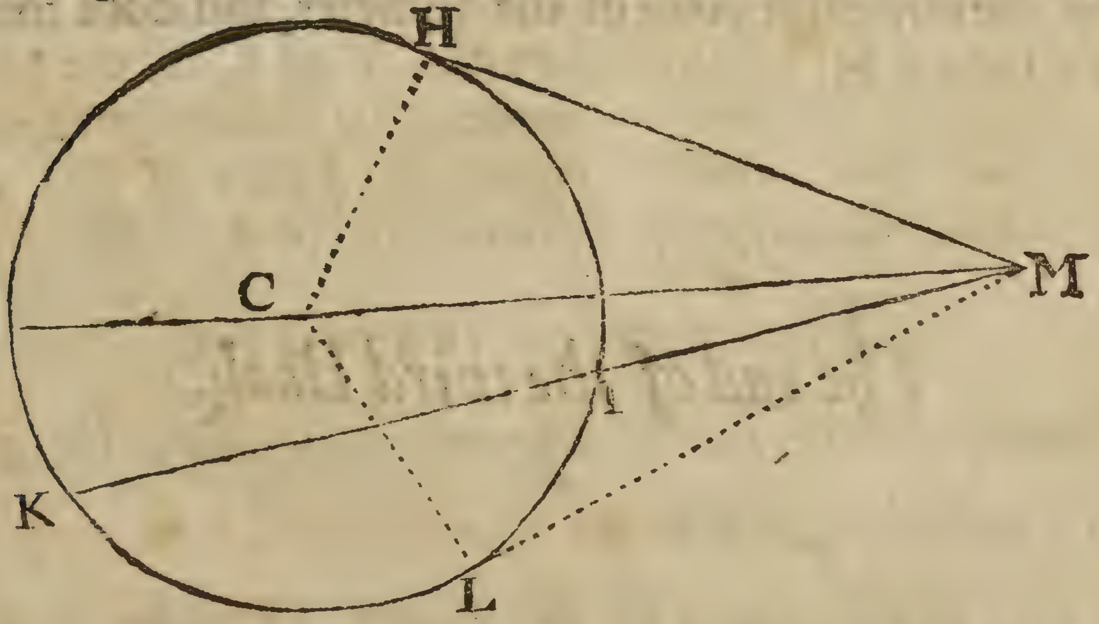
Theor. 31. Prop. 37.

If without a Circle, a point be taken by adventure, and from that point be drawn to the Circle two right lines, of which the one

one doth cut the Cicle, and the other doth fall upon the Circle, and that in ſuch ſort, that the right angle figure or Parallelogram, which is contained under the whole line, which cutteth the Circle, and that part of the ſame line, that is between the point and the utter Circumfeernce of the Circle, is equall to the ſquare of the line falling upon the Circle, then that line which ſo falleth upon the circle, ſhall touch the circle.

Conſtruction.

The point without the Circle is M, the line which doth



cut through the Circle is K M, and the line that doth fall upon the Circle is M H, the ſquare of the line M H is e-equal to the right angle Parallelogram conteined under the whole line M K, and the part M I, therefore the line M H doth touch the Circle.

Demonſtration.

Let there be drawn from the center C, theſe lines C M and C H, and *(by the 17 Propoſition of this third book)* draw from the point M, the line M L, which ſhall touch the Circle in the point L, and from L draw a line to C, as C L, and foraſmuch as the right angle Parallelogram, contained under M K and M I is equall to the ſquare of M L: it followeth, that M H and M L are equall, and becauſe the two ſides H M and H C are equall to the ſides L M and L C, and the baſe M C common to them both, the angle M B C is equall to the angle M L C *(by the eight of the firſt boook)* and the angle M L C is a right angle, and the line M L doth fall upon the uttermoſt of the diameter *(by the 6 and 18 Propoſition of this book)* therefore is the angle M H C alſo a right angle, and the line M H doth fall upon the uttermoſt point of the diameter, and doth touch the Circle in H.

The end of the third Book.

EUCLIDES ELEMENTS.

The Fourth Book.

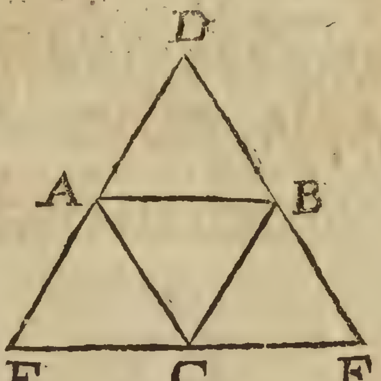
THE ARGUMENT.

This Book treateth of the Inſcription and Circumſcription of rectline Figures, how one right lined Figure may be inſcribed within another, & how one right lined Figure may be circumſcribed about another, for all rightlined Figures cannot be ſo inſcribed or circumſcribed within or about the other. Alſo it teacheth how a Triangle, a Square, or other Figure may be inſcribed within a Circle, and alſo how they may be circumſcribed, and how a Circle may be inſcribed within them, and becauſe the manner of intreaty in this Book, is divers from that of the former, he uſeth in this other terms, the *Definitions*, of which in order follow.

DEFINITIONS.

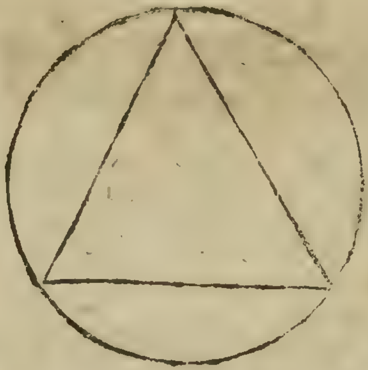
1 *A Rectline ſigure is ſaid to be inſcribed in a Rectline figure, when every one of the angles of the inſcribed figure toucheth every one of the ſides of the figure wherein it is inſcribed.*

As the Triangle A B C is inſcribed in the Triangle D E F, becauſe that every angle of the inſcribed triangle toucheth every ſide of the triangle in which it is inſcribed, namely, the angle A toucheth the ſide ED, the angle B toucheth the ſide D F, & the angle C toucheth the ſide E F.



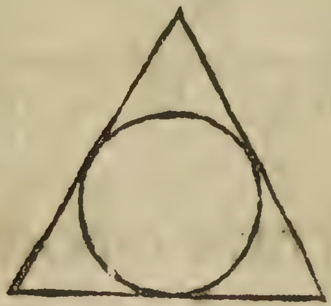
2 ***A Rectline figure is ſaid to be circumſcribed about a Rectline figure, when every one of the ſides of the figure circumſcribed toucheth every one of the angles of the figure about which it is circumſcribed.***

As the Triangle E D F is circumſcribed abont the Triangle A B C, becauſe the three ſides of the Triangle EDF do touch the three Angles of the Triangle A B C.



3 A rectline Figure is ſaid to be inſcribed in a Circle, when every one of the angles of the inſcribed figure touch the circumference of the Circle.

4 A Circle is ſaid to be circumſcribed about a rectline figure, when the circumference of the Circle toucheth every one of the angles of the figure about which it is circumſcribed.

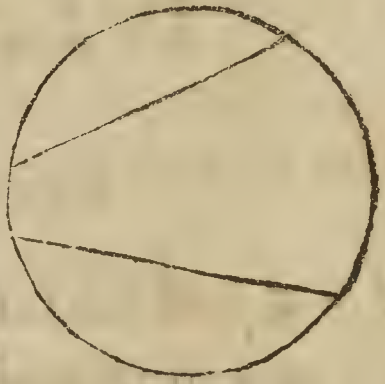


5 A Circle is ſaid to be inſcribed in a rectline figure when the circumference of the Circle toucheth every one of the ſides of the figure within which it is inſcribed.

6 A rectline figure is ſaid to be circumſcribed about a Circle, when every one of the ſides

ſides of the figure circumſcribed toucheth the circumference of the Circle.

7 *A* right line *is ſaid to be* co apted *or* applyed *in a* Circle , *when the extreams or ends thereof fall upon the* circumference *of the* Circle.



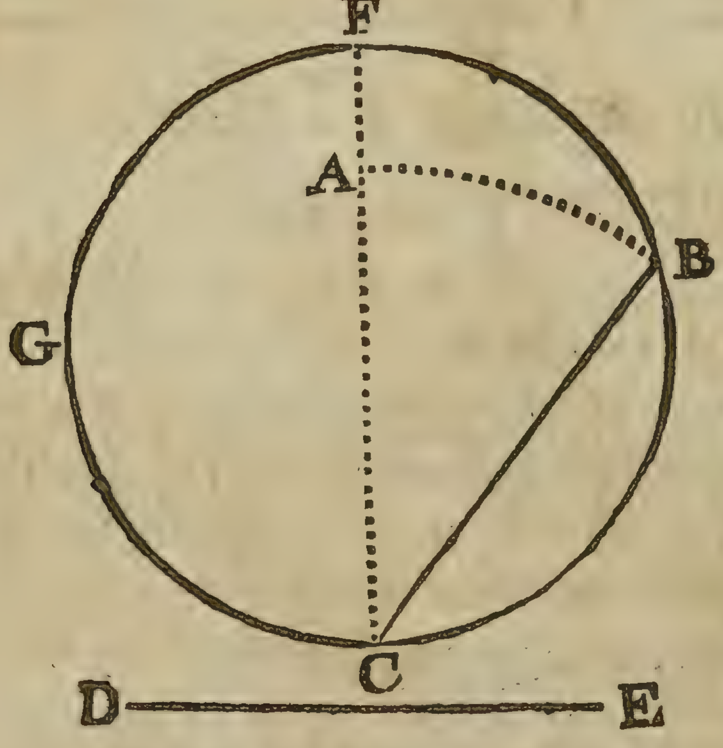
EV-

EUCLIDES ELEMENTS.

Probl. 1. Prop. 1.

In a given Circle, to apply a right line, equall to a right line given, which doth not exceed the diameter of the Circle.

Conſtruction.



The given Circle is F G C B, the right line given is D E, draw from one point of the Circumference as from C, a diameter of the Circles, as C F, and from the point C *(by the ſecond of the firſt book)* cut off a line with your compaſſes, equall to D E, as C A, then ſet one foot of your Cõpaſſes in C, & with the other foot draw the Arch A B, cutting the Circumference in the point B, from thence draw a right line to C, that line C B is in the Circle, and alſo equall to the given line D E.

Demonſtration.

Foraſmuch as A C is equall to D E, and alſo *(by the Circles Definition)* equall to C B, then is that line C B equall to the given line D E, this is manifeſt by the work it ſelf, For C B is taken equall to D E.

Another way.

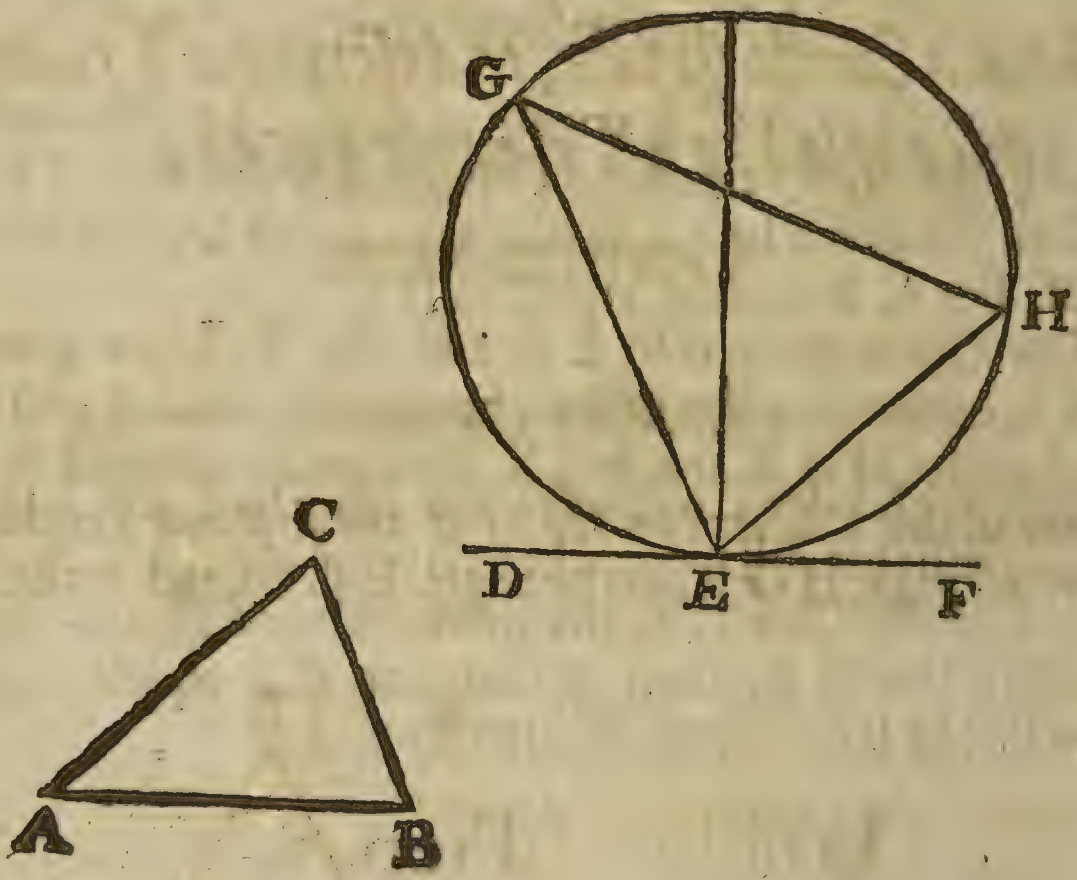
Take with your Compaſſes the length of the line D E, and with that diſtance make two points in the Circumference of the Circle, as the points C and B, and draw the right line C B, then is that line C B equall to the given line D E.

Probl. 2. Prop. 2.

In a given Circle, to deſcribe a Triangle equiangled to a given Triangle.

Conſtruction.

The Circle given is E G H, the Triangle given is A B C, draw firſt a right line which doth touch the Circle *(by the 17 of the third book)* as D E F, and from the point E upon the line D F, make an angle equall to the angle A, of the given Triangle A B C *(by the 23 of the firſt book)* as the angle E F H, and from the ſame point E uppn the other ſide of the angle F E H draw the line through the circle making an angle equall to the angle B, as HE G, and draw the line E G; And laſtly, draw the line G H. Then is the triangle E G H, equiangled to the given triangle A B C.



Demonſtration.

For as much as all the three angles of any triangle, are equall to two right angles (by the 32 of the 1 Booke) and that the angle E G H is equall to the angle F E H; (by the 32 of the 3 Book) That is, equall to the angle A and the angle G E H, by the working is equall to B. It followeth that the angle G H E, and the angle C are alſo equall, and that the Triangle within the Circle is like formed to the Triangle.

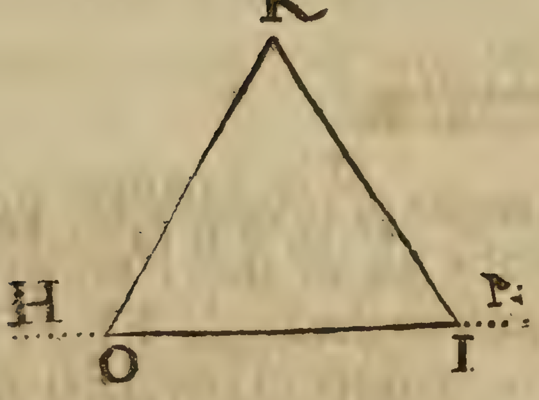
Pro-

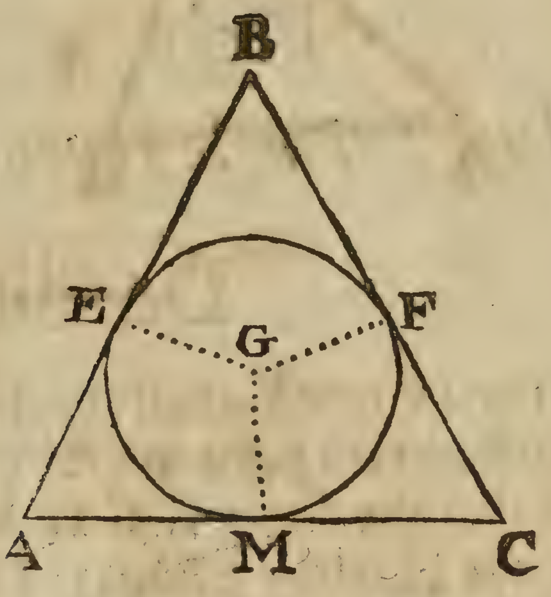
Probl. 3 Prop. 3.

About a Circle given, to deſcribe a Triangle equiangled to a Triangle given.

Conſtruction.

The Circle given is M E F, and the Triangle given is O K I, extend the line O I on both ends, making two outward angles H O K and N I K, then make an angle in the center of the Circle, *(by the 23 of the firſt book)* equall to outward angle H O K, as the angle E G M, and make alſo M G F in the Center equall to the outward N I K, and from theſe three points E M F, draw three lines touching the Circle, which ſhall come right angled upon E G, F G and M G *(by the 18 of the third book)* and theſe lines ſhall cut one another in the points A B C, making ſo the Triangle A B C, which hath angles equall to the given angles O K I.





Demon-

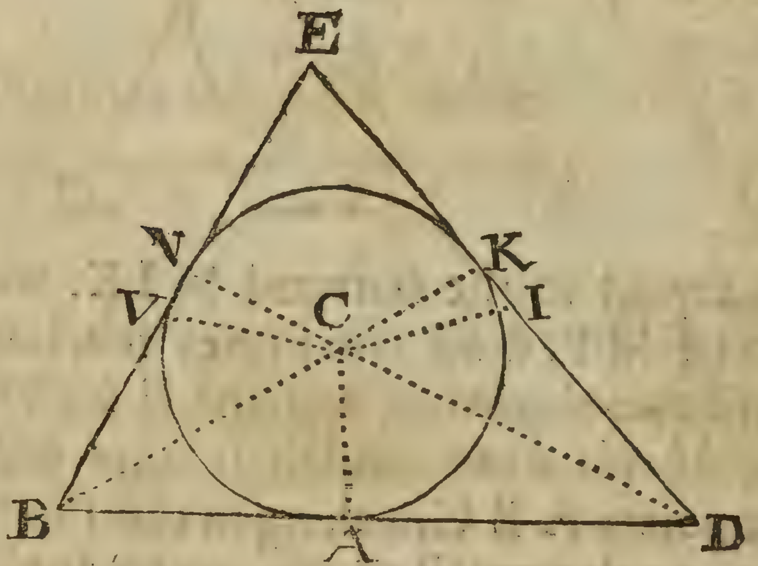
Demonſtration

Foraſmuch as the four angle figure A E G M may be inſcribed in a Circle, as may be underſtood *(by the 31 of the third book)* ſeeing that the angles A E G and E M G are right angles, the angles oppoſite, as E G M and E A M are together equall to two right angles, it followeth that the angle A is equall to I O K, being witn the outward angle H O L (equall to E G M) equall to two right angles *(by the 13 of the firſt book)* in like manner alſo may be ſhewed, that the other angles are one equall to the other, (that is) K I O equall to C, and therefore is O I K equall to B.

Probl. 4. Prop. 4.

In a Triangle given, to deſcribe a Circle.

Conſtruction.



The Triangle given is E B D, divide two of the angles in two equall parts, as the angles B and D *(by the 9 of the 1 book)* with the lines B C and D C, which ſhall concurre in C, from thence draw a line perpendicular upon the ſide, as upon BD, or any of the other ſides, which is the line A C, and this line A C is the half diameter of the Circle, which can be inſcribed within the Triangle, and C is the center thereof, which Circle ſhall touch the other ſides of the Triangle E B and E D in the points N and K.

Demonſtration.

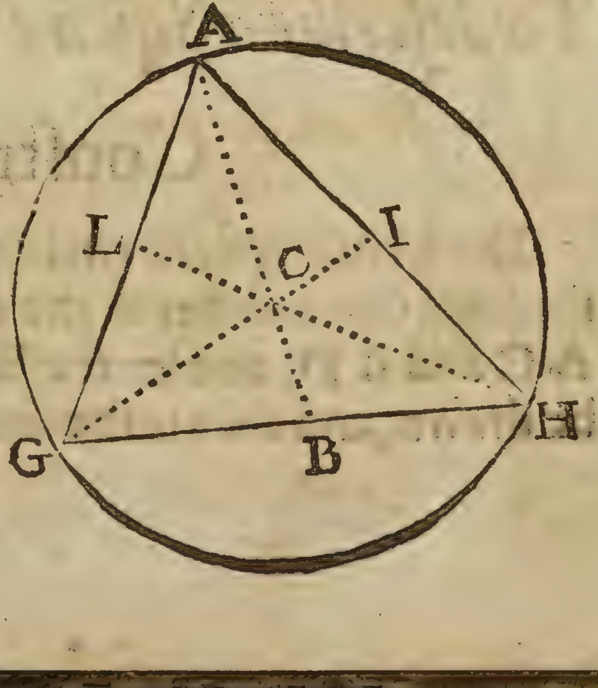
Let there be drawn out of the center C, theſe perpendicular lines, C A, C N and C K, which being equall, then is C the center of the circle, which can ſtand in the Triangle, and touch all the ſides (which is to be underſtood *by the 9 of the third book)* foraſmuch then as the angles C B N and C B A are eqnall, as alſo C N B and C A B, which are both right angles, it followeth *(by the 32 of the firſt book)* that the angles B C N and B C A are alſo equall, and therefore is C N equall to C A *(by the 26 of the firſt book)* and ſo is A C and C K found to be equal alſo, wherefore the three perpendiculars are the one equal to the other, and C the center of the Circle.

Probl. 5. Prop. 5.

About a Triangle given, to deſcribe a Circle.

Conſtruction,

The given Triangle G A H divide two ſides as A G and A H in two equall parts in L and I, and from theſe points draw two perpendicular lines in the Triangle as L C and I C cutting through one



one another in C, which is the center of the Circle, which ſhall go about the Triangle, wherefore ſet one foot of the Compaſſes in C, and with other foot deſcribe the Circle, with the diſtance A C or C G, and it will juſt incompaſſe the Triangle A H G.

Demonſtration.

Let there be drawn the lines C G, C A and C H, we muſt prove that theſe three lines be equall, if each of them be the half diameter of the Circle, which being ſo, then is C the center of the Circle that will go about the Triangle G A H *(by the 9 of the third book)* foraſmuch as the lines G L and L C are equall to A L aud L C, and that the angles in L are equall: it followeth that C G and C A are equall *(by the 4 of the firſt)* and by the ſame reaſon ſhall be proved, that A C and C H are alſo equall, and therefore all three are equall, and each of them the half of the diameter of the Circle, and that C is the center of the ſame circle, wherefore *(by the 47 of the firſt book)* as alſo in divers other manners may be demonſtrated,

Probl. 6. Prop. 6.

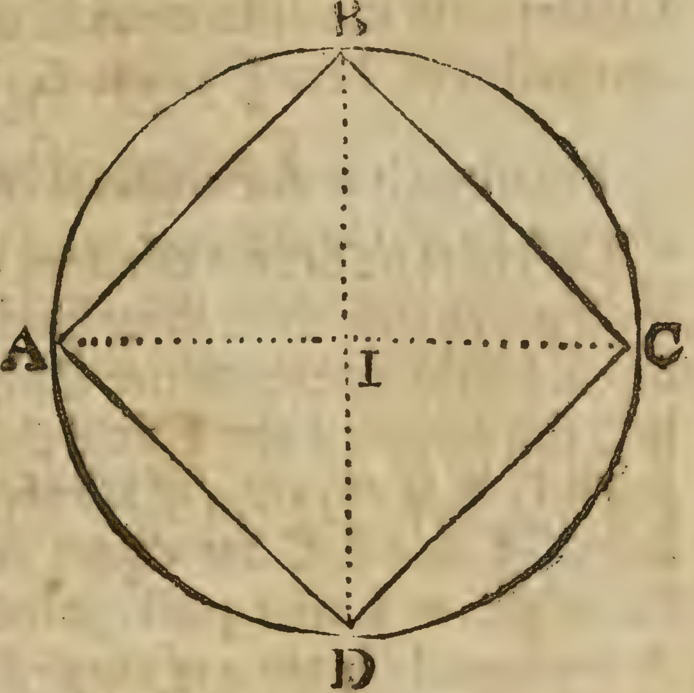
To deſcribe a Square within a given Circle.

Conſtruction.

Draw through the circle two diameters, which ſhall cut through one another in the center, with right angles, as A C and B D, and let the lines A B, B C, C D, and D A be drawn, and then is the ſquare made as was required.

Demonſtration.

Foraſmuch as the diameters AC and BD doth cut one another in right angles, and alſo in equall parts, then the four ſides are equall *(by the fourth of the firſt book)* and *(by the 31 of the third book)* are the angles A B and C D alſo right angles, therfore is the four angle figure A B C D a ſquare.

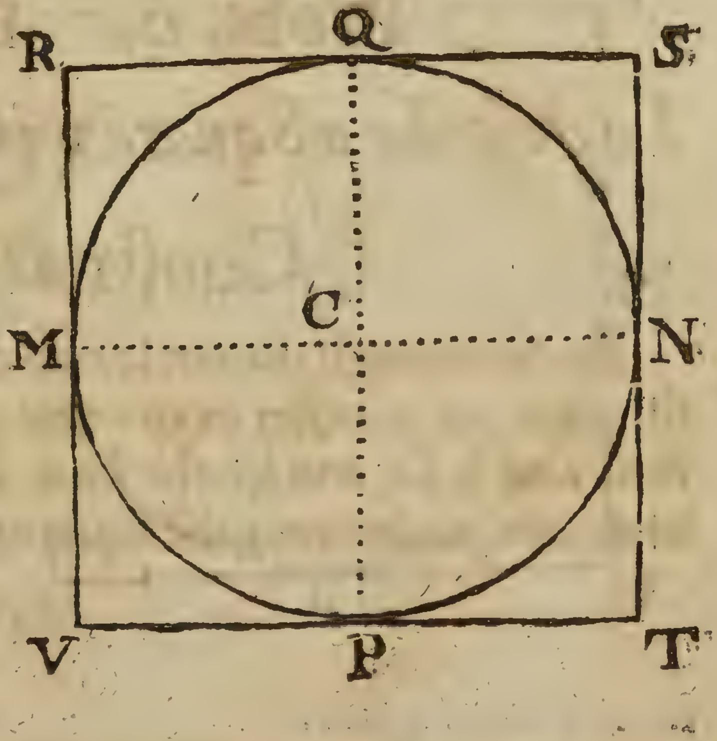


Probl. 7. Prop. 7.

About a Circle given, to deſcribe a Square.

Conſtruction.

Draw through the circle two diameters, as M N and Q P, which ſhall cut one another through in right angles in the center C, & from the uttermoſt points M N draw lines parallel to Q P, (that is perpendicular [illegible] N) as R V an[illegible] alſo two more



mqre,by the poiuts Q P parallel to M N, as R S and T V, theſe four lines cut through one another in the points R S T V, then is the ſquare V R S T deſcribed about the Circle.

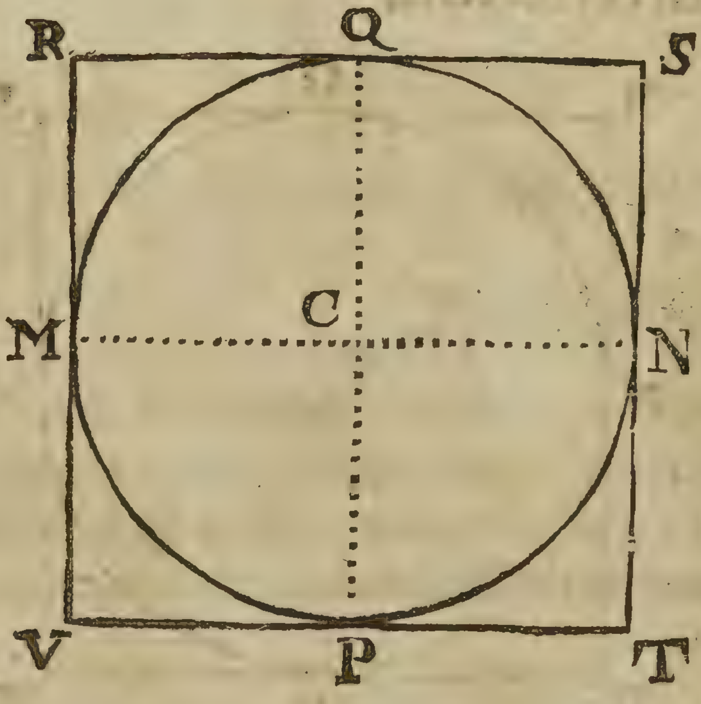
Demonſtration.

Foraſmuch as the angles in M Q N P are right angles *(by the 29 of the firſt book)* and that it followeth *(by the 34 of the ſame book)* that the angles R S T V are alſo right angles, and that the lines R S and T V are equall to M N, it followeth, that (R V and S T being equall to Q P, that is equall to M N) the four ſides are equall, and that the four angled figure V R S T is a ſquare circumſcribed about a circle.

Probl. 8 Prop. 8

In a given Square, to inſcribe a Circle.

Conſtruction.



The ſquare given is R S T V, divide the four ſides, each into two equall parts, and draw the lines Q P and M N cutting one another in C, which is the center of the circle, which can be inſcribed in the ſaid ſquare, and C M, C Q, C N and C P are, each of them ha [illegible] rs- of the cir [illegible] in the ſaid ſquare.

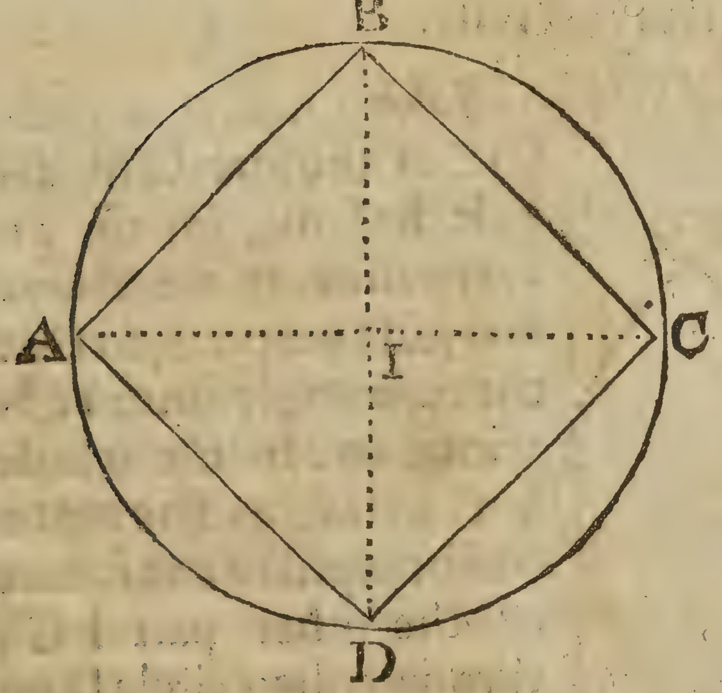
Demonſtration.

Foraſmuch as *(by the 34 of the firſt book)* Q P is equall and parallel to R V or S T, and M N is alſo equall and parallel to V T or R S, the right line angles M Q & N P, are then alſo right angles, and C M, C Q, C N and C P are *by the aforeſaid Propoſition)* equall, if then the Circle be deſcribed about the center, ſo that it touch the one ſide of the ſame ſquare, the ſame ſhall touch all the four ſides, like as is to be underſtood *by the definition of the Circle.*

Probl. 9. Prop. 9.

About a ſquare given, to deſcribe a Circle.

Conſtruction.



The ſquare given is A B C D, draw through the ſquare the diameters or diagonall lines A B and C D, which ſhall cut through one another in the point I, which is the center of the circle, and I A, I C, I B, and I D, are each of them a half diameter of the circle, which may be deſcribed about the ſame ſquare, as *by the ſixt Propoſition of this book)* may plainly be ſeen.

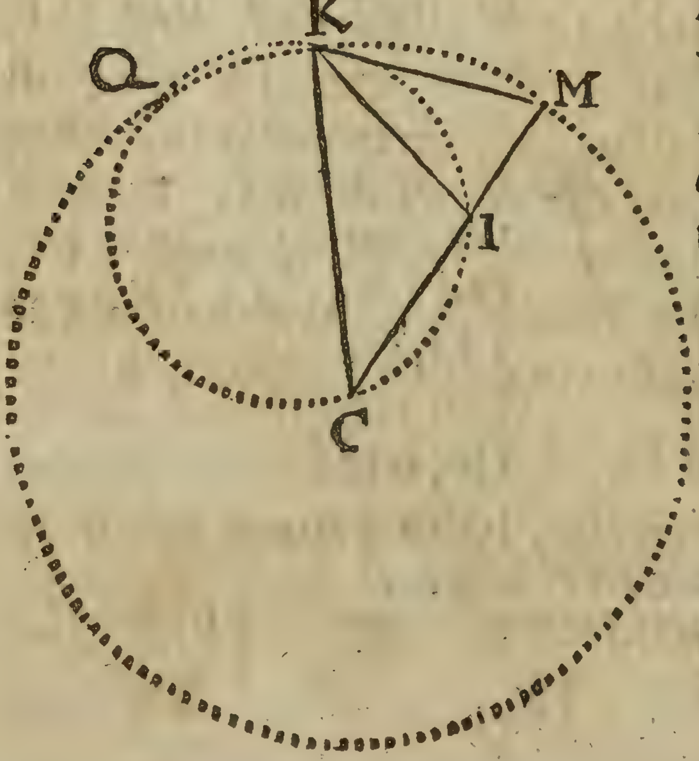
Demonſtration.

Foraſmuch as *(by the 9 and 10 Propositions of the firſt book)* the angles and the diagonals are divided into two equall parts, then is I the center of the circle *(by the 9 of the third book)* which is alſo manifeſt, ſeeing that the diagonals of the ſquares is alſo the diameter of the circle, *(by the 31 of the third book.)*

Probl. 10. Prop. 10.

To make a Triangle of two equall ſides, called Iſoſceles, which ſhall have either, or each angle at the baſe, double to the other angle.

Conſtruction.



Take a line at a pleaſure, as the line C M divide the ſame, *(by the 11 Proposition of the ſecond book)* in two parts, ſo that the right angle figure conteined under the whole line C M, and the part M I be equal to the ſquare of the other part I C, then deſcribe a circle from the center C, whoſe half diameter is M C, which is the circle M Q N, in the ſame circle apply a line *(by the firſt of the third book)* equall to the line I C which is K M, and draw the lines K I and K C , then have you two Triangles as C M K, and K I M each of them having the angles at the baſe, as C M K and C K M each double to the other angle, namely, to the angle M C K, and the angles K I M and K M I, alſo each double to I K M.

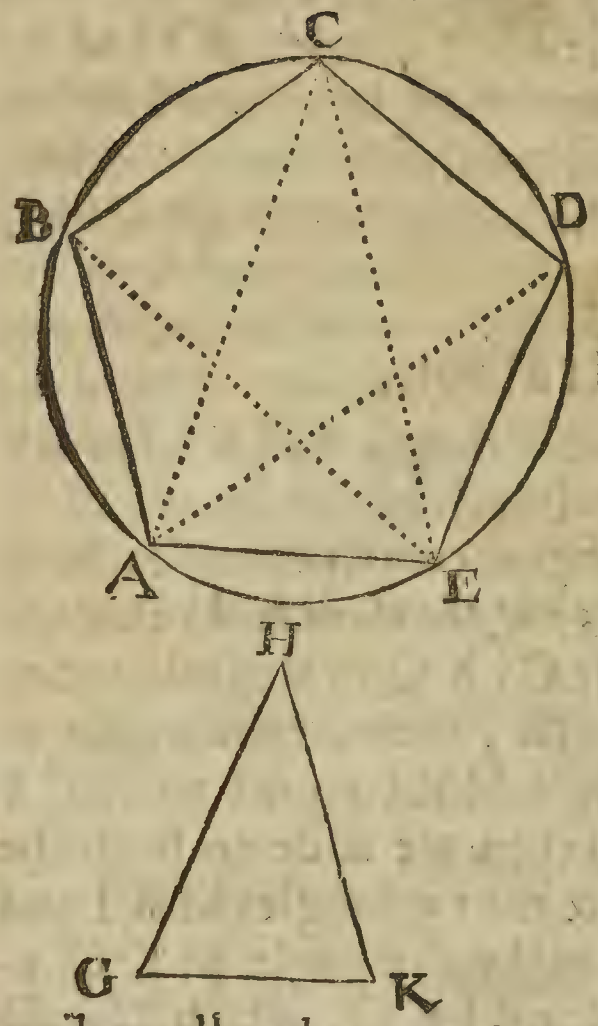
Demonſtration.

Deſcribe a Circle about the Triangle C I K *(by the 5 of this book)* as C I K Q, foraſmuch as the ſquare of M K is equall to the right angle figure conteined under M C and M I, it followeth *(by the 37 of the third book)* that the line M K doe touch the Circle C I K Q in K, ſeeing then that the line K I doe cut the circle , then are the angles of the touch line M K, as the angle M K I equall to K C I *(by the 37 of the third book)* if then we adde to both the common angle I K C, then are the two angles M K I and I K C (which together doe make the angle M K C equall to the two angles I C K and I K C, which *(by the 32 of the firſt book)* are alſo equall to K I M, and K M C is equall to M K C, and therefore alſo equall to K I M, it followeth *(by the ſixt of the firſt book)* that I K is equall to M K, and M K is equall to I C, therefore is the angle I K C equall to I C K, and the angles M K C, K M C, and K M I, each double to the angles I C K angles I C K and I *K* M, and the two Triangles C M *K* and K I M are made according to the intent of this Propoſition.

Probl. 11. Prop. 11.

In a Circle given, to deſcribe a pentagon figure, or an equilaterall and equiangled five angle.

Conſtruction.



The Circle given is A B C D E, make firſt a Triangle in form, *according to the contents of the former Propoſition* which Triangle is C H K, and deſcribe ſuch a Triangle in a Circle *(by the ſecond of this book)* which is the Triangle A C E, of which divide the two angles at the baſe, each in two equall parts *(by the 9 of the firſt book)* with the lines A D and E B, theſe lines doe cut the Circumference in the points D and B, and draw the lines A B, B C, C D, D E and E A, and then is the equilaterall and equiangled five angle made.

Demonſtration.

Foraſmuch as the angles A E B, B E C, C A D, D A E and E C A are all equall one to another, then are the parts of the circumference alſo equall (*by the 26 of the third book*) as the parts A B, B C, C D, D E and E A, and (*by the 29 of the third*) the lines which doe ſubtend equall angles have alſo equall parts of the Circumference, (which lines are the ſides of the pentagon) and (*by the 21 of the third book*) are the angles A B E, A C E, A D E, B C A B E A, B E C, B C E and D A E, all of them equall one to another, and each of thoſe angles, of the ſaid five angle, or *Pentagon*, equall to three ſuch angles, and therefore are the ſame equiangled.

Another way.

Draw from a point in the circumference, as from C, two lines by adventure, making an angle at the circumference, equall to the angle H, of the triangle G H C, as the lines CD and C E, which lines touch the Circumference in the points D and E. Then is the arch D E a fifth part of the Circumference by the former Demonſtration, then divide the whole circumference into 5 ſuch parts, and from thoſe points draw lines as A B, B C, C D, D E and E A, and the 5 angle finiſhed.

The Circumference may alſo be divided into 5 equall parts, by making the angles in the centre, equall to the angles G or K of the triangle G H K, which by the former Demonſtration, (*with the help of the 20 Propoſition of the 3 book*) may be wel underſtood.

Probl. 12. Prop. 12.

About a given Circle, to deſcribe an Equilaterall, and equiangled Pentagon or five angle figure.

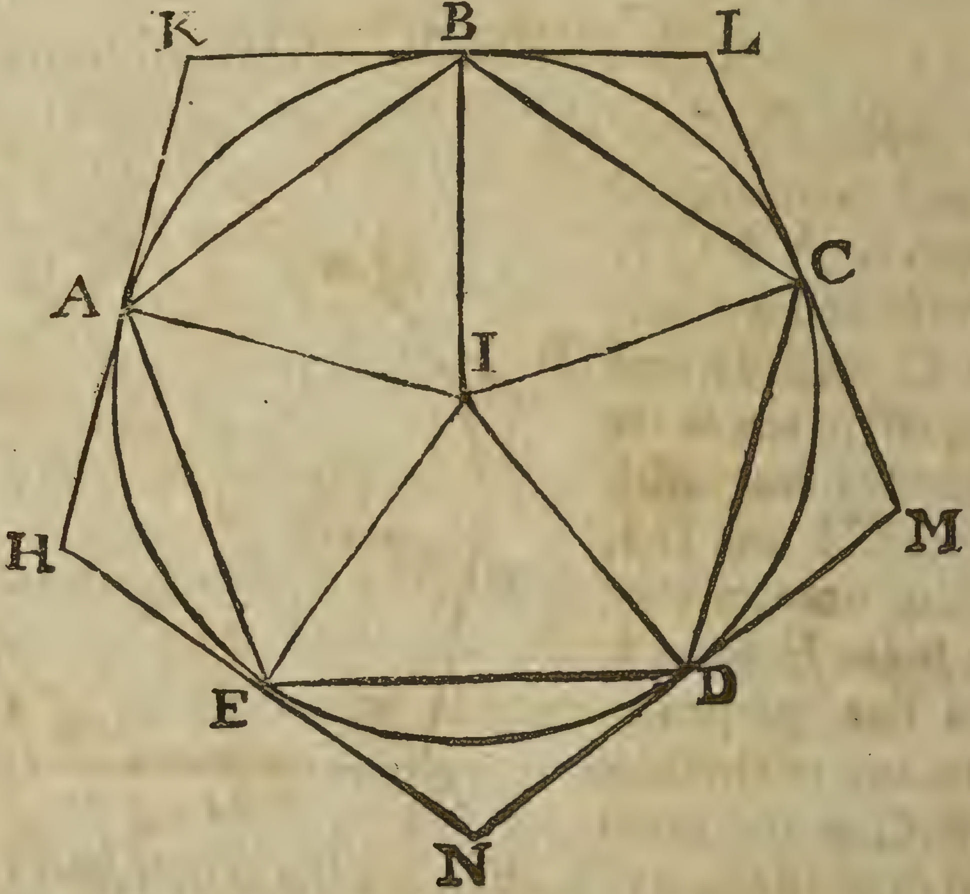
Conſtruction.

Within this given Circle, deſcribe firſt (*by the former Propoſition*) a *Pentagon* or 5 angle, as A B C DE, and from the points where the angles doe touch the Circle, draw lines to the centre I, which are half Diameters of the Circle, and from the ſame points draw other lines, which ſhal come right angled upon thoſe half Diameters, thoſe lines ſo drawn ſhal cut one another through in the points H K and L M N. Then is that five angle H K and L M N, equilaterall, & equiangled, deſcribed about the given circle.

De-

Demonſtration.

Foraſmuch as the angles at the touching of the Circle, at the ſides of the circumſcribed Pentagon, are right angles theſe four angle figures, as I E H A, I A K B, I B L C, I C M B, & I D N E, each of them may be inſcribed in a Circle, as can be underſtood *(by the 31 of the 3 Book)* ſeeing that the angles at the centre I, are all equall *(by the 4 of the 1 Book)* then are the angles H K L M N (which are the angles of the 5 angle alſo equal *(by the 32 of the 3 Book)*

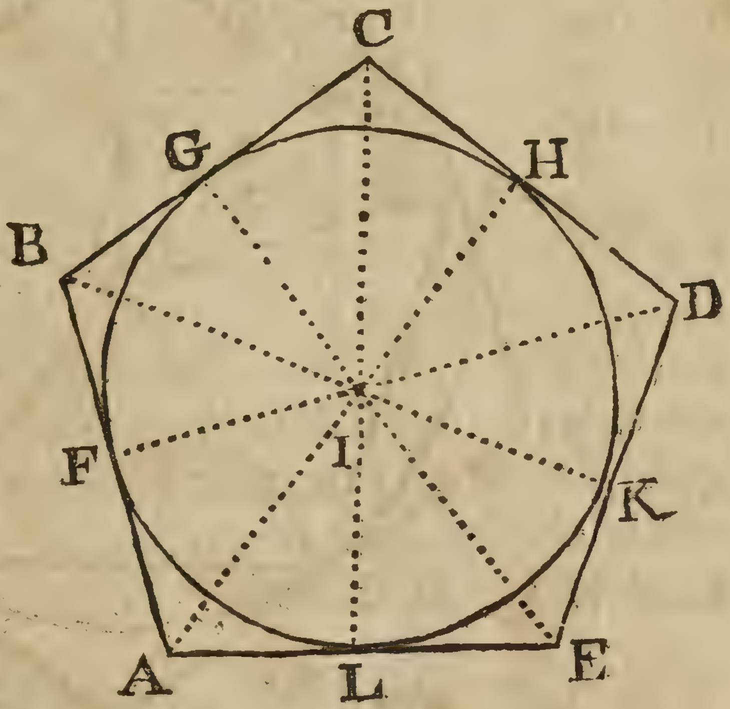


Secondly, ſeeing that the inſcribed Pentagon, are divided by the half diameters, in ſuch five Iſoſceles triangles, which have their angles at the baſe, (that is) at the ſides of the ſame pentagon, all equall *(by the 5 of the firſt book)* the angles of the other ſides of the ſame pentagon, (that is the ſides remaining of the right angle) are alſo one equal to another *(by the third common ſentence)* therefore *(by the* *ſixt of the firſt book*) are the lines A K, K B, B L, L C, C M, M D, D N, N E, E H and H A alſo equall, as alſo the ſides of the circumſcribed pentagon H K L and M N, each alſo in length equall to two of the other lines.

Probl. 13. Prop. 13.

An equilaterall and equiangled pentagon figure being given, to inſcribe in it a Circle.

Conſtruction.



The given equilaterall and equiangled pentagon is A B C D E, divide the two angles as C and D into two equall parts (*by the 9 of the firſt book*) with the lines C I and D I, theſe cut one another in the point I, from I, draw a line perpendicular to one of the ſides as to B C in the point G, and draw the line G I, then is I the center, and I G the half diameter of the Circle, which can be inſcribed in the pentagon, therefore take the diſtance I G, and from the center I, deſcribe a circle, which ſhall touch all the ſides of the pentagon in the points G H K L F.

Demonſtration.

The lines which divideth the angles in two parts doth paſſe through the center of the circle)*by the 4 of this book)* and *(by the 9 of the third book)* thoſe lines which paſſe through the center, doe cut one another in the center, therefore is that point of the croſſing, the center of the circle, and the perpendicular I G the half diameter of the circle, which may be underſtood (*by the 18 Propoſition of the third book*) which otherwiſe may alſo be thus proved. Let there be drawn from the point I, lines to all the angles of the pentagon, and alſo perpendiculars to all the ſides.

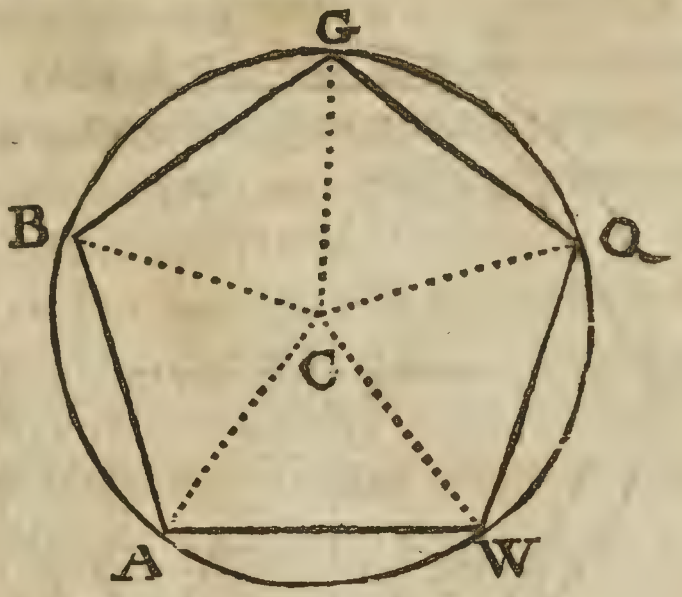
Firſt, by the equall dividing of the angles I C B, I C D and I C E, are they all equall, and *by the 6 propoſition of the firſt book*) is I C and I D equall, therefore foraſmuch as the lines C B and C D are equall, then are the three ſides of the Triangles C I B and C I D, that is I B and I D alſo equall (*by the 4 of the firſt book*) and ſo is I B equall to I D or I C, and by theſe and ſuch like reaſons is found that all the lines from I to the angles of the pentagon, are equall, and making upon the ſides of the pentagon Iſoſceles triangles, and therefore all the perpendicular lines of the ſame triangles (being half diameters of the Circle) muſt be equall, like as may be underſtood *by the aforeſaid* 4 *propoſition of the firſt book*, wherefore it is manifeſt, that the circle doth touch all the ſides of the pentagon.

Probl. 14. Prop. 14.

About a pentagon or figure of five angles given, being equilaterall and equiangled, to deſcribe a Circle.

Conſtruction.

In this pentagon ABGQW *(by the former Propoſition)* is



is the center found, as C, from thence lines drawn to the angles of the five angle, theſe lines are half diameters of the circle, which will circumſcribe the circle about the ſame pentagon.

Demonſtration.

In the former propoſition, it is proved, that the lines drawn from the center to the angles of the equall ſides, and equall angled five angle, as C A, C B, C G, C Q, and C W are all equall, and therefore is C the center of the circle, which can circumſcribe the ſaid pentagon, and the ſaid lines are half diameters *(by the 9 propoſition of the third book.)*

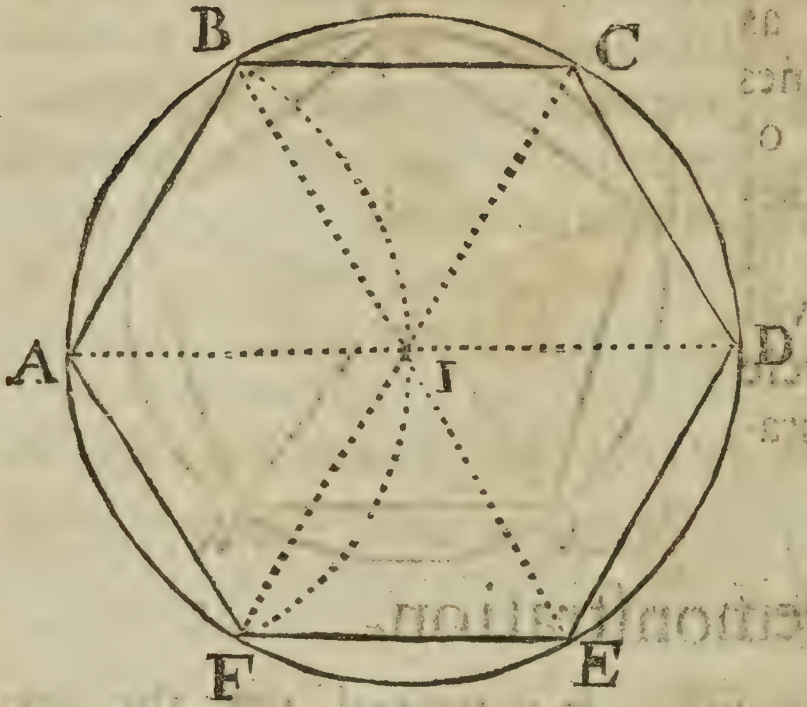
Porbl. 15. Prop. 15.

In a Circle given, to deſcribe a Hexagon, or a figure equilaterall and equiangled of ſix angles.

Conſtruction.

The circle given is A B C D E F, the center is I, and the diameter is A D, firſt draw from the point A an arch of a Circle equall to the former circle, which ſhall paſſe through the center I, and cutting the Circumference in the points B and F, and from thoſe points draw right

lines



lines through the center I, to the circumference, as the right lines B E and F C, and then draw the lines A B, B C, C D, D E, E F & F A, then is the Hexagon of ſix angle A B C D E F inſcribed in the circle.

Demonſtration.

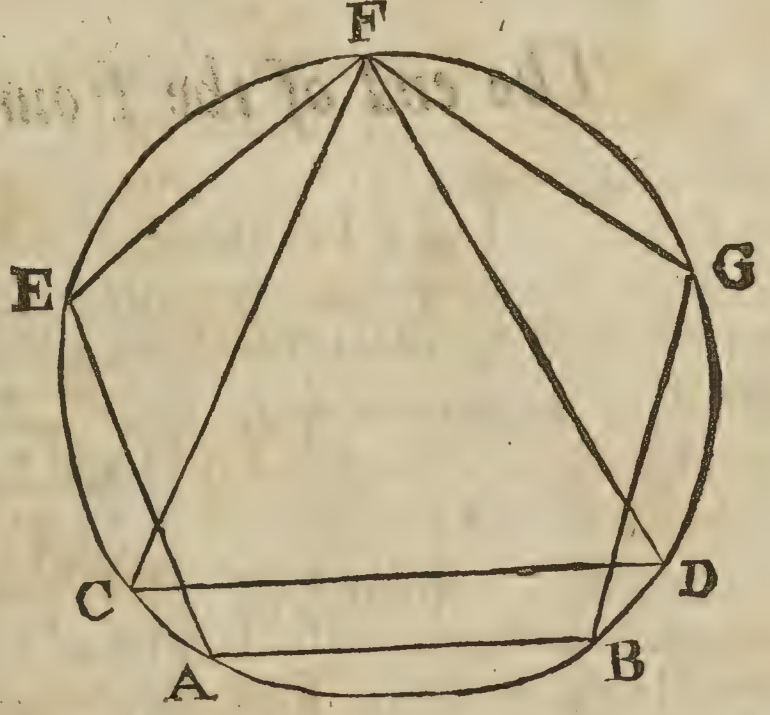
It is evideut *(by the Circles definition)* that the two Triangles A B I and A F I are equall ſided, and equall angled, ſeeing that *(by the 15 propoſition of the firſt book)* the angles F I D and B I A are equal, as alſo D I C and AIF, and that the lines that doe contein thoſe angles are equal, it followeth *(by the 4 Propoſition of the firſt book)* that the ſide E D is equall to A B, and C D equall to A F, and therefore the Triangles E I D and D I C are equall ſided alſo.

Secondly, *(by the 32 of the firſt book)* is the angle A I C equall to the two angles I A F and I F A, which are alſo equall to the angle A I B each of them, which angle A I B being taken from both, it is evident that the angle B I C is equall to I A F or I F A, and by the ſame reaſon is F I E alſo equall to one of the other angles of the equilaterall Triangles, and therefore the ſides B C and F E are equall to the other ſides, and the whole Hexagon equilaterall, and *by the ſixt common ſentence* are the angles of the Hexagon alſo equall, being each of them double to one of the angles of the equilaterall Triangles, whereby it is manifeſt, that the ſides of a regular Hexagon, or ſix ſided figure inſcribed in a circle is equall to the half diameter of the Circle, wherein the ſame is inſcribed.

Probl. 16. Prop. 16.

In a given Circle, to inſcribe a Quindecagon, or a figure of fifteen angles, equilaterall and equiangled.

Conſtruction.



In this circle given, deſcribe an equilaterall Triangle (*by the 2 of this book*) as F C D and *by the 11 Propoſition* make an equilaterall pentagon, or figure of five angles, but in ſuchſort make them that one angle of the Triangle, and of the five angle come both in one point of the circumference of the circle, as here in the point F, and this five angle or pentagon is A E F G B, and the Triangle is F C D, then is the arch or part of the circumference C A or B D one fifteenth part of the circumference of the circle, and then a right line drawn from A to C, ſhall be a ſide of the fifteenth angle equilaterall and equiangled within the circle, whereby the whole figure may be perfected.

Demonſtration.

The arch F E A is two fift parts of the whole circumference, that is ſix fifteenth parts, and the arch F E C is one third part, or five fifteenth parts of the ſame circumference, it followeth then, that the arch A C is one fifteenth part of the ſame Circumference.

The end of the Fourth Book.

EUCLIDES ELEMENTS.

The Fifth Book.

THE ARGUMENT.

This fifth *Book* is of very great uſe in *Geometry*, as alſo in *Muſick*, *Aſtronomy*, *Perſpective*, *Arithmetick*, &c. and ought of all the other *Books* of *Euclide*, to be moſt practiſed, becauſe all his following *Books* without it cannot be underſtood. It intreateth of *Analogie*, or *Proportionality*, which pertaineth not onely to *Lines*, *Figures*, and *Bodies* in *Geometry*; but alſo unto *Sounds* and *Voices*, of which *Muſick* intreateth. Alſo the whole Art of *Aſtronomy*, teacheth to meaſure *Proportions* of *Times* and *Motions*. The uſes indeed of this *Book* are infinite, though *Euclide* hath applyed them onely to *Geometry*. The firſt *Authour* of this *Book* was *Eudoxus*, one of *Plato*'s Scholers, but framed and put into order by *Euclide*.

DE-

DEFINITIONS.

1 *A Part is a leſſer magnitude, in reſpect of a greater magnitude when the leſſer meaſureth the greater.*

2 *Multiplex is a greater magnitude in reſpect of a leſſer, when the leſſer meaſureth the greater.*

3 *Proportion is a certain reſpect of two Magnitudes of one kind according to quantity.*

4 *Proportionality is a ſimilitude of proportions.*

5 *Thoſe magnitudes are ſaid to have proportion the one to other, which being multiplyed, may exceed the one the other.*

6 *Magnitudes are ſaid to be in one and the ſelf-ſame proportion, the firſt to the ſecond, and the third to the fourth, when the equimultiplices of the firſt and of the third, being compared with the equimultiplices of the ſecond and fourth, according to any multiplication either together exceed the one the other, or together are equall the one to the other, or together are leſſe the one then the other.*

7 *Magnitudes which are in one and the ſame proportion, are called proportionall.*

8 *When equimultiplices being taken, the multiplex of the firſt exceedeth the multiplex of the ſecond, and the multiplex of the third exceedeth not the multiplex of the fourth: then hath the firſt to the ſecond a greater proportion then hath the third to the fourth.*

9 *Proportionality conſiſteth at the leaſt in three terms.*

10 *When there are three magnitudes in proportion, the firſt ſhall be unto the third in double proportion that it is to the ſecond, but when there are four magnitudes in proportion, the firſt ſhall be unto the fourth in treble proportion that it is to the*

ſecond, and ſo always in order one more, as the proportion ſhall be extended.

11 *Magnitudes of like proportion are ſaid to be antecedents to antecedents, and conſequents to conſequents.*

12 *Proportion alternate, or proportion by permutation, is when the antecedeut is compared to the antecedent, and the conſequent to the conſequent.*

13 *Converſe proportion, or proportion by converſion is when the conſequent is taken as the antecedent, and ſo compared to the antecedent as to the conſequent.*

14 *Proportion compoſed, or compoſition of proportion, is when the antecedent and the conſequent are both as one compared to the conſequent.*

15 *Proportion divided, or diviſion of proportion is, when the exceſſe wherein the antecedent exceedeth the conſequent, is compared to the conſequent.*

16 *Conversion of proportion (which of the ancient is commonly called eversion of proportion, or everſe proportion) is when the antecedent is compared to the exceſſe, wherein the antecedent exceedeth the conſeqnent.*

17 *Proportion of equality is where there are taken a number of magnitudes in one order, and alſo as may other magnitudes in another order, comparing two to two, being in the ſame proportion, it commeth to paſſe that as in the firſt order of magnitudes, the firſt is to the laſt, ſo in the ſecond order of magitudes, is the firſt to the laſt, or otherwiſe, it is a compariſon of extreams together, the middle mag. being taken away.*

18 *An ordinate proportionality is, when as the antecedent is to the conſequent, ſo is the antecedent to the conſequent, and as the conſequent is to another, ſo is the conſequent to another.*

19 *An inordinate Proportionality is, when as the antecedent is to the conſequent, ſo is the antecedeut, to the conſequent, and as the conſequent is to another, ſo is another to the antecedent.*

EUCLIDES ELEMENTS.

Theor. 1. Prop. 1.

If there be a number of magnitudes, how many ſoever, equimultiplices to a like number of magnitudes, each to each, how multiplex one magnitude is to one, ſo multiplex are all the magnitudes, to all.

Conſtruction.

THe magnitudes in the firſt number, are ABC, and in the ſecond number, D E F, and A is ſuch part of D as B is of E, and C of F. Therefore are thoſe magnitudes A B C together, ſuch part of the magnitude is D E F, together.

3	9
A	D
5	15
B	E
2	6
C	F
10	30
ABC.	DEF

Demonſtration.

Foraſmuch as the magnitudes in the firſt number, each of them are unequall part of his next in order, of the ſecond number, That is, A is as many times conteined in D, as B in E, and C in F. It is evident, that A B C together, is alſo as many times conteined in D E F together; wherefore, the magnitudes A B C added together, are ſuch part of the magnitudes D E F added together, as A is of D, and B of E, or C of F, &c.

Theor. 2. Prop. 2.

If the firſt be equimultiplex to the ſecond, as the third is to the fourth, and if the firſt be alſo equimultiplex to the ſecond, as the ſixth is to the fourth. Then ſhall the firſt & the fifth added together, be equimultiplex to the ſecond as the third and ſixth added together is to the fourth.

Conſtruction.

2	3	4	6
A	B	C	D
18		3	
E		F	
20	3	40	6
AE	B	CF	D

The firſt proportioned magnitude A is to the ſecond B, as C the third is to the fourth D. To which there comes E the fifth, and F the ſixth. Now is E to B as F to D. Therefore, the magnitudes together, as A and E are proportioned together, to B, as C and F together is to D.

Demonſtration.

Foraſmuch as E hath proportion to B, like as F hath to D, and that A is to the ſame B alſo as C is to D. It is manifeſt, that if equall things be added to equall things, the magnitudes together A E ſhall have the ſame proportion to B, that C F together hath to D.

Theor. 3. Prop. 3.

If there be four magnitudes in proportion, ſo that the firſt is to the ſecond, as the third is to the fourth: & if there come two more magnitudes to the ſame, in ſuch ſort, that the proportion of the fifth is to the firſt, as the ſixth is to the third, then is the firſt alſo to the ſecond, as the ſixth is to the fourth.

Conſtruction.

3	5	9	15	6	18
A	B	C	D	E	F

The four magnitudes, equimultiplices, are ABCD of which, A is to B as C is to D, and there comes to the ſame two more magnitudes equimultiplices, As E and F, ſo that E is to A as F is to C. Therefore is E alſo to B as F is to D.

Demonſtration.

Foraſmuch as E is to A, as F is to C, and A to B as C is to D. Therefore is E as many times conteined in A, as F is in C, and alſo A in B as often as C in D. Wherefore E is to B as F to D. It is then manifeſt, that E hath proportion to B as F hath to D: (*by the ſixth Definition.*)

Theor 4. Prop. 4.

If the firſt be to the ſecond, in the ſame proportion that the third is to the fourth, then alſo the equimultiplices of the firſt and third, unto the equimultiplices of the ſecond, and of the fourth, (according to any multiplication) ſhall have the ſame proportion, being compared together. The fifth to the firſt, as the ſixth to the third, and the ſeventh to the ſecond, as the eighth to the fourth. Then are the four laſt magnitudes alſo proportioned.

Conſtruction.

12	3	16	4
A	B	C	D
6	8	9	12
E	F	G	H

The four proportioned magnitudes, are A B C D, of which A the firſt is to B the ſecond, as C the third is to D the fourth, and to A and C take equimultiplices, as G and H. Then is E to G, as F to H, alſo in the ſame proportion.

Demonſtration.

Foraſmuch as *(by the ſixt definition)* E is as many times conteined in A as F is in C, and A is to B, as C is to D. Therefore, is E to B as F is to D. And according to this propoſition is G to B, alſo, as H to D. Seeing then E to F is in the ſame proportion, as G to H. It is evident then, that the four magnitudes, E F G H, are proportioned alſo according to the propoſition.

Theor. 5. *Prop.* 5.

If the magnitude be equimultiplex to a magnitude, as a part taken away of the one, to a part taken away of the other. The reſidue of the one to the reſidue of the other, ſhall alſo be equimultiplex, as the whole is to the whole.

Conſtruction

8 12
A B
2 3
C D
———
6 9
E F

The whole magnitude A is equimultiplex to the whole magnitude B, as the part taken away C, is to the part taken away D, and the part taken away, as C from the whole, as A, there ſhall remain the reſidue E, and the part D, being taken from the whole B, there ſhall remain the reſidue F. Then is the reſidue E to the reſidue F, as the whole A is to the whole B.

Demonſtration.

Foraſmuch as C is ſuch a part of A, as D is of B. It is evident, that the reſidue, as E, is alſo ſuch a part of A, as F is of B, and therefore hath E ſuch a proportion to F, as A hath to B, or C to D.

Theor. 6. Prop. 6.

If four magnitudes be proportioned, and from the firſt and the third, a part be taken away, ſo that the firſt part be to the ſecond magnitude, as the ſecond part is to the fourth magnitude, then is the firſt reſidue to the ſecond magnitude, as the ſecond reſidue is to the fourth magnitude.

Conſtruction.

The four magnitudes, are A to B as C to D. The parts taken away, are E from A, and F from C. The reſidue, as G from A, and H from C. Then hath E proportion to B as F hath to D. Therefore is G alſo to B, as H to D.

20	3	40	6
A	B	C	D
18		36	
E		F	
2	3	4	6
G	B	H	D

Demonſtration.

Foraſmuch as E is to B as F to D, and A is to the ſame B as C is to the ſame D. It followeth, that B is as many times conteined in A as D is in C, and B as many times conreined in E, as D is in F. If then we take equal things from equall things, there ſhall remain B in the ſame proportion to G as D to H. Therefore hath G proportion to B, as H hath to D.

Theor. 7. Prop. 7.

Equall magnitudes have to one and the ſame magnitude, one and the ſame proportion, and one and the ſame magnitude, hath to equall magnitudes one and the ſame proportion.

Con-

Conſtruction.

12	12	12	4
A	B	C	E
7	15	15	15
D	G	H	K

The magnitudes A B C are equall, and each of them compared to the magnitude E. Therefore are the proportions of A B C each of them to E equall, and the magnitude D compared to each of the equall magnitudes G H K, the proportions are alſo equall.

Demonſtration.

Foraſmuch as A B C are equall one to another, then is E in each of them equally conteined, as many times in the one, as in the other, and therefore is the proportion alſo equall: and the magnitude D is for the ſelfe ſame reaſon alſo to G as to H or K.

Theor. 8. Prop. 8.

Unequall magnitudes being taken, the greater hath to one and the ſame magnitude a greater proportion then hath the leſſer, and that one and the ſame magnitude, hath to the leſſe a greater proportion then it hath to the greater.

Conſtruction.

18	14	18
A	B	C

The unequall magnitudes are A and B, the greateſt is A, and the leaſt B, and the other magnitude to which thoſe are compared, is C, now is the propoſition of A greater to C, then B is to the ſame C.

Demon-

Demonſtration.

Foraſmuch as A is greater then B, therefore is C alſo found more times in A then in B (or conteined in A) wherefore it followeth, that A hath a greater proportion to C, then B hath to C: and if C be compared to A and B, then hath C a greater proportion to B, then to A, becauſe C cannot ſo many times be conteined in B as it can in A

Theor. 9. Prop. 9.

Magnitudes which have one and the ſame magnitude, one and the ſame proportion, are equall the one to the other. And thoſe magnitudes unto whom one and the ſame magnitude hath one and the ſame proportion, are alſo equall.

Conſtruction.

The magnitudes ABC hath each of them to D one & the ſame proportion, therefore they are equall, and the magnitude G hath the ſame proportion to H that it hath to K or M. Therefore are thoſe magnitudes H K M equall.

8	8	8	3
A	B	C	D
45	20	20	20
G	H	K	M

Demonſtration.

Foraſmuch as A B C hath to D one and the ſame proportion, then is the ſame D even as many times conteined in the one of them as in the other. And thoſe things which do contein another thing, the one, even as many times, as each of his fellowes doth, are by common reaſon accounted equall. Wherefore alſo, are the magnitudes A B C equall: and for the ſame reaſon are H K M equall.

Theor. 10. Prop. 10.

Of magnitudes compared to one and the ſame magnitude, that which hath the greater proportion, is the greater, and that magnitude whereunto one and the ſame magnitude hath the greater proportion, is the leſſe.

Conſtruction.

20 16 6
A B C

The two magnitudes are A and B, the other whereunto they are compared, is C. Now A hath a greater proportion to C then B hath to the ſame C. Therefore is A alſo greater then B, and C hath a greater proportion to B then it hath to A, Therefore is B leſſe then A.

Demonſtration.

The truth of this is evident, *by the eighth propoſition.*

Theor. 11. Prop. 11.

Proportions which are one and the ſelfe ſame, to any one proportion, are alſo the ſelfe ſame the one to the other.

Conſtruction.

4 7
A B
8 14
E F
12 21
M W

The proportion of E to F, and of M to W, are each of them equall to the proportion of A to B. Therefore is E to F, as M to W.

Demonſtration.

Foraſmuch as here two things are each of them equall to another thing, they are alſo equall one to another, *by the firſt common ſentence.*

Theor. 12. Prop. 12.

If there be a number of magnitudes, how many ſoever, proportionall, as one of the antecedents is to one of the conſequents, ſo are all the anteccdents to all the conſequents.

Conſtruction.

Theſe magnitudes each antecedent, hath to his conſequence an equall proportion, that is, G hath proportion to H, like as I hath to K, and L to M, and N to O. Therefore hath the antecedents G I L N, together, alſo ſuch proportion to all the conſequences H K M O together.

3	5
G	H
9	15
I	K
15	25
L	M
12	20
N	O
39	65
GILN	HKMO

Demonſtration.

This is evident *by the firſt propoſition*, which otherwiſe might be thus proved, H is as many times included in G as K in L, and ſo of the reſt, It is manifeſt, that when equall things are added to equall things, that H K M O together, are as many times included in G I L N. Therefore hath G I L N together alſo ſuch proportion to H K M O together, as as each Antecedenr hath to his conſequent.

Theor. 8. Prop. 8.

If the firſt hath to the ſecond the ſame proportion that the third hath to the fourth: and if the third hath to the fourth a greater proportion then the fifth hath to the ſixth: then ſhall the firſt alſo have unto the ſecond a greater proportion, then hath the firſt to the ſixth.

Conſtruction.

7	21	5	15
A	B	C	D
7	28	6	15
Q	R	W	X

The four magnitndes proportioned, are A B C D, whereof A is to B as C to D, and Q hath to R a leſſe proportion then C to D. Therefore hath the ſame Q to R a leſſe proportion alſo, then A to B.

Demonſtration.

Foraſmuch as the proportion of A to B, and C to D, are equall, and that the proportion of Q to R is leſſe then C to D. It followeth *by the eleventh propoſition*, That the ſame is alſo leſſe then A to B.

But if the ſame had a greater proportion then C to D, as hath W to X, for W hath to X a greater proportion then hath C to D. It followeth alſo *by the ſaid eleventh propoſition*, that W to X hath alſo a greater proportion then A hath to B.

Theor. 14. Prop. 14.

In four magnitudes, If the firſt have to the ſecond the ſelfe ſame proportion that the third hath to the fourth, and if the firſt be greater then the third, the ſecond is alſo greater then the fourth, and if it be equall, then equall, if leſſe, then leſſe.

Conſtruction.

8	24	5	15
G	H	I	K
4	16	7	28
L	M	N	O
6	6	6	6
P	Q	R	S

In theſe four magnitudes G H I K, G is the firſt greater then I the third, therefore is H the ſecond alſo greater then K the fourth.

Demon-

Demonſtration.

Foraſmuch as G is to H as I is to K, then is G as many times conteined in H, as I is conteined in K, and ſeeing that G is greater then I, it followeth that H is alſo greater then K. And of the four other magnitudes, L M N O, is L the firſt, leſſe then N the third, therefore is M the ſecond leſſe then O the fourth.

Demonſtration.

Foraſmuch as L is leſſe then N. Then *(by the eighth propoſition)* the proportion of L to M, is leſſe then N to M, and the proportion of L to M, and of N to O, are equall. Therefore is the proportion of N to M, greater then N to O. It followeth *(by the tenth propoſition)* that M is leſſe then O, and by the like reaſon *(by the firſt Demonſtration)* may alſo be proved of the four magnitudes P Q R S, That P is equall to Q, and R equall to S.

Theor. 15. Prop. 15.

Like parts of multiplices, and alſo their multiplices compared together, have one and the ſame proportion.

Conſtruction.

The whole magnitude W doth as many times contein his part Y, as X doth contein his part Z, therefore are the parts in the ſame proportion, one to another, as the whole, that is, Y to Z, as W to X.

12	18
W	X
4	6
Y	Z

Demonſtration.

Foraſmuch as there are as many maguitudes conteined in W, or W doth contein Y as many times, as X doth Z. It followeth (*by the twelfth propoſition*) that Y is to Z, as W to X, and by the ſame reaſon, may be proved, that as Z is to Y, ſo is X to W.

Theor. 16. Prop. 16.

If four magnitudes be proportionall, then alternately, they are alſo proportionall.

Conſtruction.

12	20	9	15
A	B	C	D

Theſe four magnitudes A B C D, are proportionall, that is, A is to B, as C is to D. Therefore hath A alſo proportion to C, like as B hath to D.

Demonſtration.

Foraſmuch as A is to B, as C is to D. Then (*by the fourteenth propoſition*) it is true, that if the antecedents A and C be equall, that then the conſequents B and D, ſhall alſo be equall, whereby may plainly be underſtood, That as many time as the one antecedent, is conteined in the other antecedent, ſo many times alſo is the one conſequent, conteined in the other conſequent: whereby it is evident, (*by the fifth Definition*) that the firſt, as A, hath proportion to the third C, as the ſecond, as B, hath to the fourth D.

Theor. 17. Prop. 17.

If four magnitudes be proportionall, being compoſed, then are the ſame alſo proportionall, being divided.

Conſtruction.

The magnitudes G and H are in proportion (being compoſed) to H, as I and K compoſed, are to K. Therefore is G to H, (divided, or apart) as I to K.

4	6	8	12
G	H	I	K
10		28	
GH		IK	

Demonſtration.

Foraſmuch as G H, hath proportion to H, as I K, hath to K. It followeth, that H is as many times conteined in G H, as K is conteined in I K. Therefore is H ſuch part of GH, as K is of IK: wherefore *(by the fifteenth Propoſition)* is the proportion of H to K, as GH is to I K, and *(by the fifth propoſition).* is G alſo to I, as GH to I K, or H to K. It is then evident *(by the former propoſition)* that G is to H, as I is to K.

Theor. 18. Prop. 18.

If magnitudes divided, be proportionall, then alſo compoſed, they ſhall be proportionall, 10.

Conſtruction

The magnitudes proportionall divided, are K L M N, ſo that K hath proportion to L, as M hath to N. Therefore K L being compoſed, hath ſuch proportion to L, as M N compoſed, hath to N.

6	9	12	18
K	L	M	N
15		30	
KL		MN	

Demonſtration.

Foraſmuch as L is conteined in K, even as N is in M, and L in himſelfe, as N in himſelfe, (that is) each once. It followeth, if to equall things, be added equall things, that L is as many times conteined in KL, as N is in MN. Therefore KL is in the ſame proportion to L, that MN is to N, the like is to be proved, that KL is to K, as MN is to N.

Theor. 19. Prop. 19.

If the whole be to the whole, as the part taken away, is to the part taken away, then ſhall the reſidue be to the reſidue, as the whole is to the whole.

Conſtruction.

14	42
A	W
6	18
Q	R
8	24
S	T

The magnitudes are A and W, the parts taken away, Q and R, the reſidues are, S and T, and from A is taken away the part Q, and from W is taken away the part R, and from A reſt S, and from W reſt T, and Q hath proportion to R, as A to W, therefore hath S alſo to T the ſame proportion.

Demonſtration.

This propoſition is in effect all one with the fifth propoſition, going before, where the Demonſtration is done, ſerving to this alſo.

Theor. 20. Prop. 20.

If there be three magnitudes in one order, and as many other magnitudes in another order, which being taken two and two in each order, are in one and the ſame proportion, and if of equality in the firſt order, the firſt be greater then the third, Then in the ſecond order, the firſt ſhall be alſo greater then the third, And if it be equall, then equall, and if it be leſſe, then leſſe.

Conſtruction.

The three magnitudes in one order, are A B C, and three in another order, D E F, and A hath proportion to B, as D hath to E, and B to C, as E to F, And A is greater then C, therfore is D alſo greater then F.

16	6	4	24	9	6
A	B	C	D	E	F
3	5	12	6	10	24
G	H	I	K	L	M
5	12	5	15	36	15
N	O	P	Q	R	S

Demonſtration.

Foraſmuch as *(by the quality of proportion)* A doth as many times contein B, as D doth E, and that the ſame B doth as often contein C, as E doth F. It followeth, that A. doth contein C, as many times as D doth F. Therefore is A proportioned to C, like as D to F. It is evident, if A be greater then C, then D is alſo greater then F. And if A be leſſer, or equall to C, then by the ſame reaſon ſhall D alſo be leſſe, or equall to F. Behold the other magnitudes G H I K L M, and N O P Q R S, for the better underſtanding of the propoſition.

Theor. 21. Prop. 21.

If ſix magnitudes be proportioned without order, that is, that the firſt be to the ſecond, as the fifth is to the ſixth, and the fourth be to the fifth, as the ſecond is to the third, Then (like as in the former propoſition) ſhall the firſt be to the third, as the fourth is to the ſixth.

Conſtruction.

18 12 8 9 6 4.
A B C D E F

The ſix magnitudes proportioned, without order, are, A B C D E F, and as A is to B, ſo is E to F, and as D is to E, ſo alſo is B to C. And A is greater then C, therefore is D alſo greater then F.

Demonſtration.

Foraſmuch as C is leſſe then A, then is the proportion of C to B, leſſe then A to B, and becauſe the proportion of B to C, and of D to E, is equall, then is C as many times in B, as E in D. Therefore is C alſo to B, as E to D. The proportion then of E to D, is leſſe then E to F. For E to F is equall, to that of A to B. It followeth, (*by the eighth propoſition*) that D is greater then F, with theſe, and the like reaſon, is alſo proved, That if the firſt be leſſer, or equall to the third, that then the fourth is alſo leſſer, or equall to the ſixth.

Theor. 22. Prop. 22.

If there be ſix magnitudes, in proportion one to another, (according to the contents of the twentieth propoſition) *Then hath the firſt ſuch proportion to the third, as the fourth hath to the ſixth.*

Con-

Conſtruction.

The ſix magnitudes in proportion, one to another, *according to the* 20 *propoſition*, are G H I K L M, wherefore G is to I, as K is to M.

8	6	12	9	16	12	24	18
G	H	I	P	K	L	M	N

Demonſtration.

This is ſhewed *in the* 20 *Propoſition* which otherwiſe might be thus proved, adde beſides I one magnitude more as P, ſo that as G is to H, ſo let I be to P, and by M adde alſo a magnitude as N, ſo that as K is to L, let M be to N, then are thoſe four magnitudes G H I P, proportionall, as alſo K L M N, and *(by the 16 propoſition)* is G to I, as H is to P, and K to M, as L to N, and G H I P in the ſame order, and proportion, that K L M N is in, wherefore the proportion of G to I, is as K to M.

Theor. 23. Prop. 23.

If there be three magnitudes in one order, and as many in another order, which being taken two and two in each order are in one and the ſame proportion, and if alſo their proportion be perturbate, then of equality, they ſhall be in one and the ſame proportion.

Conſtruction

The ſix magnitudes N O P and Q R S are in proportion *according to the contents of the 21 propoſition*, that is as N is to O, ſo is R to S, and as Q to R, ſo O to P, therefore is N in proportion to P, as Q to S.

18	12	8.	9	6	4.	$2\frac{2}{3}$
N	O	P.	Q	R	S.	T

Demonſtration.

Let there yet one magnitude more, come in order, namely T, ſo that S is to T, like as O is to P, thē is Q to R, as S is to T, then are thoſe four magnitudes Q R S T in proportion, therefore *(by the 16 Propoſition)* is Q to S, as R is to T, and R is to T, *by the former* like as N is to P, it is then evident, that Q is to S, as N is to P. And by this manner may the 21 Propoſition alſo be demonſtrated.

Theor. 24. Prop. 24.

This propoſition is in effect of the ſame contents of the former ſecond of this book) and therefore needeth not to be repeated, onely there is ſpoken of multiplices, and here of magnitudes.

Theor. 25. Prop. 25.

If four magnitudes be in proportion, then are the greateſt and leaſt, (or the firſt and fourth) greater than the other two.

Conſtruction.

12	8	9	6
A	B	C	D
9	6		
C	D		
3	2		
E	F		

The four magnitudes in proportion are A B C D, and A is greateſt, then ſhall D be leaſt, therefore is A and D together greater then B and C together.

Demonſtration.

Subſtract C from A reſt E, and alſo D from B reſt F, now *(by the 19 Propoſition)* E hath proportion to F, like as A hath to B, wherefore ſeeing that A is greater then B, then is E alſo greater then F. Now D C and F together, are equall to B and C together, and C D and E together, are equall to A and D together, and greater then B and C together, becauſe E is greater then F. It is manifeſt that A and D together are greater then B and C together.

The end of the fifth Book.

EUCLIDES ELEMENTS.

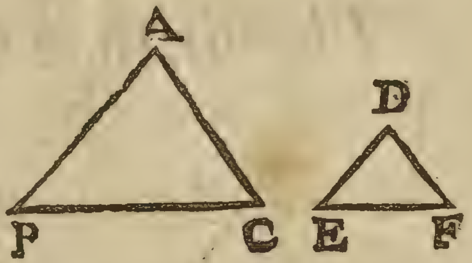
The Sixth Book.

THE ARGUMENT.

THis ſixt Book is for Vſe and Practiſe moſt excellent, for in it are taught the proportion of one figure to another, and of their ſides the one to the other and of the ſides and angles of one to the ſides and angles of the other. Moreover, it teacheth the deſcription of figures like to figures given, and marvellous applications of figures to lines, evenly, or with decreaſe, or exceſſe: with many other *Theoremes*, not onely of the proportions of right lined figures, but alſo of *Sectors* of *Circles* with their Angles. On the *Theoremes* and *Problemes* of this *Book*, depend for the moſt part the *Compoſition* of al *Inſtruments* for meaſuring of *Heights*, *Depths* and *Diſtances*, and alſo the reaſon of the uſe of the ſame *Inſtruments*, and alſo of others for the raiſing up, and moving huge things incredible to the *Ignorant*. The *Invention* of which are of inſpeakable *Profit*, beſides the ineſtimable *Pleaſure* which is in them.

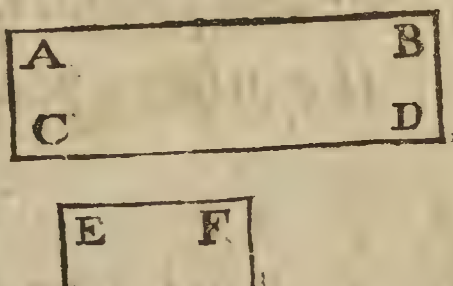
DEFINITIONS.

1 *Like rectline figures are ſuch, whoſe angles are equall the one to the other, and whoſe ſides about the equall angles are proportionall.*



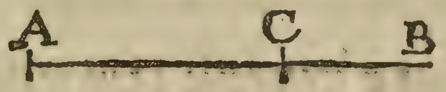
There are in this Definition two conditions required. 1 That the angles of each Triangle be equall one to the other, as here A to D, B to E, and C to F. 2 That the ſides about the equall angles be proportionall, as B A is to A C, ſo is E D to D F.

2 *Reciprocall figures are thoſe when the terms of proportion, are both antecedents and conſequents in either figure.*



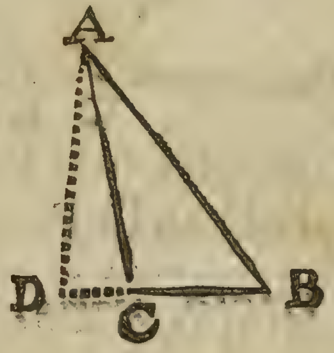
As if you have two Parallelograms A B C D, and E F G H, if the ſide A B to the ſide E F, an antecedent of the firſt figure to a conſequent of the ſecond figure have mutually the ſame proportion which the ſide E G hath to the ſide A C an antecedent of the ſecond figure to a conſequent of the firſt figure, then are thoſe two figures reciprocall.

3 *A right line is ſaid to be divided by an extream and mean proportion, when the whole is to the greater part, as the greater part is to the leſſe.*



As if the line A B be ſo divided in the point C, that the whole line A B hath the ſame proportion to the greater part A C, which the ſame greater part A C hath to the leſſer part thereof C B, then is the line A B divided by extream and mean proportion.

4 *The altitude of a figure is a perpendicular line drawn from the top to the baſe.*



As the line A D is the altitude of the Triangle A B C.

5 *A Proportion is ſaid to be made of two proportions or more, when the quantities of the proportions multiplyed the one into the other, produce another quantity.*

Suppoſe three quantities A, B, C, ſo that let A have to B *ſeſquialtera* proportion, namely 6 to 4, and let B to C have *ſeſquitertia* proportion, namely 4 to 3. Now the proportion of A to C, the firſt to the third, is made of the proportion of A to B, and of the proportion of B to C added together.

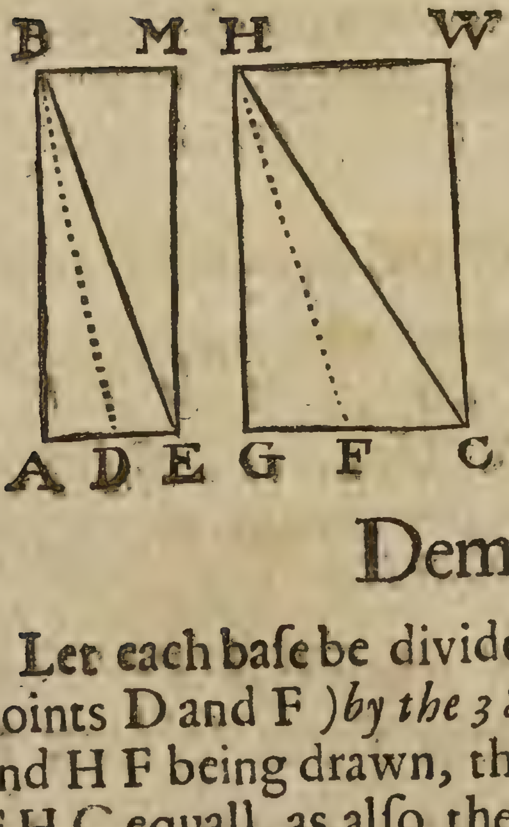
A	6
B	4
C	3

EUCLIDES ELEMENTS:

Theor 1. Prop. 1.

Triangles and Parallelograms which are under one and the ſelf ſame height, are in proportion as the baſe of the one is to the baſe of the other.

Conſtruction.



Theſe Triangles A B E and G H C have equall height, and ſtand between two parallel lines A C and B W, therefore is the Triangle A B E in proportion to the Triangle GHC as the baſe A E is to the baſe G C.

Demonſtration.

Let each baſe be divided into two equall parts in the points D and F)*by the 38 of the firſt book*) the lines B D and H F being drawn, then are the Triangles G H F and F H C equall, as alſo the Triangles A B D and D B E, wherefore (*by the 17 of the 5 book*) may be underſtood, that
the

the Triangles are in proportion one to another, that is A B E to D B E, as G H C to F H C, and in like manner, the proportion of the baſe A E is to D E, as G C is to F C, and *(by the 16 of th e fifth book)* are each of theſe four magnitudes proportioned, the firſt to the third, as the ſecond to the fourth, it followeth *(by the 11 of the 5 book)* that the Triangle A B E is in proportion to the Triangle G H C, as the baſe A E is to the baſe G C according to the intent of this Propoſition.

Secondly, the parallelograms A M and G W have equall height, therefore are they in proportion the one to the other, as their baſes.

Demonſtration.

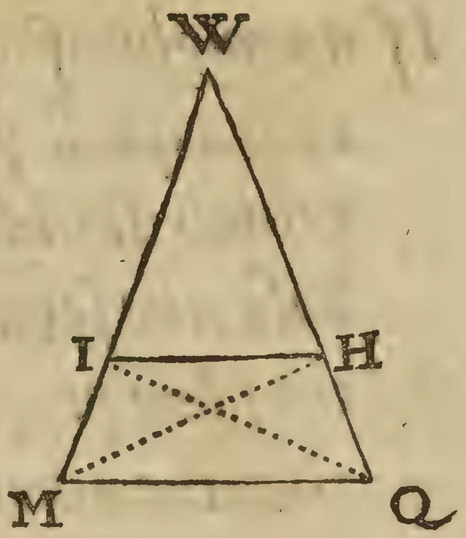
Foraſmuch as *(by the 41 of the firſt book)* the parallelograms are double to the Triangles that have one and the ſame baſes, and do ſtand between two parallel lines, or have the ſelf-ſame height, it is then evident, that parallelograms alſo are in proportion one to another, as their baſes.

Theor. 2. Prop. 2.

If to any one ſide of a Triangle be drawn a parallel right line, it ſhall cut the ſides of the ſame Triangle proportionally, and if the ſides of a Triangle be cut proportionally, a right line drawn from ſection to ſection, is a parallel to the other ſide of the Triangle.

Con-

Conſtruction.



In this Triangle M Q W is drawn the line I H parallel to M Q, therefore the ſame line I H doth divide the ſides M W and Q W proportionally, or in equall proportion.

Demonſtration.

Let there be drawn the lines I Q and H M, then *(by the 37 of the firſt book)* are the two Triangles I H M and H I Q, (ſtanding between the two parallel lines I H and M Q) equal, and *(by the former Propoſition)* the Triangle W I H hath ſuch proportion to the Triangle H I Q as the baſe W H, hath to the baſe H Q, and the ſame Triangle W I H to I H M, as W I to I M; it followeth that W H is to H Q, as W I to I M *(by the 11 of the fift book.)*

Secondly, if the line I H do divide the ſides W M and W Q in eqnall proportion in the points I and H, then is the ſame line parallel to M Q.

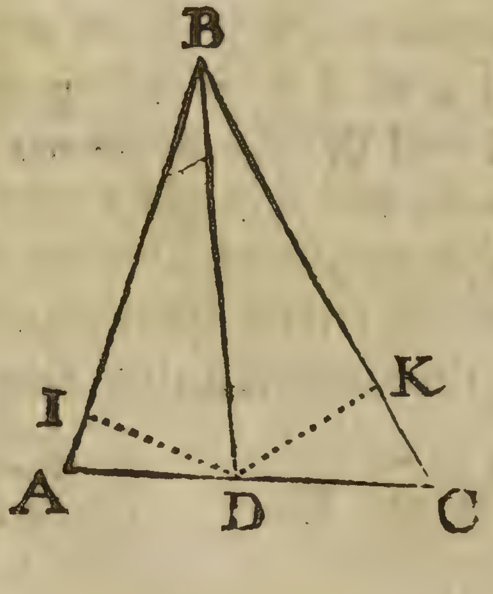
Demonſtration.

Foraſmuch as the two Triangles H I Q and I M H have equall proportion to the Triangle W I H, which proportion is as H Q to H W, or M I to I W) *by the former propoſition)* it followeth *(by the 9 of the fift book)* that the Triangles H I Q and I H M are equall, and ſeeing they ſtand between two parallel lines, and ſtand upon one baſe *(by the 39 of the firſt book)* it is then manifeſt that the line I H is parallel to the line M Q.

Theor. 3. Prop. 3.

If an angle of a Triangle be divided into two equall parts, and if the right line which divideth the angle, divide alſo the baſe, the ſegments of the baſe ſhall be in the ſame proportion the one to the other, that the other ſides of the ſaid Triangle are, and if the ſegments of the baſe be in the ſame proportion that the other ſides of the ſaid Triangle are: a right line drawn from the top of the triangle to the ſection, ſhall divide the angle of the Triangle into two equall parts.

Conſtruction



In this Triangle A B C is the angle B divided into two equall parts, with the right line B D, which doth alſo cut the ſides A C in D, then I ſay that A D hath proportion to D C, as the ſide A B hath to the ſide B C.

De-

Demonſtration.

Let there be drawn from the point D two perpendiculars, as D I upon A B, and D K upon B C, foraſmuch as the angles D B I and D I B are equall to the angles D B K and D K B, then are the angles B D I and B D K alſo equall *(by the 32 of the firſt book)* and *(by the 26 of the firſt book)* in regard of the common line B D are the two perpendiculars D I and D K alſo equall, wherefore, *by the firſt propoſition of this book*, hath AB ſuch proportion to C B, as the Triangle A B D to the Triangle D B C, and A D to D C, as the ſame Triangles hath, that is, as A B D to D B C, it followeth *(by the 11 of the fift book)* that A D hath ſuch proportion to D C, as hath A B to B C.

Secondly, if A D be to D C, as A B to B C, then is the angle B divided into two equall parts, with the right line B D.

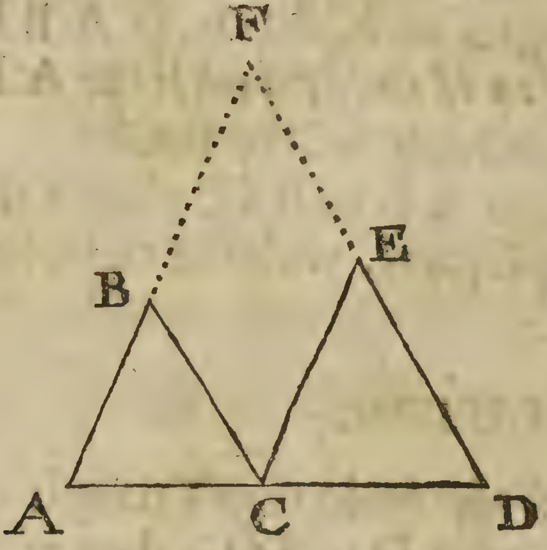
Demonſtration.

Foraſmuch as) *by the firſt of this book)* the Triangle A B D is to the Triangle B D C, as A D to D C, that is as A B to B C, it followeth (as may be underſtood *by the 1 Propoſition*) that the two Triangles perpendiculars, as D I and D K are equall, ſeeing then that *(by the 47 of the firſt book)* the two ſquares of D I and I B are equall to the ſquares of D B, as alſo the ſquares of D K and K B, it followeth that I B is equall to K B, and *(by the 4 and 8 of the firſt book)* the angle I B D is equall to the angle K B D, then doth the line B D divide the angle B in two equall parts.

Theor. 4. Prop. 4.

In equiangled Triangles, the ſides which contein the equall angles are proportional, and the ſides which are ſubtended under the equall angles, are of like proportion.

Conſtruction.



The Triangles A B C and C E D are equiangled (that is) B A C, B C A and C B A are equall to E C D, E D C and D E C, therefore are the ſides proportionall, that is as A C to C D, ſo C B to D E, and A B to C E.

Demonſtration.

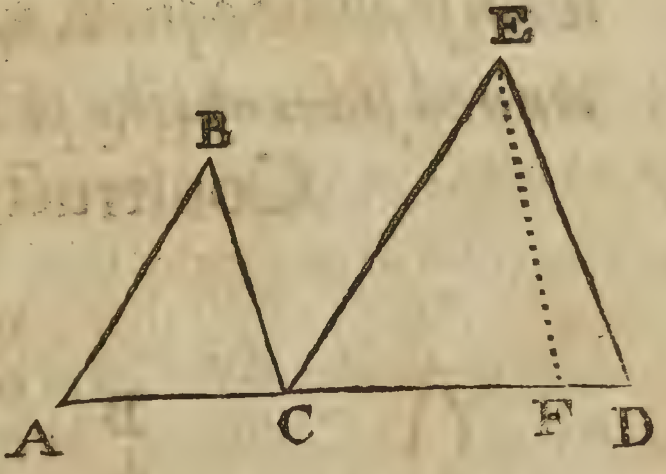
Let the baſes of both the Triangles, as A C and C D come together in one right line, as A C D, and extend the ſides A B and D E, til they concurre together in F, foraſmuch as the angles B A C and E C D are equall, as alſo B C A and E D C, then the ſides A B and C E *(by the 28 of the firſt book)* are parallel, as alſo the ſides B C and E D, and the ſame ſides ſo extended, are alſo parallels, then is the figure C B F E a parallelogram, and the ſides oppoſite are equall *(by the 34 of the firſt book)* that is B F equall to C E, and E F to C B, foraſmuch then as the line C B (in the Triangle A F D is parallel to D F, it followeth *(by the ſecond Propoſition)* that as A B is to B F or

C E (being equall) ſo is A C to C D, and ſeeing that C E is parallel to A F, then is A C to C D, as E F (being equall to C B) is to D E, and as D C is to C A, ſo alſo B F (being equall to C E) is to A B, ſo that this propoſition is found true in all his working.

Theor. 5. Prop. 5.

If two Triangles have their ſides proportionall, the Triangles are equiangled, and thoſe angles in them are equall, under which are ſubtended ſides of equall proportion.

Conſtruction.



In theſe two Triangles A B C and C E D are the ſides proportionall, namely, A B is to B C, ſo alſo E C to E D, and as C A to B C, ſo alſo D C to E D, therefore are the angles included with proportionall ſides alſo equall the one to the other, namely E C D, E D C, and C E D equall to B A C, B C A and A B C.

Demonſtration.

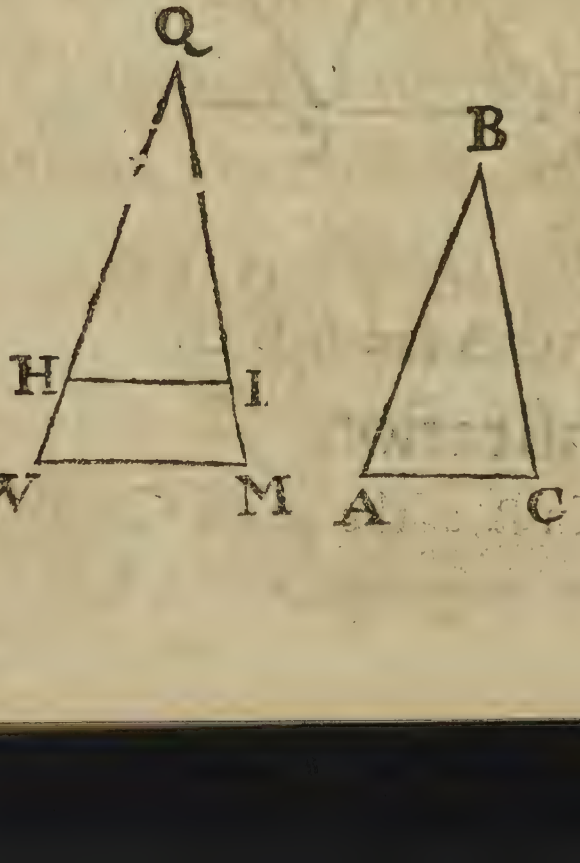
If the angle E be leſſe then B, as the angle C E F, the lines E C and E F do include a leſſe angle then is C E D, there-

D, therefore is the baſe alſo ſhorter *(by the 24 of the firſt book)* and then ſhall the ſides C E and E D have alſo a greater proportion to the baſe C F then to C D *(by the 8 of the fift book)* the like is to be underſtood if the angle E were greater, then ſhould the baſe be greater, and the proportion of the ſides leſſe, but the proportion cannot be either greater or leſſer, for the angles *(by this Propoſition)* are equall.

Theor. 6. Prop. 6.

If there be two Triangles, whereof the one hath one angle equall to one angle of the other, and the ſides including the equall angles, be proportionall, the Triangles ſhall be equiangled, and thoſe angles in them ſhall be equall, under which are ſubtended ſides of like proportion.

Conſtruction.



In theſe two Triangles W Q M and A B C are the angles Q and B equall, and the ſides Q W is in proportion to Q M as AB to BC, therefore is the angle Q W M equal to the angle B A C and Q M W equal to B C A.

Demonſtration.

Take *(by the third of the firſt*

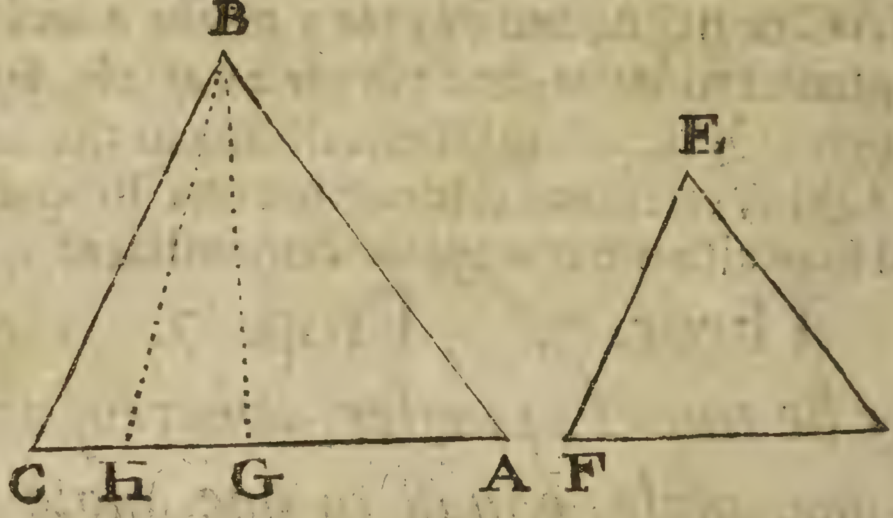
firſt book) the length of the ſide AB cut the ſame off from the point Q upon the ſide QW, which comes to H, do the ſame with B C upon the ſide QM, which comes to I, and draw the line H I, then) *by the 4 of the 1 book*) is the line H I equall to the ſide A C, and the Triangle Q H I equall to the Triangle A B C, and (*by the 4 & 8 of the firſt book*) the angles of the one triangle equall to the angles of the other triangle, namely the angles QHI, & QIH equal to the angles BAC & BCA, and (*by the 2 of the 6 book*) is the line H I parallel to W M, and (*by the 29 of the firſt book*) are the angles QHI, and QIH equall alſo to the angles QWM, & QMW; it is then evident that this Propoſition is true, and may alſo other ways be demonſtrated.

Theor. 7. Prop. 7.

If there be two Triangles, whereof the one hath one angle, equall to one angle of the other, and the ſides which include the other angles, be proportionall, and if either of the other angles remaining be either leſſe or not leſſe, then a right angle: then ſhall the Triangles be equiangled, and thoſe angles in them ſhall be equall, which are conteined nuder the ſides proportionall.

Conſtruction.

In theſe two Triangles A B C and D EF are the angles A and D equall, and the ſides which do include the angles B & E are proportional, that is as A B to B C, ſo is D E to E F, the angles which are proportioned one to another, as the ſides one to another, (or which are ſubtended with ſides proportionall) are the angles C as F, and B as E, if then the angles C and E, each of them be leſſe, or greater then a right angle(like as in this and the following, of both ſorts are to be ſeen) they are notwithſtanding equall, and the Triangles equiangled.

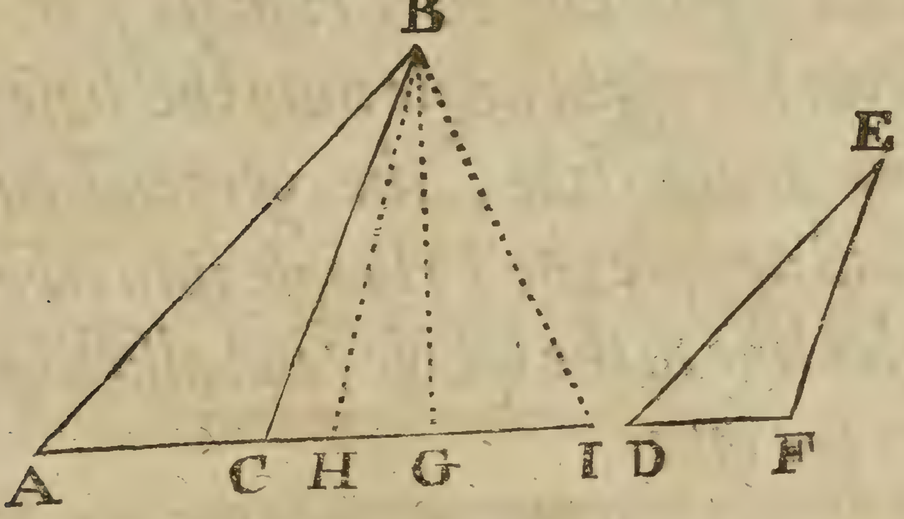


Demonſtration.

Let there be drawn the perpendicular line B G, now if the angles C and F be equall, then are the angles B and E alſo equall *(by the 32 of the firſt book)* becauſe the angles A and D are ſuppoſed equall.

Now, ſuppoſe (if it were poſſible that the angle C (in this ſharp angle Triangle) ſhould be leſſe then the angle F, the angle B ſhall then be greater then the angle E, let the line B H be drawn, and take the angle A B H to be equall to the angle E, that the angle B H A may be alſo equall to E F D, that line B H (becauſe the angle B H A is leſſe then a right angle) ſhall fall between B G and B C (as may be underſtood *by the 13 of the ſecond book)* and the angle B H A ſhall be equall to E F D, and greater then B C H, wherefore *(by the 18 of the firſt book)* is B C longer then B H, it followeth *(by the 8 of the 5 book)* that the proportion of A B to B C is leſſe then it is to B H, and A B to B C, then D E to E F, and ſo the proportion of A B to B H (being unequall) muſt be equall to the ſame, which is impoſſible, it is then evident, that the angle A B H cannot be eqnall to D E F, but A B C, and the angles C and F are then equall, and the Triangles equiangled.

But if the Triangle have an obtuſe angle, as in this, the angle C greater then the obtuſe angle F, and therefore the



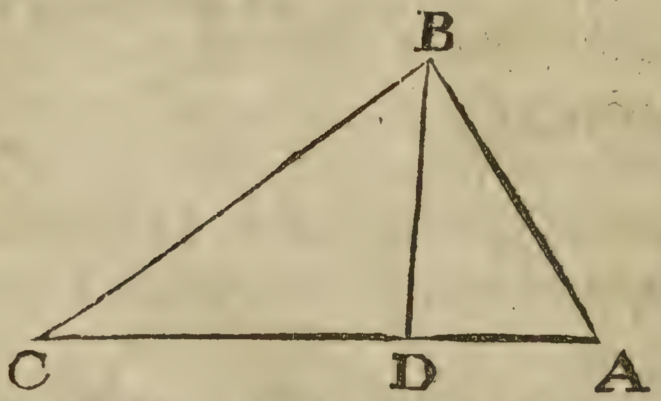
angle B leſſe then E, and A B H made equall to D E F, then ſhall B H, by the aforeſaid reaſons, be ſhorter then B C, and *(by the 12 of the ſecond book)* ſhall fall without the Triangle, and that between C and G, the proportion then of A B, (as is aforeſaid) is leſſe to B C then to B H, and the angles A B C and A C B equall to D E F and D F E, as well in the obtuſe, as in the acute angled Triangles.

And whereas in the propoſition it is ſaid, whether the angles be greater or leſſe then a right angle, that is meant, becauſe that not on that ſide of the perpendicular B G, but on the other ſide there might another line be drawn equall to B C, as the line B I, ſo that in both ſorts of theſe Triangles, A B hath ſuch proportion to B I, as the ſame A B hath to B C, and yet are not thoſe Triangles A B I and D E F equiangled, but the Triangle B I A is in the acute angled Triangle as much greater then right, as E F D is leſſe then right, and in the obtuſe angled Triangle, the ſame B I A is ſo much leſſe then right, as E F D is greater, the which might divers and ſundry ways be proved.

Theor. 8. Prop. 8.

If in a right angled Triangle, be drawn from the right angle unto the baſe, a perpendicular line, the perpendicular line ſhall divide the Triangle into two Triangles like to the whole, and alſo like the one to the other.

Conſtruction.



The Triangle ABC is right angled in B, the ſide ſubtending the right angle is A C, upon the ſide A C from the point B is drawn a line perpendicular to the line A C, as B D, which divideth the Triangle A B C into two Triangles A D B and B D C, each of them like to the whole Triangle A B C, and alſo each of them like one another, that is equiangled one to another,

De-

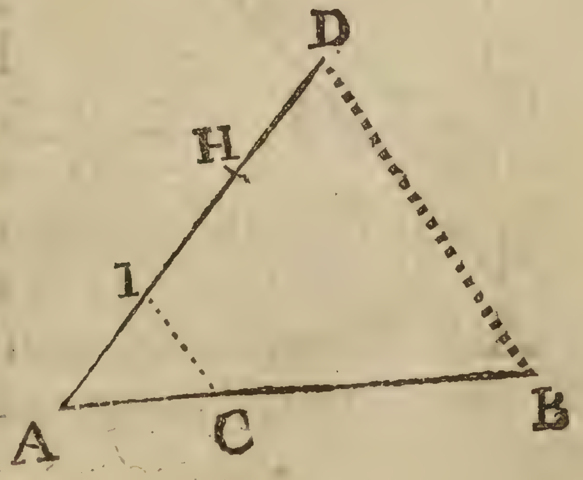
Demonſtration.

Foraſmuch as the two angles A B D and A D B (by reaſon of their common angle A) are equall to the two angles A C B and A B C, which are alſo equall to the two angles B C D and B D C, it followeth *(by the 32 of the firſt book)* that the third angle in thoſe three Triangles as B A D, C A B, and C B D are alſo equall to the three Triangles A B D, B D C and A B C being equiangled, whereby it is manifeſt, that the ſame perpendicular ſtandeth as a mean proportionall between the two ſegments of the baſe, as alſo each ſide of the Triangle A B C is a mean proportionall between the whole baſe and that part of the baſe, to which the ſide is joyned (or doth touch) ſo that A D is to D B, as D B is to D C, and A D is to A B, as A B is to A C, and alſo D C is to B C, as B C is to A C, which may better be perceived *(by the 4 Propoſition of this book.)*

Probl. 1. Prop. 9.

A right line being given, to cut off from the ſame any part required.

Conſtruction.



The right line given is A B, from which it is required to cut off a certain part, namely, a third part from the point A towards B, draw from the point A another right line by chance, as A I which line A I taken three times, comes to D, as A I, I H, and H D, and from D, draw a right line to B, and then from the point I draw a right line parallel to D B, as the line I C, then is A C a third part of the line A B.

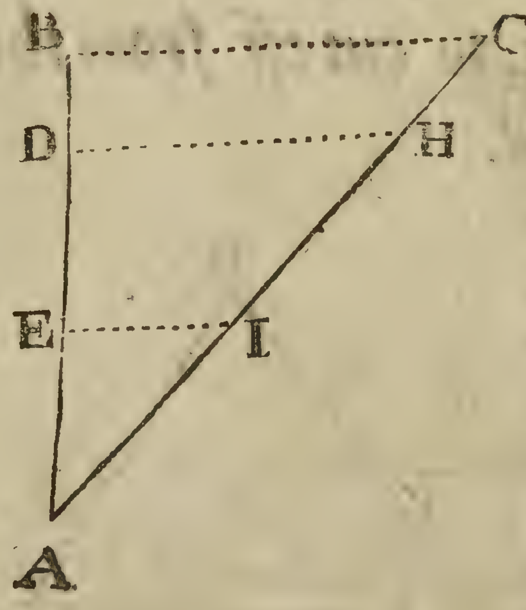
Demonſtration.

By the ſecond propoſition of this book, B C hath proportion to A C, as D I hath to A I, and *(by the 18 of the 5 book)* C B and C A together, and D I and I A together, as A B to A C, ſo A D to A I, it followeth, that A C is ſuch a part of A B, as A I is of A D, and A I is one third part of A D, therefore is A C alſo one third part of A B.

Probl. 2. Prop. 10.

To divide a right line given, not divided, like unto a right line given being divided.

Conſtruction.



The divided right line given is A B, the parts A E, E D and D B, and the line given, which ſhould be divided according to the proportion of the divided line A B is A C, ſet the one end of the line A C to the point A, and draw a line from A equall to A C, the which ſhall make with the line A B an angle by chance, as the angle B A C, then draw a right line from B to C, and then from the points D and E draw lines parallel to B C, as the lines D H and E I, which doe divide the line A C in ſuch proportion, according to the parts of the line given, namely, as B D ſo H C, and as D E ſo H I, and as E A, ſo A I.

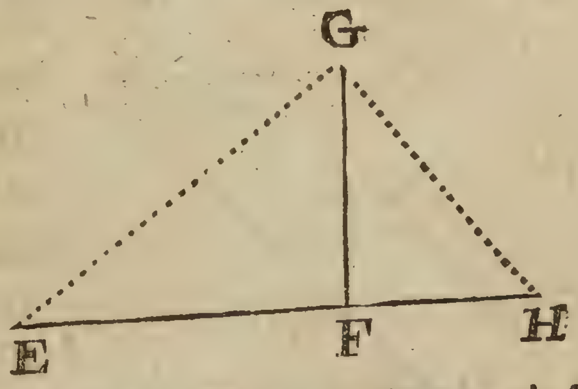
Demonſtration.

By the ſecond of the ſixt book A I is to I C, as A E to E B, and A I to I H, as A E to E D, and A H to H C, as A D to D B, and *(by the 18 of the fift book)* A I and I H, and A E and E D added, hath A H alſo ſuch proportion to I H, as A D to E D; it followeth *(by the 11 of the fift book)* that I H is to H C, as E D to D B, and ſo is A I to A E, it is then evident, that the given line A C is divided according to the proportion of the divided line A B, and his parts, which *(by the 5 of the 5 book)* and otherwiſe alſo might be proved.

Probl. 3. Prop. 11.

Unto two right lines given, to finde a third in proportion to them.

Conſtruction.



The two right lines given are E F and F G, it is required to finde a right line, which ſhall have ſuch proportion to the line F G, as F G hath to E F, ſet the lines E F, and F G together in ſuch ſort, that at the point F they make together a right angle, at their uttermoſt points or ends *(by the 11 of the* *firſt book)* as in F, and produce E F to H, and draw the line F G, and) *by the 23 of the firſt book)* draw a line from the point G, upon the line F G make an angle equal to the angle FEG, which line is GH cutting the produced line FH in the point H, then is the line F H the third line proportionall required.

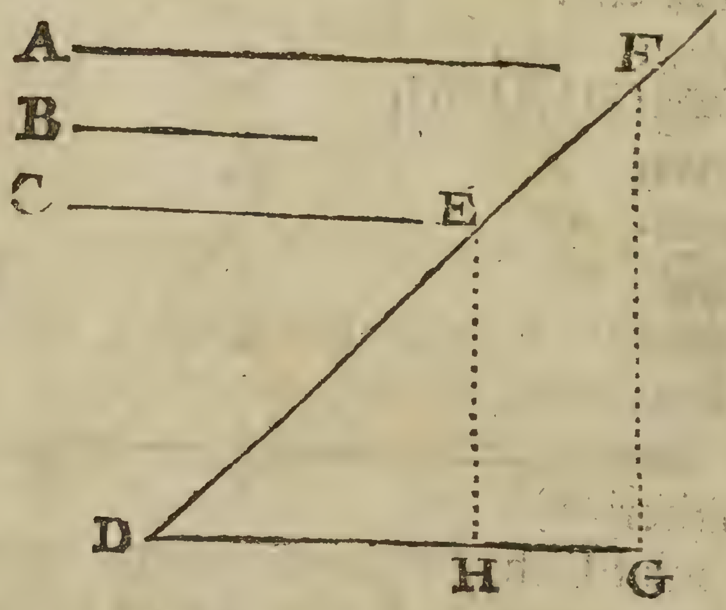
Demonſtration.

For as much as the two angles G F E and F E G are equall to H F G and F G H, it followeth *(by the 32 of the 1 Book)* that the angles F G E and F H G, are alſo equall, and *(by the 4 of this Book)* are the ſides of the triangles H F G and G F E, which are ſubtended under equall angles proportionall, that is, as F H to F G, ſo the ſame F G to F E, according to the propoſition.

Probl. 4. Prop. 12.

Unto three right lines given, to find a fourth in proportion with them.

Conſtruction



The three given lines, are A B C, it is required to finde a right line, which ſhall have proportion to C like as B hath to A, ſet the two right lines A and B together in one right line, as D F, ſo that D E be equall to A, and E F equall to B, then draw a right line from D equall to C, which in the point D, with the line D F, ſhall make an angle by chance, which line is D H. Then from H to E draw the line H E, and another line from F, parallel to H E, and prolong D H (til it interſect the line parallel to H E: namely, the line F G) in the point G, then is the line H G a fourth proportionall line required, that as D E to EF, ſo D H to H G.

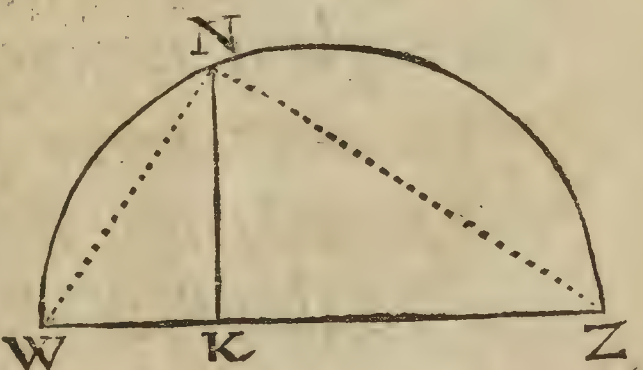
Demonſtration.

By the ſecond propoſition of this book, is H G to H D, like as E F to E D, which being equall to the given lines C, B A, it followeth that H G is to C, as B is to A.

Probl. 5. Prop. 13.

Unto two right lines given, to finde out a mean proportionall.

Conſtruction.



The given right lines are W X and K Z, ſet them together in one right line, in the point K, and upon the line ſo ſet together, deſcribe a half Circle, as the half Circle W Z X, and upon the line W Z from the point K raiſe a perpendicular line to the circumference, in the point X, then is the line K X the mean proportionall line between W K and K Z.

De-

Demonſtration.

Let the be drawn the lines X W and X Z, and foraſmuch as *(by the 31 of the third book)* the Triangle W X Z in X, then *(by the 8 of this)* the Triangles W K X, and X K Z are equiangled, and *(by the 4 of this book)* the ſides which have equall angles ſubtended are proportionall, namely, as W K to K X, ſo the ſame K X to K Z, it is then manifeſt, that the right line K X is the mean proportionall between W K and K Z.

Theor. 9. Prop. 14.

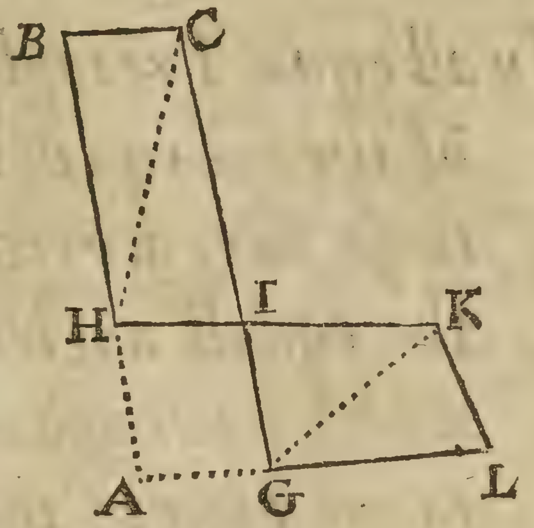
In equall parallelograms, which have one angle of the one equall to one angle of the other, the ſides ſhall be reciprocall, namely thoſe ſides which contein the equall angles, and if parallelograms which have one angle of the one, equall to one angle of the other, have alſo their ſides reciprocall, namely thoſe which contein the equall angles, they ſhall alſo be equall.

Conſtruction.

Theſe parallelograms I B and I L, are equall, and have the angles C I H, and K I G, equall, Therefore are the ſides reciprocally proportionall, namely as C I to I G, ſo K I to I H.

De-

Demonſtration.



Let the ſides H I and I K come togetherin one right line in the point I, then ſhall I C and I G (by reaſon of their equall angles) alſo make one right line(as may be underſtood *by the 13, 14 and 15 propoſitions of the firſt book)* then produce the ſides L G and B H, till they concurre in the point A, making the parallelogram A I, foraſmuch then, as the parallelogram I L is to the parallelogram I A, as I B is to the ſame I A, and that *(by the firſt propoſition of this book)* is C I to I G, and K I to I H, in one & the ſame proportion, it followeth *(by the 11 of the 5 book)* and *(by the ſecond Definition of this book)* that the ſides of the parallelograms are reciprocally proportionall.

Secondly, if the angles be equall, and the ſides reciprocally be proportionall, then are thoſe parallelograms equall.

Demonſtration.

Foraſmuch as C I is to I G, as K I is to I H, that is *(by the firſt of this book)* as the parallelogram I B is to I A, and I L to the ſame I A, it followeth *(by the 9 of the 5 book)* that the parallelograms I B and I L are equall.

Theor. 10. Prop. 15.

In all eqnal Triangles which have one angle of the one equall to one angle of the other, thoſe ſides are reciprocall, which include the equall angles, and thoſe Triangles, which having one angle of the one equall to one angle of the other, have alſo their ſides which doe include the equall angles reciprocall, are alſo equall.

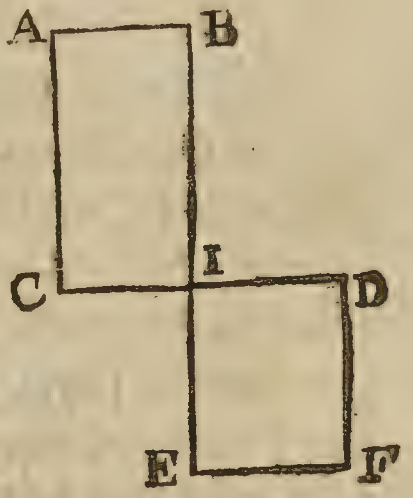
Demonſtration.

This propoſition ſpeaketh of Triangles, like as the former of Parallelograms, wherefore, foraſmuch as the Triangles I H C and I K G, in the former diagram, are the half of the parallelograms I B and I L *(by the 34 of the firſt book)* and that the ſides which doe include the equall angles are the ſame, it followeth *(by the 15 of the 5 book)* that the Triangles have the ſame proportion the one to the other, that the parallelograms have the one to the other, it is then evident that the Propoſition is true.

Theor. 11. Prop. 16.

If there be four right lines in proportion, the right angle figure made of the extreams, is equall to the right angle figure which is conteined under the means, and if the right angle figure conteined under the extreams, be equall to the right angle figure which is conteined under the means, then are thoſe four lines in proportion.

Conſtruction.



The four right lines C I, I D, E I and I B are proportionall, namely, as C I to I D, ſo E I to I B, therefore the right angle figure which is made, or included with the firſt and the fourth, namely, with I C and I B is equall to the right angle figure included of the two means, namely, of I D and I E, as I B A C equall to I D F E.

Demonſtration.

Foraſmuch as the right angle figures are reciprocally proportioned, therefore are they equall, (*by the 14 of this book.*) Secondly, if the right angle figures I B A C and I D F E be equal, then are they reciprocally proportioned, *by the aforeſaid 14 of this book*: it is then evident, that the four lines C I, I D, E F and I E are proportioned.

Theor. 12. Prop. 17.

If there be three right lines in proportion, the right angle figure that is included of or under the extreams, is equall to the ſquare that is made of the mean, and if the right angle figure which is made of the extreams, be equall to the ſquare of the mean, then are thoſe three right lines proportionall.

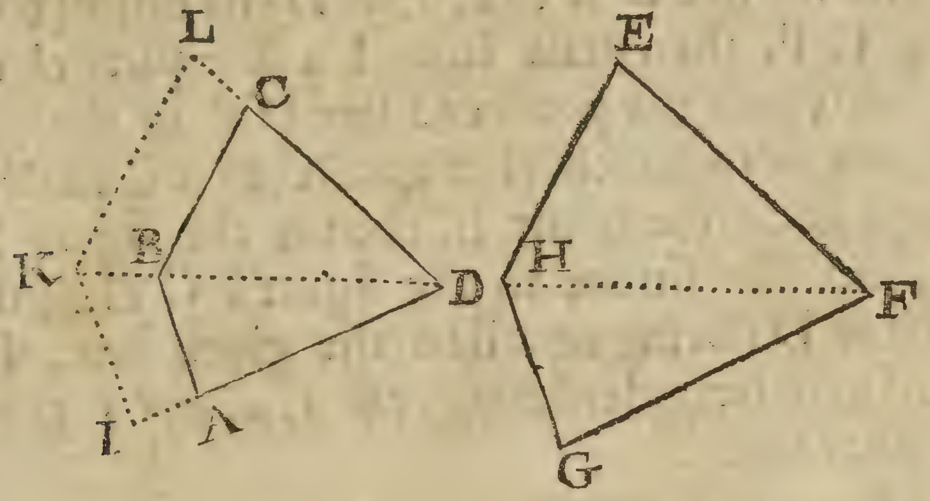
The Demonſtration of this Propoſition is in effect all one with the former, the difference is, that here is ſpoken of three lines, and there of four, which if we here take the mean twice (whereof commeth the ſquare) then is the work all one with the former propoſition, to bring four lines proportioned, where the two means are equall.

Probl. 6. Prop. 18.

Upon a right line given, to deſcribe a right line figure like, and in like ſort ſituate, to a right line figure given.

Conſtruction.

The right line given is G F, the right line figure is A B C D, B C D, divide the ſame figure into two Triangles (which may be done in all right line figures, of how many ſides ſoever, & the number of triangles wil be leſſe by two then the number of ſides) which are D B A & D B C, then *(by the 23 of the firſt book)* make upon the line G F an angle



in the point G, equall to the angle B A D, and in F make another angle equall to B D A, the lines which make theſe angles are G H and F H, which concurre in the point H, make alſo upon the line H F two angles, as in H an angle equall to C B D, and in F, another equall to C D B, the lines which make this angle concurre in the point E, then is the figure G H E F deſcribed upon the line G F, and in like ſort ſituated unto the figure A B C D.

Demonſtration.

Foraſmuch as the angles H G F and H F G are equall to the angles B A D and B D A *(by the 32 of the firſt book)* the angles G H F and A B D, are alſo equall, and *(by the fourth of this book)* are the ſides including equall angles proportioned, that is G F to G H, & H F to A D, as A D to B A and B D, the Triangles G H F and A B D, then are alike ſituate, and by the ſame reaſons are alſo proved, that the Triangles H E F and B C D are alike ſituate, and therefore the whole right line figure G H E F, alſo alike ſituated to the given figure A B C D.

Another way.

Extend the line D A to I, that D I be equall to the given line G F, then draw from I a line parallel to A B, which ſhall touch the produced line D B in the point K, and from K draw another line parallel to B C touching the produced line in the point L, then is the figure D I K L equall, and like ſituate to the figure F G H E, made upon the line D I being equall to the given line, the truth whereof may be underſtood *by the 2, 4, and 6 propoſitions of this ſixt book.*

Theor. 13. Prop. 19.

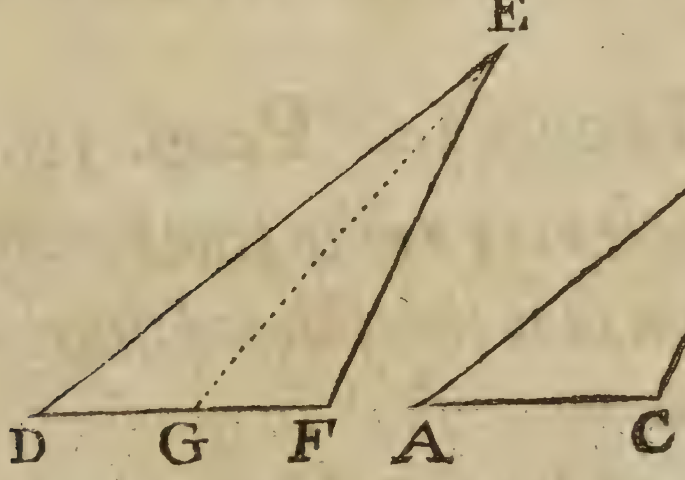
Like Triangles are one to another in double proportion that the ſides of like proportion are.

Conſtruction.

Theſe two Triangles A B C and D E F are alike ſituate, therefore is the proportion of the Triangle D E F to the Triangle A B C double to the proportion of the proportioned ſides, namely, as D E to A B, and D F to A C.

Demonſtration.

Seek firſt of all, *(by the 11 propoſition of this book)* a line which ſhall be in proportion to A C, as A C is to D F, (that is in a third proportion) which line is D G, and draw the line E G, then *(by the ſecond part of the 15 propoſition)* are the Triangles D E G and A B C equall, in reſpect of the equality of the angles A and D, wherefore *(by the 7 of the fift book)* hath the Triangle D E F, ſuch proportion to the Triangle A B C, as to D E G, and the Triangle D E F hath to the Triangle D E G, as the line D E to D G *(by the firſt propoſition)* therefore, foraſmuch as the three lines D F, A C, and D G are proportionall, then *(by the 10 Definition of the fift book)* the proportion of D F to D G (being equall to the proportion of the Triangle D E F, to the Triangle A B C) is double to that of D F to A C: it is then manifeſt, that the propoſition is true, and thereto doth *Euclide* ſet down this Corallary.



Hereby it is manifeſt, that if there be three right lines in proportion, as the firſt is to the third, ſo is the Triangle deſcribed upon the firſt, to the Triangle deſcribed upon the ſecond, if the ſame Triangles be like and in like ſort ſituate, for it hath been proved, that as the line D E is to the line D G, ſo is the Triangle D E F, to the Triangle A B C.

And hereby it is to be noted, that that which in the 15 Definition and more following Propoſitions is ſaid o double proportion, muſt be underſtood as well of a leſſer, or a greater proportion, for *by the intent of this Propoſition* is the proportion of the Triangle A B C to the Triangle D E F, double to the proportion of the proportioned ſides A C to D F, although the proportion of the Triangle A B C to D E F is leſſe then the ſides A C to D E, wherefore, foraſmuch as the leſſer Triangle is compared to the greater, like as alſo the leſſer ſide to the greater, it is *(by the 10 of the 5 book)* that the proportion is leſſe then a whole, wherefore, *by the intent of this propoſition*, it is to be underſtood, that the proportion of this Triangle ABC to the Triangle D E F is double as little, as the proportion of the ſide A C to the ſide D F, and likewiſe to the contrary, the proportion of the Triangle D E F to the Triangle A B C, is double as great as the line D E to the line A C.

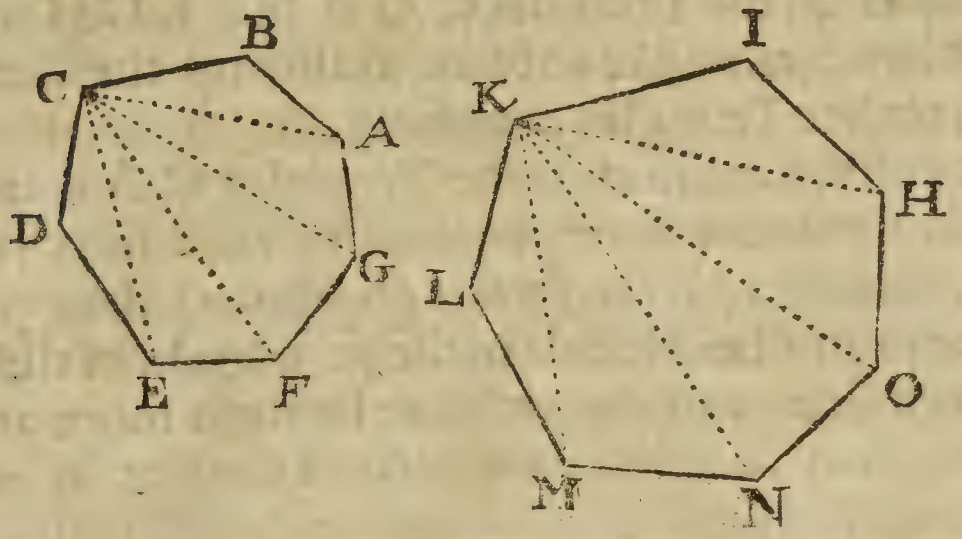
Theor. 14. Prop. 20.

Many ſided figures right lined, which are like ſituate, may be divided into like Triangles ſituated, equall in number, and of like proportion to the whole, and the one of thoſe many ſided figures is to the other figure in double proportion, that the one of the ſides of like proportion, is to one of the ſides of like proportion.

Conſtruction

Theſe two many ſided figures A B C D E F G and H I K

H I K L M N O are like ſituate, therefore they may be both of them divided into as many Triangles, and like ſituate, the one as the other, which to do, draw from one angle in each figure, (which are compared the one to the other) his lines to each angle of the figure, as from C to A, to G, to F, and to E, and from K the like to H, O, N, M, L, ſo is each figure in five Triangles divided, which in the firſt figure are C A B, C G A, C F G, C E F and C D E, and in the ſecond figure K H I, K O H, K N O, K M N and K L M, theſe Triangles have equall proportion, the firſt Triangle of the firſt figure, to the firſt Triangle of the ſecond figure, and the following to the following.



Demonſtration.

Foraſmuch as the ſide B C is to the ſide B A, as I K is to I H, and that thoſe ſides include equall angles B and I, thoſe Triangles C A B and K I H are equiangled *(by the 6 of this book)* and *(by the fourth propoſition)* hath B C or B A ſuch proportion to A C, as I K or I H hath to K H, wherefore the Triangles C A B and K H I are alike, and like ſituate, and by the ſelf ſame reaſon alſo, the Triangles C D E and K L M, furthermore ſeeing that the angles B A G and I H O are equall, as alſo B A C and I H K, the angles remaining of the Triangles C G A and K O H, namely, C A G and K H O are alſo equall, and they being included with proportioned ſides A C and A G, and H K and H O, the Triangles C G A and K O H are alſo proportioned, and a like ſcituate, as alſo by the ſame reaſons the other Triangles remaining.

Secondly, foraſmuch as the Triangles C A B and K H I are like ſcituate in forme, equiangled with ſides proportioned, hath alſo *(by the former propoſition)* their proportion one to another, double to the proportion of their proportioned ſides, and that each like figure in forme and ſcituation, may be divided into like Triangles in number and form, it followeth *)by the former Propoſition)* and *(by the 12 of the firſt book)* that the proportion of the whole figure A B C D E F G, is to the whole figure H I K L M N O alſo double to the proportion of the ſides, which ſtand in like proportion one to another.

Theor. 15. Prop. 21.

Right line figures, which are like to one and the ſame right line figure, are alſo the one to the other alike.

Demonſtration.

The truth of this Propoſition is evident, *by the firſt common ſentence*, look farther hereof in the 8 Propoſition of this book.

The-

Theor. 16. Prop. 22.

If there be four right lines proportionall, the right line figures alſo deſcribed upon them being like and in like ſort ſet, ſhall be proportionall, and if the right line figure deſcribed upon them be proportionall, thoſe right lines alſo ſhall be proportionall.

Demonſtration.

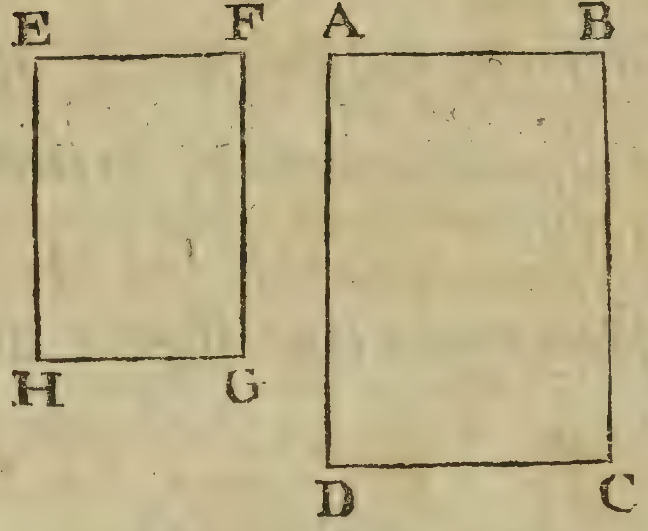
Foraſmuch, as *(by the 20 of this book)* the proportion of the figures which are like ſcituate, are double to the proportion of their proportioned ſides or lines, it followeth *(by the ſixt and ſeventh common ſentences)* that this propoſition is true.

Theor. 17. Prop. 23.

Equiangled Parallelograms have the one to the other, that proportion which is compoſed of their ſides.

Conſtrnction.

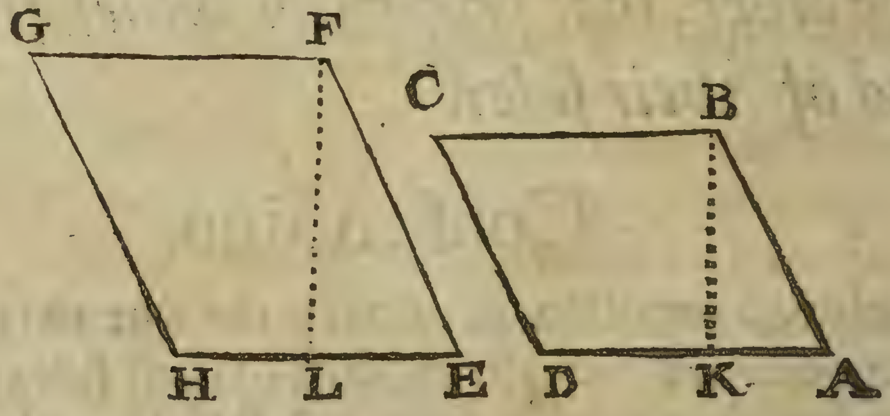
Theſe two parallelograms have the one to the other, that proportion which the product of their ſides doth compoſe, namely the parallelograms A B C D and E F G H, are equiangled, therefore is the proportion of the parallelogram A B C D to E F G H, as the proportion which is produced of their ſides, of A B to E F, and B C to F G, or as A B to F G and B C to E G.



Demonſtration.

Let us firſt take the parallelograms to be right angled, and foraſmuch as *(by the fift definition of this book)* the product of A B, with B C, to the product of E F with F G, is a proportion compoſed of the proportion of the lines A B and B C the power thereof compared to the proportion of line to line, as of F G to F E is in proportion one and the ſame to the power of parallelograms, it followeth that the parallelogram A B C D hath ſuch proportion to the parallelogram E F G H, as the lines A B with B C is to the lines E F with F G, their proportioned ſides.

But if the parallelograms be not right angled, as here, where the angles B A D and F E H are equall, let there be drawn the perpendiculars B K and F L, then hath B K ſuch proportion to F L, as A B to E F, as may be underſtood *by the fourth of this book)* ſeeing that the product of the



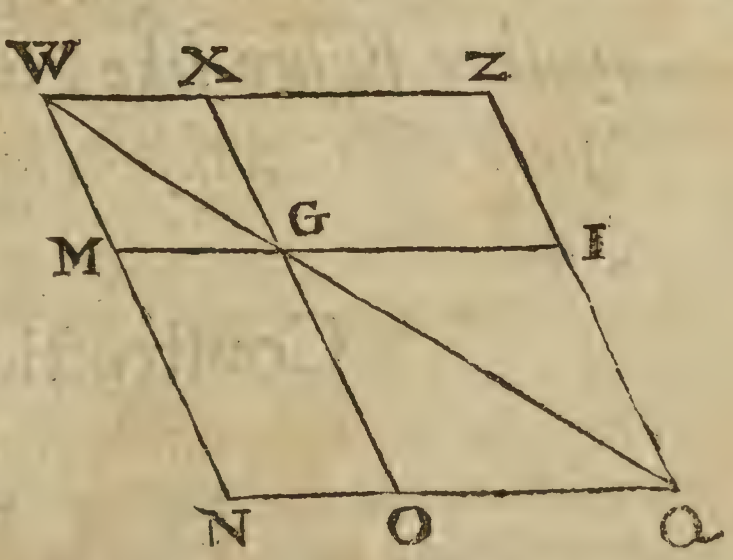
the baſe with the perpendicular doth produce the power of the parallelogram, we may conclude *(by the 12 of the fift book, and by the former demonſtration)* that the Propoſition is true in all parallelograms, which can be underſtood alſo, in regard, that Parallelograms hath two proportions to be compared the one to the other, namely, the proportion of height, and the proportion of the baſe, like as can be underſtood *(by the firſt propoſition of this book.)*

Theor. 18. Prop. 24.

In every parallelogram, the parallelograms about the diametient or diagonall, are like unto the whole, and alſo like the one to the other.

Conſtruction

In this parallelogram N W Z Q, the parallelograms O G I Q, and M W X G, doe ſtand upon the dagonall W Q, and they have the angles Q W common, therefore they are like unto the whole, and alſo one like another.



De-

Demonſtration.

Foraſmuch as M G I is parallel to W Z and X G O, parallel to W N, and that the diagonall line W Q doth paſſe through the point G, that is common, it followeth *(by the 29 of the firſt book)* that the angles X W G, W G M, I G Q, and G Q O are equall, as alſo M W G, X G W, O G Q, and G Q I, and the ſides which include equall angles, are proportionall *(by the fourth of this book)* namely, as X W to X G, ſo I G to I Q, and as W M to M G, ſo G O to O Q, the angles of the parallelograms G X W M, and Q I G O are then equall, and the ſides proportioned, and therefore alike ſcituate, and by the ſame reaſon is alſo ſhewed, that the ſame parallelograms are alike, and in like ſort ſcituate to the whole parallelogram N W Z Q, wherein they ſtand, and that the Propoſition is true.

Probl. 7. Prop. 25.

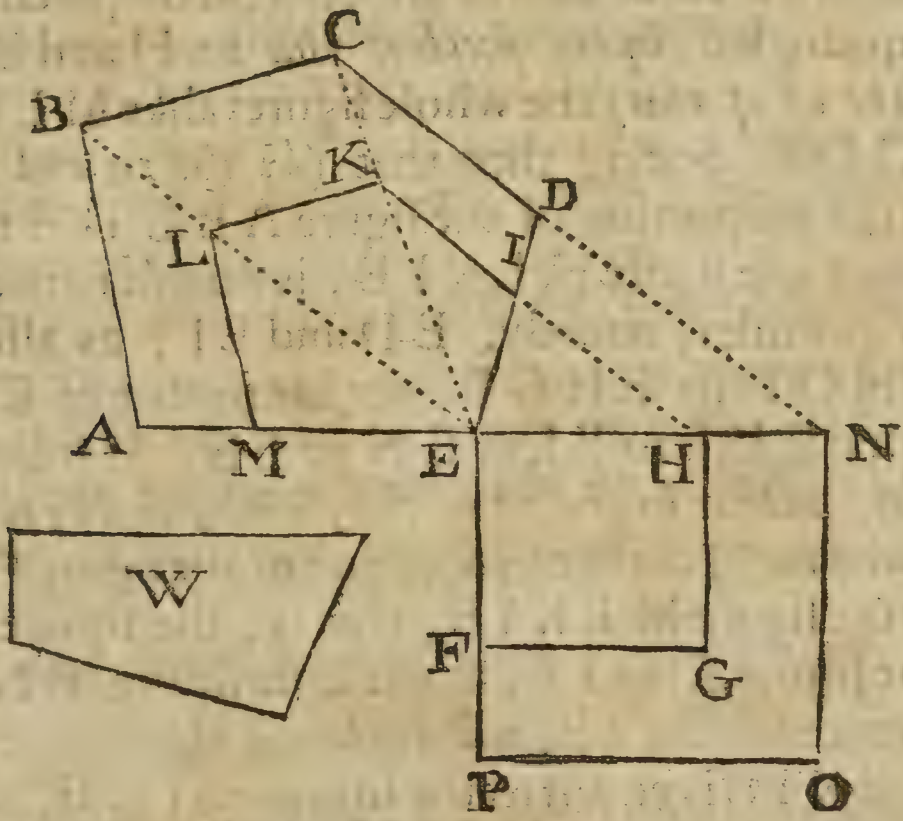
Unto a right line figure given, to deſcribe another figure like, which ſhall alſo be equall unto another right line figure gigen.

Conſtruction.

The right line figures given are A B C D E and W, it is required to make a right line figure equall to the figure W, and like, and in like ſort ſcituate, to the right line figure A B C D E. To do the ſame.

Reduce the figure W into a Triangle of the ſame quantity,

tity, and then into a Parallelogram *(by the 45 of the firſt book)* and then into a Square *(by the 14 of the ſecond book)* Reduce alſo the figure A B C D E into a ſquare, as E N O P, ſet thoſe ſquares the one within the other, *(by the working of the 15 of the ſecond book)* and ſet one ſide of the



right line figure, as A E, ſo that it doe make with the ſide of E N together, one right line, as A E N, and the leſſer ſquare within the ſame, as E H G F: then is AEHN in one right line, then from the angle E of the right line figure A B C D E. Divide the figure into three triangles, by drawing two lines from the angles B and C as E B and E C, then ſeek *(by the 12 of this Book)* a line which hath proportion to E D, like as the ſide of the ſquare E H hath to E N, which is E I, ſet that from E towards D, comes to I. And from the point I draw lines parallel to the lines of the triangles, or ſides of the figure D C, C B and B A, as the lines I K, K L and L M, then is the right line figure

M L K I

M L K I equall to the right line figure W, and like unto the right line figure A B C D E, in form and ſituation.

Demonſtration.

For as much as the ſides of the triangles of the figure M L K I E are proportioned to the ſides of the figure A B C D E (each of them to that to which it is compared) then are they equiangled *(by the ſixth of this Book)* and like ſituate, *(by the 18 of this)* the whole figures alike M L K I E like A B C D E. Seeing then that *(by the 19 and 20 of this Book)* the proportion of the figure A B C D E to the proportion of the figure N L K I E, is double to the proportion of the ſides, namely, E D and E I, as alſo the ſquares E N O P to E H G F, double to that of E N to E H, and that E D is to E I, as E N to E H. It followeth *(as we may conclude by the 11 of the 5 Book, and by the ſixt common ſentence.)* That the proportion of the figure A B C D E, is to the figure M L K I E, that is, the ſquare E N O P, to the ſquare E H G F, as the ſame ſquare E N O P is to the figure M L K I E, *and by the 9 of the 5 Book*, the figure M L K I E is equall to the ſquare E H G F, which is alſo equall to the figure W. The right line figure M L K I E is then equall to the figure W, and like ſituate to the figure A B C D E, as the Propoſition requireth.

Theor. 10. Prop. 26.

If from a Parallelogram be taken away another Parallelogram like unto the whol, and in like ſort ſet, having alſo an angle common with it, then is the Parallelogram about one and the ſame Diagonall line with the whole.

Con-

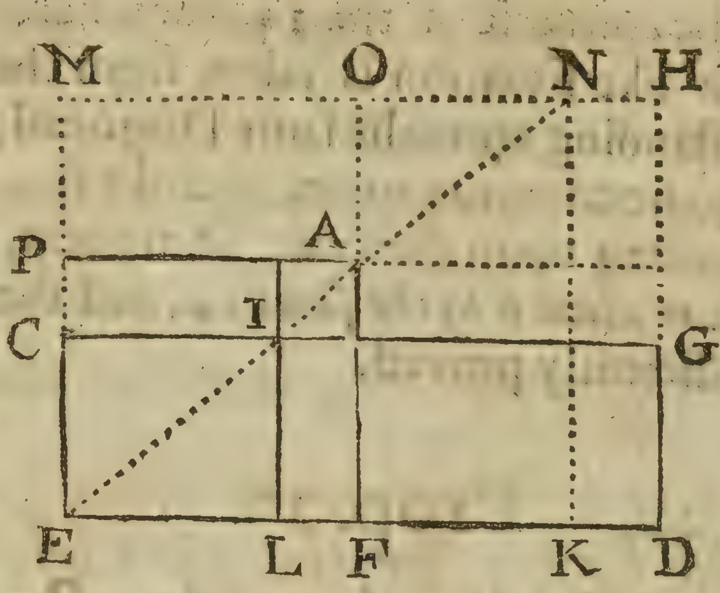
This Propoſition is the converſe of the 24, for whereas there the likeneſſe, or like ſituation is taken from the common angle, and the ſtanding upon the ſame Diagonall, or Dimetient, ſo here by the common angle, and like ſituation is proved, the ſtanding upon the ſame Dimetient, wherefore this Propoſition, then *is by the ſame 24*, and the Demonſtration there ſufficiently proved.

Theor. 20. Prop. 27.

If upon the half a right line be made a Parallelogram, and upon a part of the ſame line be made another Parallelogram, in ſuch ſort, that a Parallelogram made upon the part remaining, like unto the firſt, and in like ſort ſituate, and ſtanding upon the ſame Dimetient, then is the Parallelogram ſtanding upon the half Line, greater then that which ſtandeth upon the firſt part

Conſtruction.

The right line is E D, the midſt of the line is F, upon the half line E F is made a Parallelogram F P, and upon a part of the line, as upon D L is made the Parallelogram D I, ſo that a Parallelogram upon the part remaining.



As upon E L, namely, the parallelogram C L, being like to F P, and in like ſort ſituate, then is the parallelogram F P greater then the parallelogram D I, and alſo greater then the parallelogram D N, in regard that E A N is a right line.

Demonſtration.

For as much as the Figures F G and F C are equall *(by the 36 of the 1 book)* as alſo F I and I P equall *(by the 43 of the 1 book.)* It followeth that the parallelogram F P is greater then D I, by the parallelogram A I. Secondly, ſeeing that F M and F N are equall *(by the aforeſaid 36 of the 1 Book)* as alſo the ſuppliments A M and A K *(by the 43 of the 1 Book)* it followeth, that if we take like things from like things, that D N and N A together are equal to F P: It is then evident, that F P is alſo greater then D N, by the figure N A.

Probl. 8. Prop. 28.

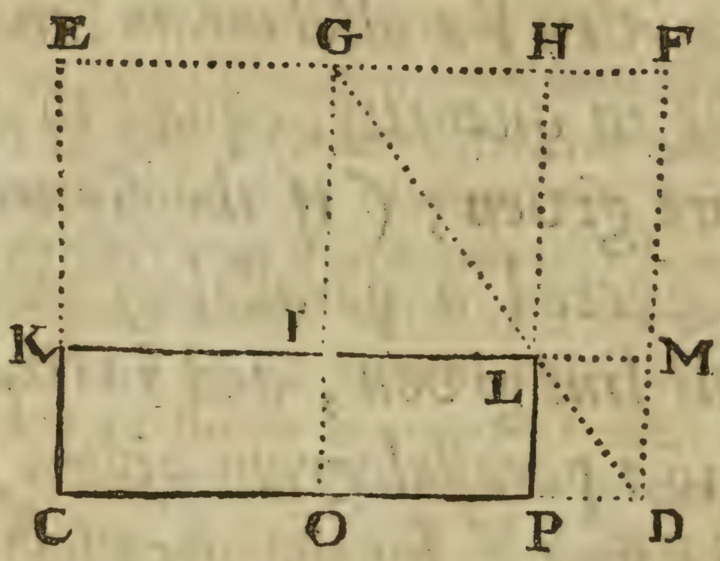
Upon a given right line, to make two Parallelograms of one and the ſame height, whereof the one ſhal be in form and ſituation,

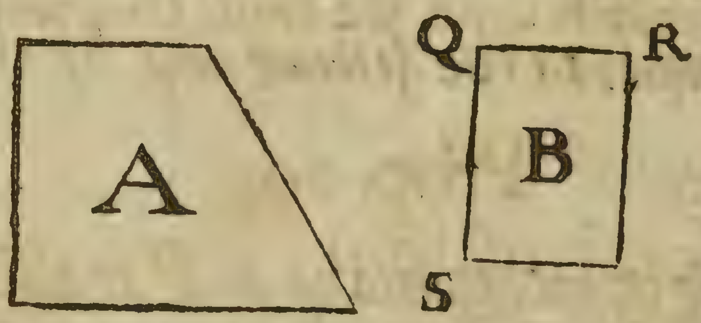
ation, like to a given Parallelogram, and the other ſhall be in quantity, equall to a right line figure given, (if the figure given doe not exceed a parallelogram, made of the half line given, and in like ſort ſituate, to a parallelogram given) and wanting in figure, by the parallelogram firſt named, to the filling up of the line given.

Conſtruction.

The given right line is D C, the Parallelogram (whereunto ſhall be made a parallelogram like, and in like ſort ſituate,) is B, and the right line figure, (whereunto ſhall be made upon the right line given, a parallelogram equal to the ſame figure) is A, which parallelogram ſhall want in figure (of the filling thereof, namely of the line given) by a parallelogram like to B, and in like ſort ſituate, to do this.

Divide the line D C, in two equall parts in the point O, and upon the half line D O (*by the 18 of this book*) deſcribe a right line figure like unto the parallelogram B, & in like ſort ſituate, whoſe ſides ſhall be in like proportion the one to the other, as the parallelogram B, namely, as Q R to G F, ſo Q S to G O, which is the parallelogram O D F G, and produce F G to E, and draw the line C E parallel to O G, making ſo the parallelogram C O G E equall to O D F G,, now if the right line figure A were





equall to C G, the work's finiſh't. But A is ſuppoſed to be leſſe then G C by a certain figure reduced into a Parallelogram, in the ſame proportion (*by the 25 of this book, and the 13 of the ſame*) which parallelogram is I H (the difference between C O and the figure A.)

This I H applyed within O F, having an angle common with the ſame in the point G, which *by the 26 of this book* ſhall ſtand upon the Diagonall D G, then produce I L to M on the one ſide, and to K on the other ſide, produce alſo H L to P, then I ſay that the parallelogram C L (ſtanding upon the one part of the given line C D, namely, upon the part C P) is equall to the right line figure A, and doth want in figure of the whole line C D, by the parallelogram L D, ſtanding upon the other part P D, which parallelogram L D is like to the parallelogram B, and in like ſort ſituate.

Demonſtration.

Foraſmuch as the parallelogram D G is equall to the right line figure A, and to the parallelogram I H together, and that the Gnomon I L H F D O is alſo equall to the ſaid figure A, the ſupplement M H is equall to the ſupplement L O *(by the 43 of the firſt book)* and M P is common to them both, wherfore the whole parallelogram D H is equall to the whole parallelogram D I But D I is equall to I C *(by the firſt of this book)* by reaſon that O C is equall to O D, and I C is then equall to D H, to this adde I P common to both, then is the whole parallelogram C L equall to the whole Gnomon I L H F D O, and it is proved that the ſame Gnomon is equall to the figure A, and upon the part of the line C D remaining, P D is made the parallelogram, P M like unto B, and in like ſort ſituate, for it ſtandeth upon the ſame Diagonall line G D, which was required to be demonſtrated.

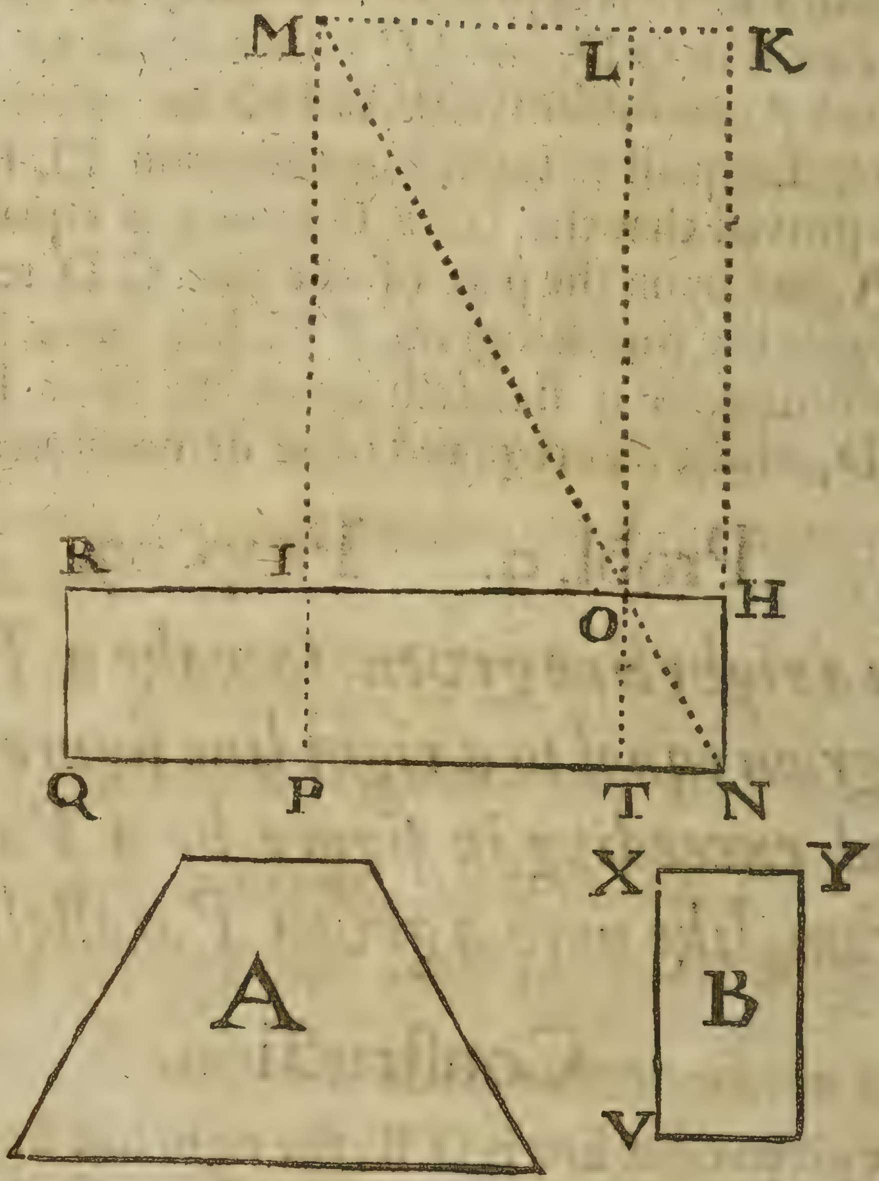
Probl. 9. Prop. 29.

Upon a right line given, to make a Parallelogram equal to a right line figure given, and exceeding in figure by a Parallelogram, like unto a given Parallelogram.

Conſtruction.

The given right line is O R, the right line figure given (whereunto is required upon the ſame line O R, to make an equal parallelogram) is A, and the parallelogram whereunto the exceſſe is required to be like, is B, it is required upon the line O R to make a parallelogram, equall to A, andexceeding in figure by a parallelogram like unto B.

Dividethe line O R into two equall parts in the point I, and upon the halfe line O I deſcribe a parallelogram, like unto B, *(by the 18 of this book)* and in like ſort ſituate, whoſe ſides ſhall have the ſame proportion, namely, as X Y to X V, ſo I O to O L, then reduce the figure A into a Triangle equall to the ſame *(by the 44 of the firſt book)*



and again that Triangle into a parallelogram equall to the ſame *(by the 42 of the firſt book)* and laſtly, reduce that parallelogram into the height of the line given *(by the 44 of the firſt book)* that is O R, and there ſhall come the breadth O T or R Q, with which breadth make the parallelogram O Q, and extend the lines R O to H, and Q T to N, extend alſo M I to P, then from the point N draw a line parallel to T L, through the point H to K, and extend

tend M L to K, to make the figure compleat: Then I ſay, that the parallelogram N R is equall to the given figure A, and exceeding the ſame by a parallelogram, namely N O, which is like and in like ſort ſituate to B.

Demonſtration.

Foraſmuch as the parallelogram N M is equall to the parallelogram O M together with the figure A, it followeth that the Gnomon K L O I P N is equall to the figure A, and *(by the 43 of the firſt book)* the parallelogram or ſupplement O K is equall to O P, which O P is alſo equall to P R *(by the firſt of this book)* therefore the parallelogram N R is alſo equall to the figure A, together with the parallelogram N O, which N O is alſo *(by the 24 of this book)* like and in like ſort ſituate to O M, and the ſame O M is proved to be like to B, therefore alſo N O is like to B, which was required to be demonſtrated.

Theor. 21. Prop. 30.

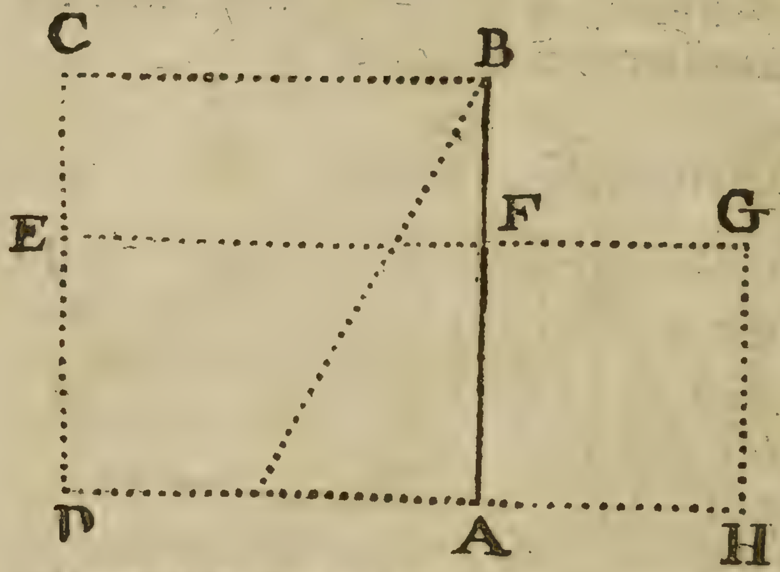
To divide a right line given, by extream and mean proportion.

Conſtruction.

The given right line is A B, divide the ſame line A B into two parts, *according to 11 propoſition of the ſecond book*, as in F, then is the ſame line A B divided in the point F, in extream and mean proportion.

De-

Demonſtration.



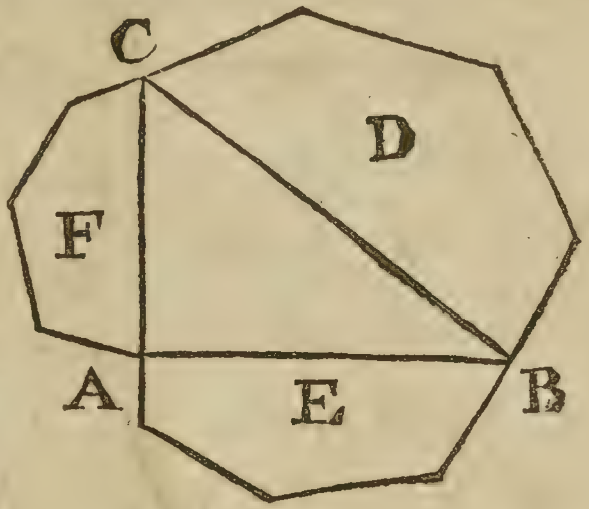
Foraſmuch as the right line figure included with A B and F B, as F B C E is equall to the ſquare of A F, as the ſquare A F G H, it followeth *(by the 17 of this book)* that the line A B is divided in extream and mean proportion, that like as A B to A F, ſo A F to F B.

Theor. 22. Prop. 31.

In right angled Triangles, the figures made of the ſide ſubtending the right angle, is equal to the figures made of the ſides comprehending the right angle, ſo that the ſaid three figures be like and in like ſort ſituate or deſcribed.

Conſtruction.

Upon this right angled Triangle A B C, doth ſtand three right line figures like and in like ſort ſituate or deſcribed, as D, E and F: the figure D doth ſtand upon the ſubtending ſide of the right angle, I ſay, that the ſaid figure D is equall to the two figures E and F, being made in like ſort to the figure D, and ſtanding upon the other two ſides of the Triangle A B and A C, comprehending the right angle.



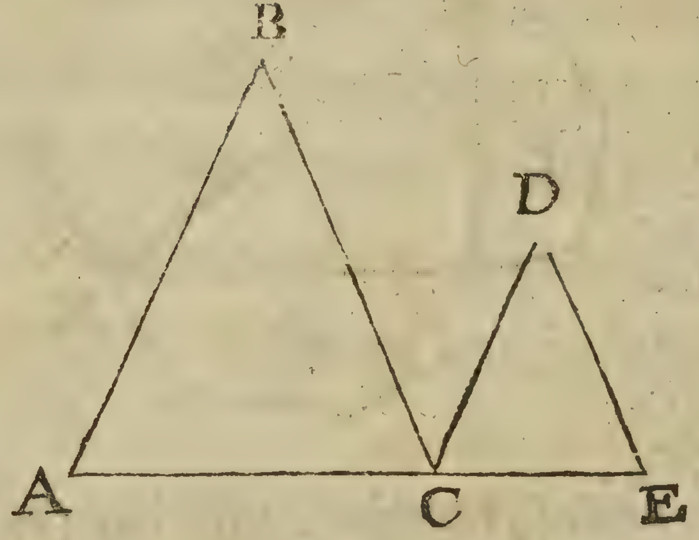
Demonſtration.

Foraſmuch as *(by the 47 of the firſt book)* the ſquare of the ſide B C is equall to the two ſquares of the ſides ſubtending the right angle, namely, of the ſides A B and A C together, and that *(by the 19 and 20 of the ſixt book)* the proportion as well of ſquares, as of like formed Triangles, and all other like formed right line figures, are double to the proportion of their proportioned ſides, it is evident *(by the 11 of the fift book)* that the figure D is equall to the two figures together E and F.

Theor. 23. Prop. 32.

If two Triangles be ſet together at one angle, having two ſides of the one proportionall to two ſides of the other, ſo that their ſides of like proportion be alſo parallels, then the other ſides remaining of thoſe Triangles ſhall be in one right line.

Conſtruction.



Of theſe two Triangles A B C & C D E, is the ſides A B proportioned to C D, as C B is to E D, and A B is parallel to C D, and B C to D E, and the angles C doe touch one another in the point C, wherefore the ſides A C and C E doe ſtand in one right line.

Demonſtration.

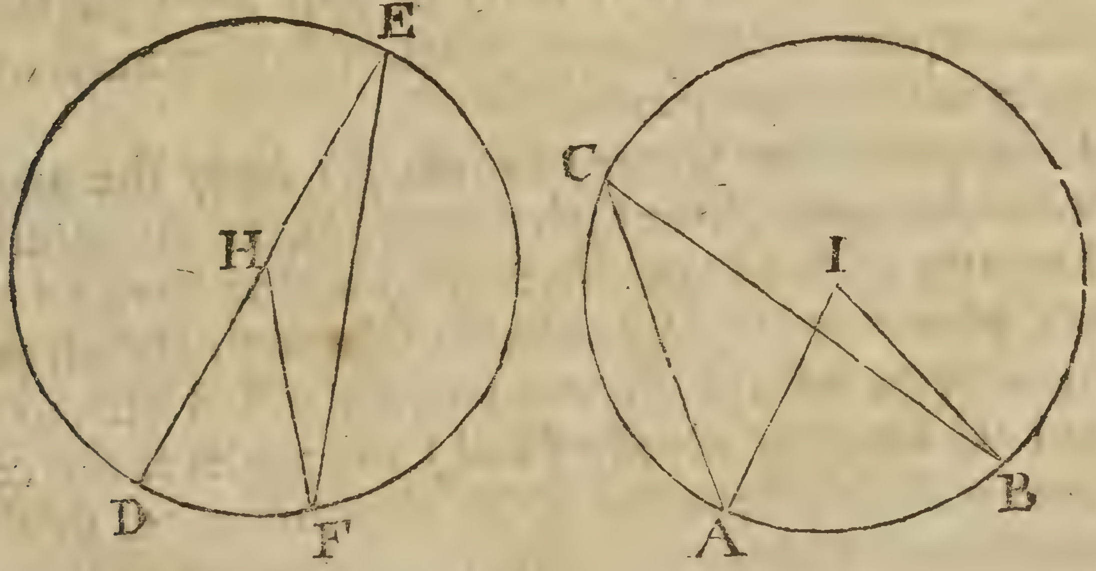
Foraſmuch as C D falleth upon the parallels C B and E D, and C B upon the parallels A B and C D: it followeth *(by the 29 of the firſt book)* that the angles C D E, B C D and C B A are equall, as alſo the angles D C E and B A C, and alſo D E C and B C A, the three angles upon the line E C A in C, as B C A, B C D and D E C, are equall to the three angles of the Triangle, any one of the two, which *(by the 32 of the firſt book)* are equall to two right angles, it followeth *(by the 14 of the ſame firſt book)* that A C E (being the ſides of the two Triangles) are one right line.

Theor. 24. Prop. 33.

In equall Circles, the angles have one and the ſame proportion that the Circumferences have, wherein they conſiſt, whether the angles be ſet at the Centers, or at the circumferences, and in like ſort are the Sectors which are deſcribed upon the centers.

Conſtruction

Theſe circles A B C and D E F are equall, therefore are the angles C to E, and I to H in like proportion the one to the other, like the parts of the circumference whereupon they ſtand, namely, as A B is to D E, and in



like manner are the angles of the Sectors in proportion, one to another, as the part of the circumference is whereupon they ſtand, namely, the angle I to the angle H, as the part A B is to the part D E.

Demonſtration.

Foraſmuch as if both the Circles whole Circumferences equally were divided into as many equall parts, as could be imagined, and thoſe equall parts in both the Circles drawn with lines from point to point, (of thoſe diviſions) thoſe lines *(by the 29 of the third book)* are all equall, and if from the Center, there be made upon the ſame lines as many Triangles, by drawing from the Center to each point half Diameters, then are thoſe Iſoſcheles Triangles equall, and the angles in the Center equall *(by the fourth and eighth of the firſt booke)* if then in each circle, there were made by chance any ſuch angles, by adding ſo many of thoſe parts together, as is required (how many ſoever) thoſe angles ſhall contein each of them ſo many parts of the circumference, ſo that thoſe angles ſo added, ſhall be ſuch parts of four right angles as the whole parts of their angles taken in the circumference, is to the whole circumference.

It followeth that *(by the 12 of the fift book)* the angles added in the center, have ſuch proportion one to another, as the parts of the circumference ſo added have to the whole, wherefore the angle I hath to the angle H ſuch proportion, as the parts of the circumference A B hath to D F, and *(by the 20 of the third book)* the angles at the center are double to the angles at the circumference, ſo that the angle I is double to the angle C, and ſo is H to E, it is then evident *(by the 15 of the fift book)* that the angles C and E alſo are proportioned like as the parts of the circumference, whereupon they ſtand, that is as A B to D E.

Secondly, the Sectors of the Circles A I B, and D H E, have ſuch proportion one to another, as the angle I hath to the angle H, or as the arch A B hath to the arch D E, and the ſame which is ſaid of arches is true *(by the 11 of the fift book)* alſo in angles.

Demonſtration.

Foraſmnch as the center in all places is equally diſtant from the circumference *(by the 15 definition)* that all half diameters of a circle are equall, and doe all of them fall (as it were) perpendicular upon the circumference, and we may conclude, that like as in the firſt propoſition, of the ſixth booke) is ſaid of Triangles of equall height, it is there ſhewed that thoſe Triangles are in proportion one to another, like as their baſes, even ſo here alſo of Circles ſectors; for they are in proportion one to another as the parts of the circumference whereupon they ſtand.

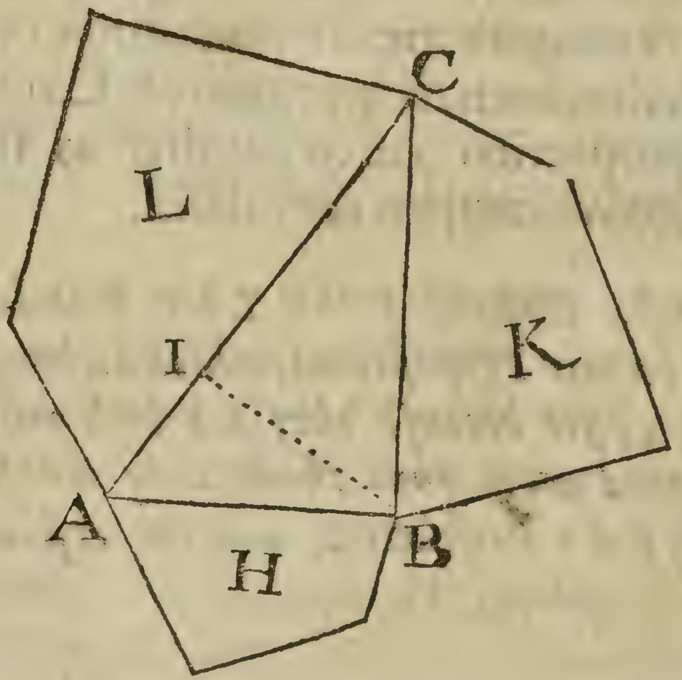
And herewith is the firſt ſix Books of EUCLIDE *ended. There be hereafter added certain Propoſitions, which although they be not* EUCLIDES, *yet becauſe they are both witty and uſefull, I thought it good not to omit them, they are ſix in number, whereof the firſt is a* Probleme, *and the reſt are* Theoremes.

The end of the ſixt Book of Euclide.

Propoſition 1.

To make two right line figures, like and equall, and in like ſort ſituate, unto a right line figure given, which ſhall alſo have a proportion given.

Conſtruction.



The right line figure (to which the other two figures ſhal be together equal, and alſo alike in form and ſituation) is L, and ſuppoſe the line A C of the figure L be divided in the point I *(by the 10 of the ſixt book)* into ſuch proportion as is required that the figures ſhould have one to another, and (*by the 13 of the ſixt book*) finde a mean proportionall line between A I and I C, which line is I B, let that line I B ſtand right angled upon A C in the point I, and draw the lines A B and B C, and upon the ſame deſcribe (*by the 18 of the ſixt book*) two right figures like to L, and in like ſort ſituate, as H and K.

De-

Demonſtration.

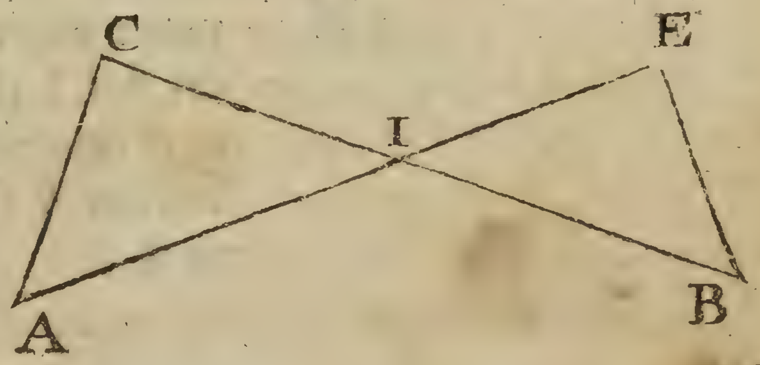
Foraſmuch as the angle A B C is a right angle, as *in the 13 propoſition of the ſixt book* is ſhewed, and that *(by the 8 of the ſixt book)* the Triangles I B A, I C B and A B C, are equiangled, and that (*by the fourth of the ſixt book*) the ſides are proportioned, namely, A B to B C, as A I to I B, and I B to I C, therefore (*by the tenth Definiti- and 19 of the fift book*) the proportion of A I to I C is double to that of A B to B C, and the proportion of the like formed figures, as of H to K is alſo double to that of the lines A B to B C: it followeth (*by the 11 of the fift book*) that the proportion of H to K, is alike, and (*by the 31 of the ſixt book*) are the right line figures H and K together equall to L, which was required to be proved.

Propoſition 2.

If two right lines cut one another obtuſe angled wiſe, and from the ends of thoſe lines which out one another be drawn lines perpendicular to either line, the lines which are between the ends, and thoſe perpendicular lines are cut reciprocally.

Conſtruction.

The right lines A E and B C doe cut one another obtuſely in the point I, and the perpendicular line from the point E falling upon the line



B C in the point B, and alſo from the point C, upon the line A E in the point A, then I ſay, that the parts of the lines A E and B C from the point I are reciprocally proportioned, namely, A I to I C, ſo B I to I E.

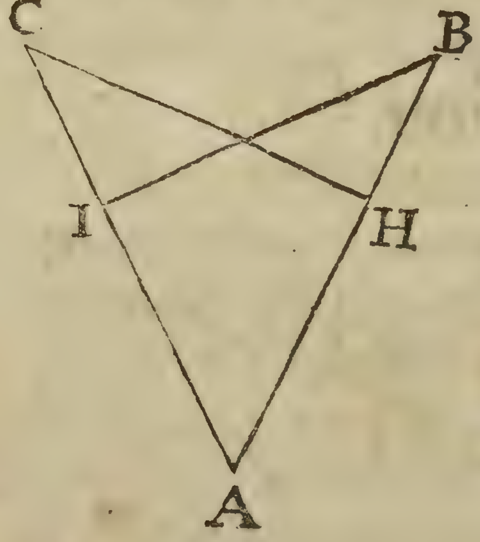
Demonſtration.

Foraſmuch as the angles C and E are right angles, and that the angles A I C and B I E are equall *(by the 15 of the firſt book)* it followeth *(by the 32 of the firſt book)* by conſequents, that the angles A C I and B E I are equall, and *(by the fourth of the ſixt book)* are the ſides proportioned, ſo that A I is to I C, as B I is to I E.

Propoſition 3.

If two right lines make an acute angle, and from their ends be drawn to each line perpendicular lines, cutting the ſame, the two right lines given ſhall be cut in reciprocall proportion as the ſegments which are about the angle.

Conſtruction.



Theſe two right lines A B and A C doe include an acute angle in A, and from C is a perpendicular line drawn upon A B in the point H, and another from B upon A C in I, then is A B to A I in proportion as A C is to A H, and as A C is to A B, ſo is A H, to A I.

De-

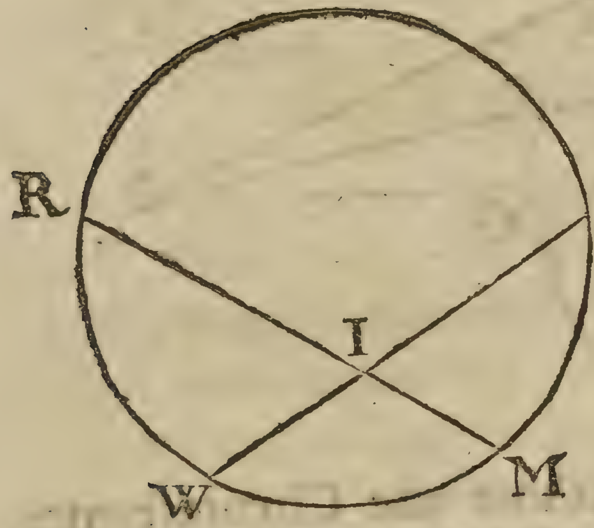
Demonſtration.

Foraſmuch as the two Triangles A I B and A H C hath the angle A common to both, and the equall right angles I and H (then as in the former is ſhewed) they are equiangled, and the ſides ſubtending equall angles are proportionall, where hence it is evident, that the Propoſition is true.

Propoſition 4.

If in a Circle be drawn two right lines cutting the one the other, the ſections of the one, to the ſections of the other, ſhall be reciprocally proportionall.

Conſtruction.



In this Circle doe the right lines R M and W F cut through the one the other in the point I, therefore are the parts of the lines reciprocally proportioned, namely, as I M to I F, ſo W I to I R.

Demonſtration.

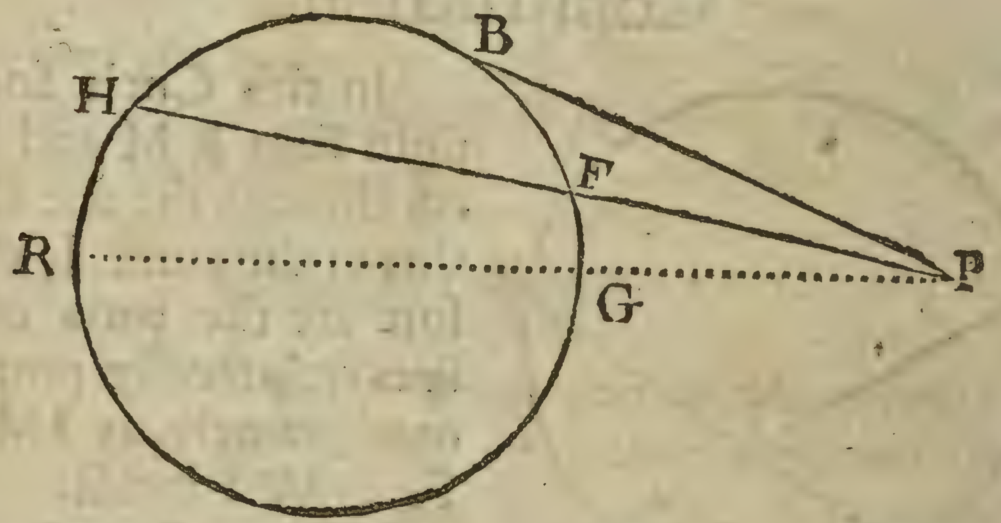
Foraſmuch as the right angle figure made of I M and I R *(by the 35 of the third book)* is equall to the right angle figure made of I W and I F, and that *(by the 14 of the ſixt book)* the parallelograms made of the ſaid lines (by this Propoſition the ſides which here are the parts or ſegments of the ſaid lines by this Propoſition) are proportioned, it is evident, that the Propoſition is true.

Propoſition 5.

If from a point given (upon a plain ſuperficies) be drawn two right lines to the concave circumference of the Circle, theſe lines ſhall reciprocally be proportioned with their parts taken without the circle, & moreover a right line drawn from the ſaid point, and touching the Circle, ſhall be a mean proportionall, between the whole line, and the utter part or ſegment.

Conſtruction.

The point without the circle is P, the lines drawn within from the pont P to the concave ſide of the circumference of this Circle, are P R and P F, and the touch line is P B, and the parts taken without the Circumference, are P G and P F, now is P R to P H, as P F to P G, and P B is a mean proportionall line, between P R and P G, or between P H and P F.



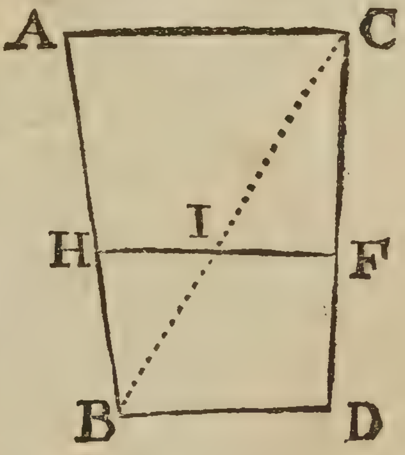
Demonſtration.

Foraſmuch as *(by the 36 of the third book)* the right angle figure made of P R and P G is equall to that of P H and F, it followeth *(by the 14 of the ſixt book)* that the lines are to their parts reciprocally proportioned according to the propoſition, and *(by the ſame 36 of the ſixt book)* is the ſquare of the line P B, equall to the right angle figure made of P R and P G, or of P H and P F, it followeth then *(by the 17 of the ſixt book)* that the line P B is the mean proportionall between P R and P G, or P H and P F.

Propoſition 6.

If two right lines doe ſtand between two parallel lines which are cut through with a third parallel line, that is done in equall proportion.

Conſtruction.



The two ſtanding lines are A B and C D, they ſtand between the two parallel lines A C and B D, and are cut through with another parallel line H F in the points H and F, that cutting doth happen in equall proportion, namely, as A H to H B, ſo C F to F D.

Demonſtration.

Let the line C B be drawn, cutting H F in I, foraſmuch as (*by the ſecond of the ſixt book*) A H is to H B, as C I is to I B, and C F to F D, as the ſame C I is to I B, it followeth (*by the 11 of the fiſt book*) that the propoſition is true.

F I N I S.

J·III·N

[illegible] per connexionem

[illegible] quod [illegible] fio [illegible]

[illegible] offensæ [illegible] remittetur, vt [illegible]

[illegible] & proposito [illegible]

[illegible] secundum quod est [illegible]

[illegible] peccatum [illegible] remittatur sine pœnitentia [illegible]

[illegible] quod est [illegible] entum autem pœnitentiæ [illegible]

[illegible] sacerdotis ligantis & soluen

[illegible] potest Deus [illegible], sicut remisit Christus mu

[illegible] adultera, vt legitur [illegible], vt legitur Lu. 7 Quibus ta-

[illegible] non remittit [illegible] mani sicut Greg. * di-

[illegible] pœnitentiam, quam

[illegible]

Ad [illegible] quod in pueris non est nisi pecca-

[illegible] deordinatione volunta-

[illegible] vt in Secunda parte

[illegible] peccatum [illegible] habituali immuta-

[illegible] autem cum actuali. Sed

[illegible] quæ consistunt in deordinatione

[illegible] peccata etiam in baptismo sine a-

[illegible] per pœnitentiam.

Ad secundum [illegible] ratio illa procedit de pœnite-

[illegible] quod [illegible] sacramentum.

528. Ad tertium dicendum, quòd misericordia Dei est maioris

[illegible] misericordia hominis, in hoc, quòd immutat volunta

[illegible] tendum, quod misericordia hominis [illegible] non

[illegible]

ARTICVLVS III.

[illegible] per pœnitentiam vnum peccatum sine [illegible]

AD [illegible] proceditur. Videtur, quod possit

[illegible] sine alio remitti. Dicitur enim

[illegible] super alteram [illegible] autem [illegible]

pars vna [illegible] quam non pluit [illegible] Qu

ens [illegible] dicit [illegible] qui proximum

[illegible] se corrigit [illegible] complur

[illegible] vitia resecant,

[illegible] Ergo potest [illegible] pœnitentiam

[illegible]

[illegible] Ambro [illegible] via. Pri

[illegible] ad [illegible] omnia peccata mortalia [illegible] parte [illegible]

[illegible] quam oportet [illegible] pœnitentiam [illegible]

[illegible] dicendum [illegible] ex [illegible]

[illegible] non contrariatur amicitiæ [illegible] remitti

potest vnum dimitti sine alio. Sed [illegible] contra-

[illegible] vna culpa vel [illegible] alter

[illegible] videmus quod etiam [illegible] aliquis veniam pet

[illegible] de alia.

Ad [illegible] dicendum, quòd dilectio qua Deus d

[illegible] naturæ [illegible] ordinatur ad bonum [illegible],

[illegible] quodlibet peccatum mortale [illegible]

[illegible] ordinat hominem ad [illegible]

[illegible] Rom. 6. Gratia Dei, vita æterna. V [illegible]

ARTIC [illegible]

[illegible] culpa per pœnitentiam [illegible]

AD [illegible] ARTICVLVM sic proceditur. Videtur [illegible]

per pœnitentiam, non [illegible]. Rem

remouetur effectus [illegible] pœ

aliquis dignus pœna [illegible] commisit. Ergo

potest remanere reatus pœnæ.

¶ 2 Præterea. Sicut Apostolus dicit [illegible] 5. Donum

[illegible] quam peccatum. Sed peccando homo simul inc

pœnæ [illegible] peccatum. Ergo multo magis per donum gratiæ

culpa & tollitur [illegible] reatus.

3 [illegible] peccatorum fit per pœnitentia [illegible]

[illegible] Quem propter [illegible]

[illegible] propter [illegible]

delicto [illegible]

peccati [illegible] p

manet aliquis reatus pœnæ [illegible] Domino

SED CONTRA est, quod [illegible] qu

[illegible] dixisset ad Nathan, Peccaui Domino

Dominus quoque transtulit peccatum [illegible]

[illegible] filius qui natus est tibi [illegible]

[illegible] peccati: vt ibidem dicitur [illegible]

[illegible] reatus pœnæ.

[illegible] S I C [illegible]